Incentive Regulation in Vertically Related Industries: Welfare Effects of Industry Structure and Institutional Coordination*

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Abstract

Economic theory has examined the impact of individual regulatory constraints on regulated naturally monopolistic network utilities. However, some regulated industries, like natural gas and electricity, are closely related: besides being substitutes for consumption, one product is an important input for producing the other. Thus, regulation of one industry necessarily interferes with the other, and these interactions should be accounted for. First, we show that under perfect information, with no regulation and linear pricing, the welfare gains due to increased competition do not outweigh the losses stemming from double marginalization, which is an argument for firm integration. We then show that informational asymmetries can reverse this result. We also show that, regardless of the prevailing industry structure, two sector specific regulators cannot achieve incentive compatibility without at least some form of coordination, even if they share their information and contract in all available variables. Worse still, if they do not share their information, miscoordination might lead to monopoly pricing. We characterize the set of globally incentive compatible linear regulatory contracts, which includes price caps, cost-plus, and other combined forms, and show how the optimal choice of regulation and industry structure varies according to the substitutability in consumption and technical dependency in production of the regulated goods and services.

Résumé

La théorie économique a étudié l'effet des contraintes de régulation individuelles sur les secteurs servis par des réseaux en monopole naturel. Cependant, parmi ces industries régulées, certaines sont étroitement liées, comme le gaz et l'électricité: les deux biens sont substituables sur le marché final de l'énergie, et en outre l'un des deux biens est un facteur essentiel pour la production de l'autre. Par conséquent, la régulation de l'un des deux secteurs interfère avec celle de l'autre, et ces interactions devraient être prises en compte. Premièrement, cet article montre que, dans un contexte de prix linéaires non régulés avec information parfaite, les gains de bien-être provenant de l'augmentation de la concurrence ne suffisent pas à compenser les pertes engendrées par les doubles marges, ce qui plaide en faveur de l'intégration des firmes des deux secteurs. Néanmoins, l'introduction d'asymétries d'information permet de renverser ce résultat. Cet article montre aussi qu'indépendamment de la structure industrielle prédominante, deux régulateurs sectoriels ne peuvent pas mettre en place des contrats incitatifs sans se coordonner, même si leur information est partagée et si les contrats portent sur toutes les variables disponibles. Si les régulateurs ne partagent pas leur information, le manque de coordination peut même induire des prix de monopole. On caractérise l'ensemble des contrats linéaires globalement incitatifs: il inclut, entre autres, des price-caps et des cost-plus. Le papier montre ensuite comment les choix optimaux du type de régulation et de structure industrielle dépendent de la substituabilité et de la dépendance technique entre les deux biens régulés.

1. Introduction

Despite the recent wave of utility restructuring, naturally monopolistic segments of these network industries, such as gas and electricity transportation or local loop telecommunications, cannot be efficiently open to competition and thus require some form of price regulation or "regulated competition". Following the standard partial equilibrium analysis, previous economic literature has examined the impact of regulatory constraints on a single isolated industry, without taking into account interactions between the different regulated industries or segments. However, some of these markets are closely related because the regulated products can be substitutes in consumption, inputs for the production of other regulated goods, or both. This is the case in various regulated network utilities, such as different forms of telecommunications (internet and telephone, for instance), or different energy sources (like electricity and gas). In particular, regulation of industries that show a double interaction, in both the final good market (the market for final consumption) and the intermediate good market (the output/input market destined for downstream production), produces externalities, which have only recently been addressed –in many different practical ways– by the numerous regulatory bodies around the world.

This paper focuses on the spillover effects from price regulation of a single product that is a substitute in consumption and vertically related to the product of another regulated industry. In our exposition here, we use the energy sector as an example: gas and electricity have regulated natural monopoly segments and present the "double link" described above. All results hold for any other vertically related pair of regulated industries or segments, but for simplicity we confine our discussion to the gas-electricity example, where institutional arrangements and industry organisation are more variable across countries and regulatory jurisdictions.[1]

In recent years, markets for gas and electricity have become increasingly intertwined. Not only do both of these products compete in the final market as sources of energy, but there is also a very direct technical link between them : electricity production is increasingly dependent on combined cycle gas turbine (CCGT) generation. The 2000 Annual Energy Review published by the European Commission states that, worldwide, "gas use [for electricity production] has more than doubled since 1980", and in the EU, "increases [of gas demand since 1990] were spectacular in the power sector (104%)". Increasing efficiency of combined cycle gas turbine (CCGT) generation has made gas the fuel of choice for electric generation capacity expansion. In fact, not only CCGT technology allows efficient construction at relatively small scales, but its emissions are also very clean when compared to other thermal generation technologies, so CCGT construction is also compatible with environmental goals. In fact, the development of this technology has been one of the key factors in the introduction of competition in electricity and the ensuing enormous transformation experienced by this industry in recent years.

Nevertheless, gas and electricity prices have traditionally been regulated independently.[2] In June 1999 the UK pioneered the fusion of its former electricity and gas

[1] For a broader perspective, just think of the gas firm as the regulated "upstream" segment, and of the electricity firm as the regulated "downstream" segment.

[2] In the US, for instance, the Federal Energy Regulatory Commission (FERC) has since long overseen gas, oil and electricity, but its authority is limited to transmission and transportation functions in the gas and electricity sectors. In contrast, generation investments, retail pricing and distribution are regulated by separate sectoral state

regulators into one single regulatory body, the Office of Gas and Electricity Markets (OFGEM), a move which most countries in the EU are likely to follow[3]. The rationale for this merger, as stated by OFGEM, was that individual regulators were not able to adequately fulfil their task, because they weren't allowed to take into account the indirect effects of their decisions in consumers other than those directly buying each regulated product. One of the main objectives underlying the merger was to broaden the scope of the "relevant market" considered by the office. This recent move toward consolidating regulation of distinct but related industries has followed a general trend of business consolidation that has touched the energy sector, through numerous mergers of electricity and gas providers around the world. Despite business models and financial theory defending specialization of firms within only one of the two sectors, the competing vision in which gas and electricity firms merge in order to provide both goods simultaneously has gained momentum in the last years, on the grounds of "convergence" and market liberalization. Not only have large gas providers acquired gas generating units, and *vice versa*, sometimes the mergers have also included oil producers. The recent trend toward consolidation leads to several questions: Should gas and electricity be regulated separately? Is global welfare greater when there are integrated gas and electricity providers, or separate, competing producers? How do price regulations of gas and electricity interact? Do the efficiency gains caused by integration outweigh the costs associated to reduced competition?

In a paper comparing regulated prices in different states serviced by two separate monopolies (one for gas, and one for electricity) or a single, integrated, dual product firm, Knittel [2000] studies regulatory imperfections in the US. He argues that, when regulation is perfect, both industry structures result in the same performance because a perfectly informed regulator would always set prices to equal marginal costs, replicating the perfectly competitive equilibrium. Knittel then presents empirical evidence to support that regulatory imperfections exist, and political economy theory predicts that regulators respond to the relative incentives of electricity and natural gas firms; both electricity prices and reliance on natural gas generation are higher when there is a single dual-product firm. While consumer surplus is decreased by higher electricity prices, the net effect on global welfare is ambiguous: "it can be viewed as equally likely that integration is a welfare gain rather than a loss". In this paper we further explore this question.

On the one hand, integration of gas and electricity supply can be desirable, on the grounds of avoided "double marginalization". Kaserman and Mayo [1991] find empirical evidence of significant economies of vertical integration between the generation and transmission/distribution stages of electricity supply, and the same could be true for energy supply in general. On the other hand, integration reduces price competition coming from substitutability of the two forms of energy, and gives rise to increased market power. Gilbert and Hastings [2001] study vertical integration in gasoline supply and find empirical evidence of higher wholesale prices, which they interpret as a strategy of raising rival's costs. Salop and Scheffman [1987] explore cost-raising strategies more generally, and show that they are more effective than predatory pricing, even in the absence of traditional market power, and under fixed coefficient technology. Gilbert and Riordan [1995] study when "bundled" supply is better than "unbundled" supply, in the context of complementary products, and use

regulatory agencies. There is, additionally, a regulatory overlap between the SEC and the state agencies in some respects. See Joskow [2001] for an in depth description of energy regulation in the US.

[3] Notably France and Spain. One interesting exception is Germany, where authorities have been reluctant to create a regulatory body for the electric sector, which remains under the jurisdiction of the *Bundeskartellamt* only.

electricity generation and transmission/distribution as an example. They find that unbundling introduces an additional component of information cost that is similar to "double marginalization" in the monopoly pricing of complementary products, and that under fixed coefficients technology, "bundled" regulation is to be preferred.

In this paper we adopt the theoretical approach of incentive regulation, comprehensively reviewed by Joskow and Schmalensee [1986] in the case of electric utilities. To compare the outcomes of the different potential regulatory institutions and industry structures, we apply the principles of mechanism design. Settings where there are either one or two regulators, and either single or multi-product firms, lead us to confront the full spectrum of possible principal-agent configurations: single-principal, single-agent; multi-principal, single-agent; single-principal, multi-agent; and multi-principal, multi-agent. Additionally, in some of these settings, firms have private information of a multidimensional nature (about the production costs of each of the two forms of energy, for example). Epstein and Peters [1999], and Martimort and Stole [2001], show some of the potential difficulties in using the revelation principle to study equilibria in common agency games. Rochet and Stole [2001] address the problem of multidimensional screening and survey the literature and the new problems that surface when extending the techniques of the one-dimensional paradigm to a more general environment. In particular, they illustrate the problems associated with the classic parametric utility approach to optimal contracting (Mirrlees [1971]) in the multidimensional case and suggest the use of demand profiles and aggregate demands, an approach we have retained in this paper.

This paper is organized as follows: in Section 2, we provide the framework used throughout the rest of the paper, in which a monopoly or a duopoly serves a single market, with two differentiated, partially substitutable, goods: gas and electricity. The setup differs from standard duopoly competition models because gas is an input to electricity production, so there is also a vertical link between firms. We then add a regulatory layer to the model, in which either one or two regulators set the pricing rules for both markets, using alternative information disclosure procedures.

In Section 3, we solve the model under perfect information and compare equilibrium outcomes in the duopoly and monopoly cases. This solution allows to gauge the relative importance of the "competition effect" and the "double marginalization effect", in order to determine the optimal industry structure in the absence of regulation. We then solve for the equilibrium under first best regulation, a benchmark for subsequent comparisons, as it represents the maximum potential gains from regulation.

In Section 4, we introduce private information about firm production costs and derive the sets of incentive compatible (IC hereafter) regulation contracts for the different industry structures: (i) two firms and two regulators; (ii) two firms and one regulator; (iii) one firm and two regulators; and (iv) one firm and one regulator. We show that in some sub-cases of the two regulators setting, there are no contracts that are globally IC, and demonstrate that at least some minimal form of coordination between the regulators is needed to achieve incentive compatibility.

In Section 5, we calculate the optimal choice of contract coefficients within the IC sets according to social optimality criteria, for a given industry structure, as a function of the values of model parameters. We also show that the optimal choice of the type of regulation to be used changes with industry structure and characteristics, so regulators should be sensitive to the intensity of the links between regulated industries when deciding about the instruments to be implemented.

In Section 6, we make the choice of industry structure endogenous by comparing the equilibrium outcomes in all the different possible settings. We confirm previous findings about coordination issues between principals and show that full coordination is necessary for incentive compatibility, which pleads in favour of regulatory integration. We then gauge welfare gains from regulation and welfare losses due to informational rents, and compare outcomes across industry structures. We show that the results obtained without informational asymmetries can be reversed.

In Section 7, we discuss shortcomings and possible extensions, and in Section 8 we conclude with policy implications.

2. The model

We consider a setting in which a single gas firm (upstream firm) sells q_g units of natural gas (intermediate good) to energy consumers (final consumers) at price p_g, and q_t units to a single electricity firm (downstream firm) at price p_t, as an input for electricity (final good) production. The gas firm buys these q_t units at price p_t, transforms them, then sells q_e units of electricity at price p_e to final consumers. We illustrate this general market structure in Figure 1.

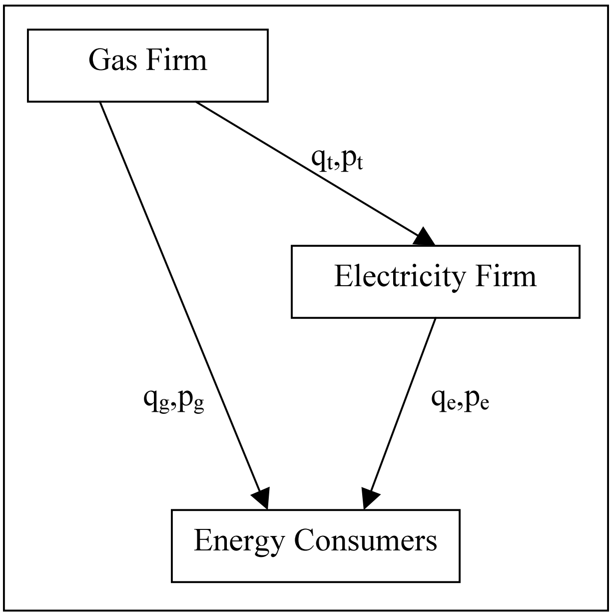

Figure 1. Market structure

Total demand at the consumer level is characterised by substitutability between both forms of energy and positive cross-price elasticities. In order to facilitate equilibrium calculations and global welfare computations, we specify a bilinear gross consumer utility function :

$$U_B(q_g, q_e) = a \cdot (q_g + q_e) - \tfrac{1}{2}(q_g{}^2 + q_e{}^2) - b \cdot q_g q_e \tag{1}$$

The corresponding linear inverse demand and demand functions are :

$$\begin{cases} p_g = a - q_g - b \cdot q_e & \qquad q_g = \frac{a}{1+b} - \frac{1}{1-b^2} p_g + \frac{b}{1-b^2} p_e \\ p_e = a - q_e - b \cdot q_g & \qquad q_e = \frac{a}{1+b} - \frac{1}{1-b^2} p_e + \frac{b}{1-b^2} p_g \end{cases} \tag{2}$$

Parameter *a* determines maximum demand and should is large enough to allow positive solutions, and parameter *b* is a substitution parameter that varies between 0 and 1 (corresponding respectively to independent goods and perfect substitutes).[4]

We assume a fixed technical transformation factor of *k*, that is, *k* units of gas are needed to produce one unit of electricity[5]. *k* must be greater than 1, because there are thermal and other technical losses when transforming gas into electricity. *k* may be thought of as an efficiency parameter for CCGT generating units with plants approaching perfect efficiency as $k \to 1$. The vertical link between gas and electricity production is captured by the following equation :

$$q_t = k\, q_e\ ;\ k > 1. \tag{3}$$

For simplicity, we assume constant marginal costs for production and delivery of natural gas, c_g, and electricity, c_e (although the latter depend on p_t). We normalize fixed costs to zero, and assume they are financed by the "fixed charge" component of the corresponding energy bill. We consider gas and electricity that are, each, vertically integrated across production and delivery.[6]

In particular, we assume that the marginal cost of electricity consists of the sum of a "full pass-through" component, corresponding to the cost of the natural gas input, and a constant, fully idiosyncratic component, denoted by ε. This relationship is described by equation (4).

$$c_e = k\, p_t + \varepsilon \tag{4}$$

[4] Observe that if $b = 1$ consumer utility can be written as a concave function of (q_e+q_g), so consumers care only about the total amount of energy consumed, and not about the "mix" of gas and electricity.

[5] Not all electricity is produced using natural gas, there is a large "base" of hydro and nuclear electricity that is selected first merit-wise. For the sake of simplicity we assume that *k* is fixed and predetermined, because it is our goal to focus on understanding the implications of different regulatory schemes for global welfare. We do not explore the extent to which regulation might change *k* and the least-cost merit order. For further comments on this, see Section 7.

[6] For a discussion of the many issues involved in considering the different stages of the value chain inside each of the two industries (namely production, transport and distribution), which can actually be produced by different firms to obtain the final "bundled" good, see Section 7.

2.1 Industry structures

Two different industry structures can be considered within this framework. The first corresponds to the one portrayed in Figure 1 above, that is, two distinct firms selling their output in a final market. The second corresponds to a single, dual-product, integrated firm selling both products to end users, as shown in Figure 2.

In the two firm setting, profits of the gas and electricity firms are respectively given by

$$\Pi_g = (p_g - c_g)\cdot q_g + (p_t - c_g)\cdot q_t \qquad (5)$$

$$\Pi_e = (p_e - c_e)\cdot q_e \qquad (6)$$

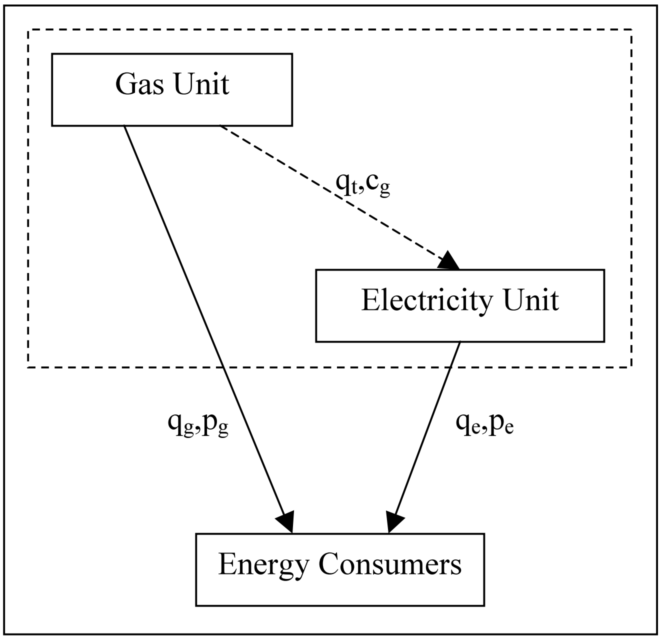

Figure 2. Dual-product firm setting

In the single, dual-product firm setting, the internal transfer price for natural gas will be marginal cost, so profits are given by:

$$\Pi^m = (p_g - c_g)\cdot q_g + (p_e - c_e)\cdot q_e, \quad c_e = k\cdot c_g + \varepsilon \qquad (7)$$

A social planner choosing between these two industry structures must consider the two main competing forces. First, vertical integration improves economic efficiency, by eliminating "double marginalization", thus giving rise to welfare gains. Second, it increases producers' market power and informational advantages, reduces competition at the final market level, and increases information rents, thus giving rise to welfare losses. The aim of this paper is to determine which of these two effects dominates in the presence of regulatory imperfections.

2.2 Regulation and informational structures

When adding regulation to the model, we must consider two primary regulatory structures: a single, separate regulator for each activity, and simultaneous regulation by a single authority. In either regulatory structure, outcomes may vary because of different regulatory objectives[7]. Regulators may also give more or less relative importance to consumer surplus versus firm profits, resulting in a wide range of possible regulatory schemes. Here, we assume that regulators care about the (unweighted) sum of profits and consumer surplus, in their relevant sector of analysis.

Firms have private information about their cost structures, and regulators (and rivals) cannot directly observe these costs, but the have prior beliefs about them. We assume that the marginal cost of gas, c_g, is equal to a random variable γ, drawn from a known distribution function g(.) over support [0,G], and that the idiosyncratic component of the marginal cost of electricity, ε, is drawn from a known distribution f(.) with support [0,E].

Regulation will be treated as a one period, two stage static game in which regulators first simultaneously propose a contract for each type of energy. The contracts are a pair of rules to compute the prices of the respective commodities as a function of the costs announced by the firms. The firms, with full knowledge of both contracts, then announce simultaneously their (privately known) costs, in order to maximize their expected profits. Firms are assumed to be risk neutral, and their cost announcements will be noted $\hat{\gamma}$ for gas and $\hat{\varepsilon}$ for electricity. Quantities, profits, and consumer surplus in the final market then follow from the resulting prices.

Game Timing

Step 1. *Regulators simultaneously announce pricing contracts* $p_g(\hat{\gamma},\hat{\varepsilon})$ *and* $p_e(\hat{\gamma},\hat{\varepsilon})$.

Step 2. *Firms simultaneously announce their idiosyncratic cost parameters* $\hat{\gamma}$ *and* $\hat{\varepsilon}$.

Step 3. *Prices are determined by the contracts. Quantities and payoffs ensue, according to market demand at these prices.*

When there are two sectoral regulators, different information structures may be considered. The first is one in which each regulator takes into account only the cost parameter corresponding to this own sector and ignores the other. In this setting regulators do not share information, regardless of the industry structure (or, equivalently, they are unable to regulate a sector by using information gathered about the other). The second is one in which both regulators do know (and are able to use) the cost announcement made to the other regulator, which allows for more complexity in regulatory contracts (that can be contingent on both $\hat{\gamma}$ and $\hat{\varepsilon}$) and leads to interesting communication issues that we will treat below.[8]

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[7] In the UK, for example, Ofgem officials confirmed the fact that Offer and Ofgas had precise mandates restricting their attention to their specific sector, taking only into account the surplus of customers in their own regulated market, without considering the indirect impact on the other sector's consumers.

[8] We could imagine, but do not take up here, other relevant information structures. For instance, in the two firm context, one in which each firm is better informed about his rival's costs than the regulator(s); or, in the two regulators, two firms context, one in which contracts are privately proposed to each firm and are not common knowledge in the second stage of the game.

3. Benchmark Case: Perfect Information

We begin with market equilibria in the absence of regulation. This will shed light on firms' incentives and the problems that the regulator will later try to prevent. Because of our interest in price regulation, we focus on Nash-Bertrand competition when there are two firms, in order to have a relevant benchmark for later comparisons.

3.1 Unregulated equilibrium with one firm

The single, dual product firm solves the standard dual product monopolist's problem:

$$\max_{p_g, p_e} \Pi^m = (p_g - c_g) \cdot q_g + (p_e - c_e) \cdot q_e \tag{8}$$

The solutions are:

$$p_g^m = \frac{a + \gamma}{2} \tag{9}$$

$$p_e^m = \frac{a + k\gamma + \varepsilon}{2} \tag{10}$$

We note that the optimal prices are similar to those of the standard dual product monopolist with two linear demands and no vertical link, that is, they are equal to one half of the demand parameter a plus the total marginal cost of each product ($c_g = \gamma$ for gas and $c_e = k\gamma + \varepsilon$ for electricity). The resulting prices in this case are independent of the substitution factor b. In other words, substitutability is not sufficient to incite the integrated firm to deviate from its optimal monopoly pricing strategy in each of the two separate individual markets, because the potential gains from such a deviation are outweighed by the losses, as all internal transfers of gas for the production of electricity are efficiently made at marginal cost.

Quantities, on the other hand, depend on b:

$$q_g^m = \frac{(a - \gamma) - b(a - k\gamma - \varepsilon)}{2(1 - b^2)} \tag{11}$$

$$q_e^m = \frac{(a - k\gamma - \varepsilon) - b(a - \gamma)}{2(1 - b^2)} \tag{12}$$

Global welfare in the monopoly situation is given by the sum of consumer surplus and profits:

$$W = S + \Pi \tag{13}$$

$$W^m = \frac{6a^2(1-b) + 3(k \cdot \gamma + \varepsilon)^2 - 6a(1-b)(\gamma + k \cdot \gamma + \varepsilon) - 3\gamma((2b \cdot k - 1)\gamma + 2b \cdot \varepsilon)}{8(1 - b^2)} \tag{14}$$

3.2 Unregulated equilibrium with two firms

The equilibrium in a market with two separate, unregulated firms, is given by the intersection of the best response functions that solve :

$$\max_{p_g} \Pi_g = (p_g - c_g)\cdot q_g + (p_g - c_g)\cdot q_t \tag{15}$$

$$\max_{p_e} \Pi_e = (p_e - c_e)\cdot q_e \tag{16}$$

Notice that we have constrained the analysis to cases in which $p_t = p_g$.[9]

The vertical relationships between both firms lead to divergence from the standard duopoly solution, because the gas firm can use its pricing power to raise its rival's costs and thus reduce demand for its primary substitute.

Equilibrium prices are[10]:

$$p_g^* = \frac{a(1-b)(2+b+k)+2(1-bk)\gamma-(k-b)\varepsilon}{k^2-b^2-4bk+4} \tag{17}$$

$$p_e^* = \frac{a(1-b)(2+b+k+k(k-b))+(1-bk)((b+k)\gamma+2\varepsilon)}{k^2-b^2-4bk+4} \tag{18}$$

And the corresponding equilibrium quantities:

$$q_g^* = \frac{a(1-b)\left(k^2-k+2+\left(k^2-3k+1\right)b-kb^2\right)-(1-bk)\left(2-bk-k^2\right)\gamma+\left(b+k-2kb^2\right)\varepsilon}{(1-b)(1+b)\left(k^2-b^2-4bk+4\right)} \tag{19}$$

$$q_e^* = \frac{a(1-b)(2-k-2bk+b)-(1-bk)(k-b)\gamma+\left(b^2+bk-2\right)\varepsilon}{(1-b)(1+b)\left(k^2-b^2-4bk+4\right)} \tag{20}$$

3.3 Optimal industry structure in the absence of regulation

We will first show that, if regulation consist of choosing the industry structure only, a benevolent regulator will generally prefer an integrated, unregulated, dual-product firm, to a pair of single product unregulated firms, because global welfare is greater in the first case.

We can see that for a given set of parameters $\{b,k\}$ the profit of the monopolist is greater than or equal to the sum of the duopolists' profits because the monopolist has greater market power in the final market, takes into account pricing externalities of gas for electricity

[9] At first glance, this constraint might seem quite strong, but practice shows that discrimination is difficult, because regulators tend to align regulated prices with deregulated industrial prices , and because discriminatory pricing would give rise to arbitrage of gas production by the electricity firm, which could then resell at a higher price. Moreover, as we will be focusing on the "regulated component" of the price of energy goods, it is reasonable to assume that the regulation of the price of gas is made in a non discriminatory way.

[10] See Appendix A for a more detailed analysis of unregulated duopoly equilibria.

production, and practises optimal internal transfer prices. However, what happens to consumer surplus is less obvious. Thus, for global welfare to improve when switching from the monopoly to the duopoly setting, the latter should entail an increase in consumer surplus large enough to offset the decrease in collective firms' profits. It turns out that within the boundaries of existence of the duopoly equilibrium (that is $k < 2$, $b < \frac{2-k}{2k-1}$, and a large enough), consumer surplus generally also increases as we move from duopoly to monopoly, so integration is better for all members of society. We summarize this result in the following Proposition: [11]

Proposition 1. *Within the boundaries of existence of an interior duopoly equilibrium (i.e. equilibria in which electricity production is positive), consumer surplus is higher in the single, dual-product firm setting than in the duopoly situation, in the absence of regulation and under perfect information.*

Proof.

Compute equilibrium consumer surplus in the monopoly case by using expressions (9) to (12) and in the duopoly case by using expressions (17) to (20). Subtract the latter from the former, and observe that this expression can be written as a parabola in a, in which the coefficient of a^2 is positive for the relevant values of b and k. Thus, there always exists a value of a large enough to make the difference in consumer surpluses positive.

The proof of the proposition lies in showing that this critical value of a is smaller than the value needed for q_e to be positive. Therefore, we show that if a is large enough for q_e to be positive, then this is a sufficient condition for the differences in consumer surpluses to be positive.[11] *Q.E.D.*

Proposition 1 above implies the following corollary:

Corollary 1. *Within the boundaries of existence of an interior duopoly equilibrium (i.e. equilibria in which electricity production is positive), global welfare improves by allowing the gas and electricity firms to merge, in the absence of regulation and under perfect information.*

In short, in the absence of informational asymmetries, the "double marginalization" effect dominates the "increased competition" effect. That is, the expected price decrease induced by competition in the two firm case is not enough to cancel out the savings that result from ownership of gas inputs in a single firm. In the following sections we will examine the influence of informational asymmetries on the relationship between these two effects.

This suggests that, if there is no merger control in the energy sector (in addition to the absence of regulation), firms will naturally seek to merge in order to maximize their joint profits, and, in merging, they also increase consumer surplus and improve global welfare.

3.4 First Best Regulation

If the regulator is perfectly informed and has price setting capabilities, she will set price equal to marginal cost and thus achieve maximum global welfare by improving

[11] Details of the demonstration of Proposition 1 and of the boundaries of existence of interior duopoly equilibria can be found in Appendix A.

efficiency and maximising consumer's surplus. This is consistent with the normalization of fixed cost to zero (or, equivalently, of the financing of the fixed cost by the "fixed component" of the energy bills). In this context, firms' profits are driven to zero by taking $p_g = c_g = \gamma$ and $p_e = c_e = k\,\gamma + \varepsilon$, and global welfare equals consumer surplus, given by:

$$W^{FB} = \frac{2a^2(1-b)+(k\cdot\gamma+\varepsilon)^2-2a(1-b)(\gamma+k\cdot\gamma+\varepsilon)-\gamma((2b\cdot k-1)\gamma+2b\cdot\varepsilon)}{2(1-b^2)} \tag{21}$$

This is the best attainable situation in the absence of informational imperfections, and it will serve as a benchmark for the regulator when assessing her possibilities in a world with asymmetric information.

In this setting, it turns out that the quantity of electricity will be inferior to the quantity of gas, and that the impact of an increase in gas costs on global welfare exceeds that of an increase in electricity costs.[12] It is also straightforward to check that global welfare decreases with k and, perhaps more surprisingly, with b.

An interesting property of this model is that welfare under first best regulation is always proportional welfare under unregulated monopoly, where it reaches its maximum in the absence of regulation. By comparing expressions (14) and (21), we obtain:

$$W^{unreg} = W^m = \frac{3}{4}W^{FB} \tag{22}$$

That is, a move from perfect regulation to unregulated monopoly results in a 25% welfare loss, or, conversely, a welfare increase of 33% when moving from an unregulated monopoly to perfect regulation. This provides a very strong argument in favour of regulation. However, when regulation is not perfect, such an increase in global welfare is not achievable. In what follows, we will explore the extent to which the existence of informational asymmetries modifies the potential welfare gains to regulation.

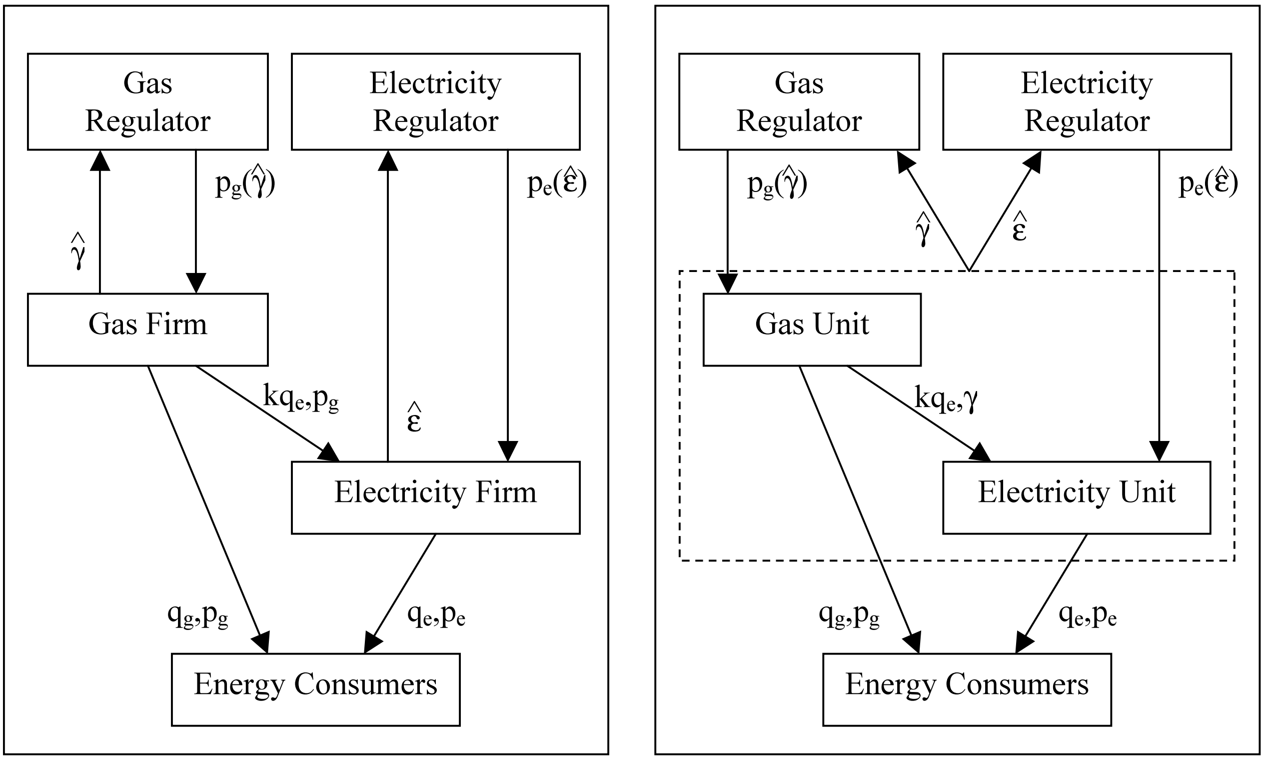
4. Incentive Compatible Regulation

In this section we will characterize the sets of IC regulatory contracts for each of the possible industry structures and institutional arrangements when firms have private information about their production costs.

When there are two regulators, the optimal strategies of all players will differ according to the prevailing information structure. In particular, the sets of actions available to players (and the resulting equilibria) will depend on whether each regulator knows and uses only the cost parameter specific to his industry (Figure 3), or whether all cost parameters are common knowledge to both regulators (Figure 4). In the first setting each regulator sets the price according only to the cost parameter of the specific industry, without taking into account the other, and in the second they can both propose pricing contracts that are contingent in both cost announcements. This information sharing among regulators brings forward many

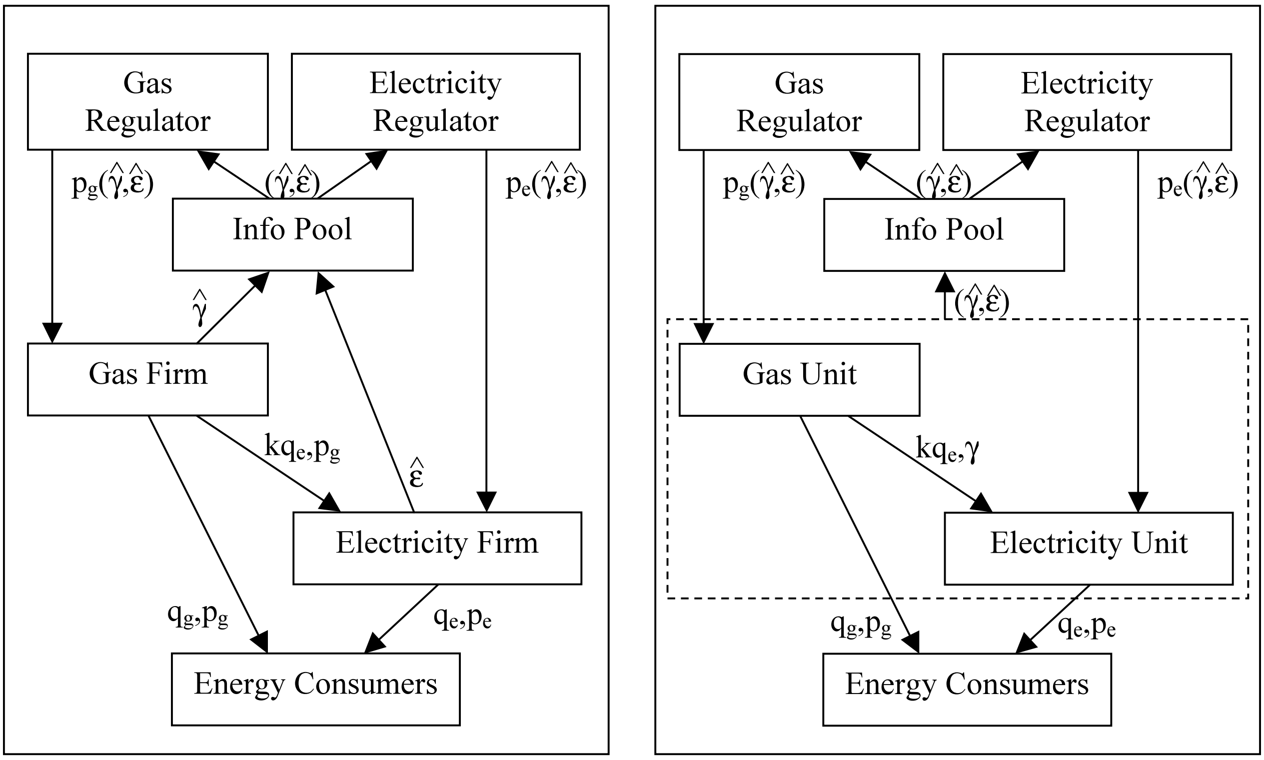
[12] This is because the latter have only a direct effect on end users, and the former have, additionally, an indirect effect, through the increase in total electricity production costs.

interesting strategic effects, which will be studied hereafter. An information pool is included between the firms and the regulators, just to make sure that the firms provide the same information to both regulators.



A. Duopoly **B.** Monopoly

Figure 3. Two regulators with private cost announcements



A. Duopoly **B.** Monopoly

Figure 4. Two regulators with public cost announcements

Looking to the literature, the revelation principle as formulated by Gibbard [1973], Myerson [1979, 1982], among others, applied to the standard principal-agent model, states that the regulator can, without loss of generality, restrict her attention to direct mechanisms in which firms reveal their true types. Given a firm of type β playing an optimal strategy, $\sigma^*(\beta)$, a direct revelation mechanism may be built, in which the firm is asked to reveal its type, and the transfers are constructed so as to offer the firm exactly the same payoffs it would get by playing its optimal strategy $\sigma^*(\beta)$ when announcing the true value of β.

However, Martimort and Stole [2001] illustrate that in general "the revelation principle cannot be applied to study equilibria of the multi-principal games" because (i) the agent has a role as a communication device between principals; (ii) the "truthful when indifferent" hypothesis is no longer valid; and (iii) out-of-equilibrium messages have a critical strategic value vis-à-vis the other principals, and they are pruned by the revelation principle.

In what follows we will focus on the set of IC contracts for each industry structure. In some cases, we show that these sets are empty. This does not mean that equilibrium outcomes do not exist in these settings, but rather that the outcomes are not IC. Following the standard argument in the regulatory literature, we restrict attention to regulatory contracts that are linear in firms' cost announcements, as shown in expressions (23) and (24).

$$p_g(\hat{\gamma},\hat{\varepsilon}) = A + B\hat{\gamma} + C\hat{\varepsilon} \tag{23}$$

$$p_e(\hat{\gamma},\hat{\varepsilon}) = X + Y\hat{\gamma} + Z\hat{\varepsilon} \tag{24}$$

Besides ease of mathematical tractability, these contracts might be intuitively interpreted as a first order approximation of more elaborate contracts, and they allow us to describe the whole range of "classic" regulatory contracts, namely (i) price-caps (or fixed price), when B = C = 0 or Y = Z = 0; (ii) cost-plus reimbursement rules, when C = 0 or Y = 0; (iii) yardstick regulation, when B = 0 or Z = 0. More complicated forms of regulation, all of which are a combination of these, are naturally also included in our approach. As stated by Laffont and Tirole, linear contracts may be a good approximation of the optimal policy in some circumstances, and they are also attractive because they are robust, and still optimal when any accounting or forecast error is added.

4.1 Two regulators and two firms

a. Industry specific information (private cost announcements)

Here, the two regulators cannot contract in each other's sector-specific information, so p_g is contingent only on $\hat{\gamma}$, and p_e only on $\hat{\varepsilon}$ (Figure 3A). If $p_g(\hat{\gamma})$ and $p_e(\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator for gas and electricity as a function of announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the incentive compatibility constraints for the firms are:

$$\frac{\partial}{\partial \hat{\gamma}} E_\varepsilon\left[\Pi_g\left(p_g(\hat{\gamma}), p_e(\varepsilon), \gamma\right)\right] \Big/_{\hat{\gamma}=\gamma} = 0, \quad \forall \varepsilon \tag{25}$$

$$\frac{\partial}{\partial \hat{\varepsilon}} E_\gamma\left[\Pi_e\left(p_g(\gamma), p_e(\hat{\varepsilon}), \varepsilon\right)\right] \Big/_{\hat{\varepsilon}=\varepsilon} = 0, \quad \forall \gamma \tag{26}$$

This brings us to the following proposition (see Appendix B for details):

Proposition 2. *Two independent, sector-specific regulators are unable to achieve incentive compatibility with contracts other than price-caps when regulating two firms, without some form of coordination between them.*

Proof

In a "truthful equilibrium", both incentive compatibility constraints must be simultaneously satisfied, and this will happen if and only if the pair of contracts takes one of the following forms:

(a) A pair of price-caps: $p_g(\hat{\gamma}) = A$ <u>and</u> $p_e(\hat{\varepsilon}) = X$; or

(b) A price-cap for gas and a standard linear regulatory contract for electricity: $p_g(\hat{\gamma}) = A$ <u>and</u> $p_e(\hat{\varepsilon}) = X(A) + \frac{\hat{\varepsilon}}{2}$; or

(c) A standard linear regulatory contract for gas and a price-cap for electricity: $p_g(\hat{\gamma}) = A(X) + \frac{\hat{\gamma}}{2}$ <u>and</u> $p_e(\hat{\varepsilon}) = X$.

In this context, the only way to achieve incentive compatibility is by choosing a fixed price for one of the goods, and a correctly tuned linear IC regulatory contract for the other (or choosing two fixed prices). *Q.E.D.*

If both regulators try to implement regulatory contracts (other than fixed prices) simultaneously, incentive compatibility will be lost. They must coordinate their action to agree on which good will have a fixed price and which will have a linear regulatory contract whose fixed coefficient will depend on the price of the other good.

Corollary 2. *If two independent, sector-specific regulators try to implement incentive compatibility without coordinating their action, the resulting pair of contracts will not induce the two regulated firms to tell the truth.*

<u>b. Public information</u>

Here, regulators can contract in both sectors' specific information, which is made available to them through an information pool, so p_g and p_e are contingent on both $\hat{\gamma}$ and $\hat{\varepsilon}$ (Figure 4A). If $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulators for gas and electricity as a function of announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the incentive compatibility constraints for the firms are:

$$\frac{\partial}{\partial \hat{\gamma}} E_\varepsilon \left[\Pi_g \left(p_g(\hat{\gamma},\varepsilon), p_e(\hat{\gamma},\varepsilon), \gamma \right) \right] \Big/_{\hat{\gamma} = \gamma} = 0, \quad \forall \varepsilon \tag{27}$$

$$\frac{\partial}{\partial \hat{\varepsilon}} E_\gamma \left[\Pi_e \left(p_g(\gamma,\hat{\varepsilon}), p_e(\gamma,\hat{\varepsilon}), \varepsilon \right) \right] \Big/_{\hat{\varepsilon} = \varepsilon} = 0, \quad \forall \gamma \tag{28}$$

From the firms' perspective, it is irrelevant whether the still unspecified regulatory contracts $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are chosen by two separate or a single regulator, so incentive compatibility constraints will be the same in this context than in the "one regulator, two firms" setting, that we discuss in detail in Section 4.3 and Appendix B, so the set of IC contracts will coincide. Here, the difference is that the choice of the coefficients in the contracts within the IC sets will be made by taking into account different criteria, because of the competing objectives of the two regulators, so parameter choice will require coordination between regulators. We will return to this problem when comparing industry structures and contract coefficients tuning in Section 5.

4.2 Two regulators and a single, dual product firm

a. Industry specific information (private cost announcements)

Here, the two regulators cannot contract in each other's sector-specific information, so p_g is contingent only on $\hat{\gamma}$, and p_e only on $\hat{\varepsilon}$ (Figure 3B). If $p_g(\hat{\gamma})$ and $p_e(\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator for gas and electricity as a function of announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the incentive compatibility constraints for the dual product firm are:

$$\begin{cases} \frac{\partial}{\partial\hat{\gamma}}\left[\Pi_d\left(p_g(\hat{\gamma}),p_e(\hat{\varepsilon}),\gamma,\varepsilon\right)\right]\Big/\left(\hat{\gamma}=\gamma,\hat{\varepsilon}=\varepsilon\right)=0, \quad \forall\gamma, \quad \forall\varepsilon \\ \frac{\partial}{\partial\hat{\varepsilon}}\left[\Pi_d\left(p_g(\hat{\gamma}),p_e(\hat{\varepsilon}),\gamma,\varepsilon\right)\right]\Big/\left(\hat{\gamma}=\gamma,\hat{\varepsilon}=\varepsilon\right)=0, \quad \forall\gamma, \quad \forall\varepsilon \end{cases} \tag{29}$$

This brings us to the following proposition (see Appendix C for details):

Proposition 3. *There is no pair of sector-specific incentive compatible regulatory contracts other than price-caps available to two sector-specific regulators that wish to regulate a single, dual product monopolist, if they do not share their information.*

Proof

The only solution to this system of differential equations is $\frac{\partial p_g}{\partial\hat{\gamma}}=0$ (which means constant gas prices) and $\frac{\partial p_e}{\partial\hat{\varepsilon}}=0$ (which means constant electricity prices), so a pair of fixed price contracts. *Q.E.D.*

Assuming $\frac{\partial p_g}{\partial\hat{\gamma}}\neq 0$ and $\frac{\partial p_e}{\partial\hat{\varepsilon}}\neq 0$, and solving for the firm's profit-maximization, yields $p_g(\hat{\gamma})=\dfrac{a+\gamma}{2}$ and $p_e(\hat{\varepsilon})=\dfrac{a+k\gamma+\varepsilon}{2}$. This is identical to the price outcome in the case of an unregulated monopoly (expressions (9) and (10)). Thus, a pair of independent regulatory contracts that does not include fixed prices is exploited by the firm to achieve monopoly pricing: this regulatory setting is equivalent to the absence of regulation.

Corollary 3. *If two independent, sector specific regulators try to implement incentive compatibility when confronting a dual product monopolist, the firm will exploit the proposed mechanisms to achieve monopoly pricing. The regulators should prefer fixed price contracts.*

This setting is a clear example of the simple proposition "no regulation is better than bad regulation", because in it, the monopoly outcome is reproduced despite the presence of a costly pair of regulators. In short, it is better not to regulate at all than trying to implement incentive compatibility with two independent regulators.

b. Public information

Here, regulators can contract in both sectors' specific information, which is made available to them through an information pool, so p_g and p_e are contingent on both $\hat{\gamma}$ and $\hat{\varepsilon}$ (Figure 4B). If $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator for gas and electricity as a function of announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the incentive compatibility constraints for the dual product monopoly are:

$$\begin{cases} \frac{\partial}{\partial \hat{\gamma}}\left[\Pi_d\left(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon}), \gamma, \varepsilon\right)\right] \Big/ \left(\hat{\gamma}=\gamma, \hat{\varepsilon}=\varepsilon\right) = 0, \quad \forall \gamma, \quad \forall \varepsilon \\ \frac{\partial}{\partial \hat{\varepsilon}}\left[\Pi_d\left(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon}), \gamma, \varepsilon\right)\right] \Big/ \left(\hat{\gamma}=\gamma, \hat{\varepsilon}=\varepsilon\right) = 0, \quad \forall \gamma, \quad \forall \varepsilon \end{cases} \tag{30}$$

From the firms' perspective, it is irrelevant whether the still unspecified regulatory contracts $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are chosen by two separate or a single regulator, so incentive compatibility constraints will be the same in this context than in the "one regulator, one firm" setting, that we discuss in detail in section 4.4 and Appendix C, so the set of IC contracts will coincide. Here, the difference is that the choice of the coefficients in the contracts within the IC sets will be made by taking into account different criteria, because of the competing objectives of the two regulators, so parameter choice will require coordination between regulators. We will return to this problem when comparing industry structures and contract coefficients tuning in Section 5.

4.3 One integrated regulator and two separate firms

We now discuss the industry structure illustrated in Figure 5. If $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator for gas and electricity as a function of announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then profits, expected payoffs and incentive compatibility constraints for the gas and electricity firms are the same as the ones in the setting with two regulators that share their information (Section 4.1.b).

We can simplify incentive compatibility constraints (27) and (28) by replacing demand profiles given by (2) and restricting our attention to linear contracts of the form (23) and (24). Thus we obtain:

$$\left[a(1-b)(1+k)+(b-k)(X+Y\hat{\gamma}+Z\hat{\varepsilon})+(b\cdot k-1)(A+B\hat{\gamma}+C\hat{\varepsilon})\right]B+(A+(B-1)\hat{\gamma}+C\hat{\varepsilon})\left[(b-k)Y+(b\cdot k-1)B\right]=0$$

$$\left[X+Y\hat{\gamma}+(Z-1)\hat{\varepsilon}-k\cdot(A+B\hat{\gamma}+C\hat{\varepsilon})\right](Z-bC)+\left[X+Y\hat{\gamma}+Z\hat{\varepsilon}-a(1-b)-b\cdot(A+B\hat{\gamma}+C\hat{\varepsilon})\right](Z-kC)=0$$

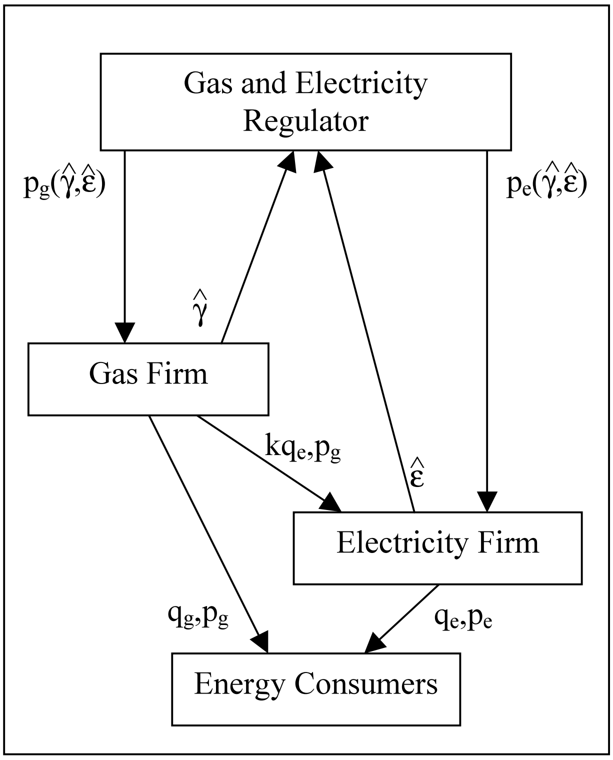


Figure 5. One regulator, two firms.

We know that these equalities must hold for every possible pair of values of γ and ε, so, by equating the coefficients of these variables in both polynomials to zero, we solved for {A,B,C,X,Y,Z}, and obtained the following 4 relevant solutions (see Appendix B for details):

1. <u>Fixed price pairs of contracts</u>

This solution corresponds to taking B = Y = C = Z = 0 and setting p_g and p_e according to a "worst case scenario". If the regulator decides that production of the two goods is required, she should set p_g and p_e equal to marginal costs when firms are most inefficient, that is, when both costs are as high as they can be, and when the electricity firm pays p_g for gas rather than c_g. This means $p_g = G$ and $p_e = E + k\,G$.

2. <u>Pairs of contracts depending only on γ</u>

These contracts correspond to those in which C = Z = 0. For the contracts to be incentive powered in $\hat{\gamma}$, we need the following conditions to be satisfied:

$$B = \tfrac{1}{2}\text{, and } X = \frac{a(1-b)(1+k) - 2A(1-b\cdot k + (k-b)Y)}{k-b}.$$

The exact values of the free parameters A and Y should be set according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

3. <u>Pairs of contracts depending only on ε</u>

These contracts correspond to those in which B = Y = 0. For the contracts to be incentive powered in $\hat{\varepsilon}$, we need the following conditions to be satisfied:

$$C > -\frac{1}{2(k-b)}\text{, } Z = kC + \tfrac{1}{2}\text{, and } X = \frac{a(1-b) + A(b + k + 2k(k-b)C)}{2(1+(k-b)C)}.$$

The exact values of the free parameters A and C should be set according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

4. Pairs of fully specified contracts

All contract coefficients are defined as follows.

First define $t = 1 + k^2 - 2bk$ and $\Delta = \sqrt{(1-b^2)t}$, then set:

$$A = \frac{a}{2}\left[1 - \frac{(k-1)(t-\Delta)}{t(k-b)}\right], \qquad B = ½, \qquad C = -\frac{t-\Delta}{2(k-b)t}, \text{ and}$$

$$X = \frac{a}{2}\left[1 + \frac{(1+k-2bk)(t+\Delta)}{t(k-b)}\right], \qquad Y = \frac{bk-1+\Delta}{2(k-b)}, \qquad Z = \frac{k\Delta - bt}{2(k-b)t}.$$

Intuition tells us that, due to the presence of individual rationality (participation) constraints for both firms, the fully specified contracts should perform better, because they minimise informational rent extraction by the firms. However, incentive compatibility is achieved at a cost, and we will return to this problem in Section 5 when tuning contract coefficients and comparing the resulting outcomes, and show that in fact this is not generally the case.

We note that there are no incentive powered contracts with "exclusive cross dependency", that is, where the price of each product depends only on the cost of the other firm, as in "yardstick competition". However, by carefully tuning the coefficients of both contracts, it is possible to regulate both industries and achieve incentive compatibility by using only one of the two cost announcements. We summarize these observations in the following lemma.

Lemma 1. *Incentive compatible regulation in the duopoly case can be implemented by using none, only one, or both cost announcements, but careful tuning of both contracts is necessary in all cases. Incentive compatibility is not achievable using yardstick contracts.*

4.4 A single integrated regulator for a single, dual product firm

Here, the relevant industry structure is illustrated in Figure 6.

If $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator for gas and electricity as a function of the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the profit of the dual product firm and the incentive compatibility constraints are analogous to those in the setting with two regulators who share their information (Section 4.2.b).

The firm will maximize its profits by announcing an optimal pair of values $\hat{\gamma}$ and $\hat{\varepsilon}$, that will determine the prices, and thus its own profits. Incentive compatibility requires that announcing the true values of the private cost parameters be an optimal choice, which means that the first partial derivatives of this expression with respect to the announced parameters must be equal to zero when the announcements are equal to the true values of the parameters. By replacing the demand functions (2) and the linear regulatory contracts (23) and (24) into (30), we obtain:

$$\left(aB-2AB-abB+2bBX+aY-abY+2bAY-2XY\right)$$
$$+\left(B-2B^2-bkB-bY+4bBY+kY-2Y^2\right)\gamma+\left(Y-bB+2bBZ+2bCY-2BC-2YZ\right)\varepsilon=0$$
$$\left(aC-2AC-abC+2bCX+aZ-abZ+2bAZ-2XZ\right)$$
$$+\left(C-2BC-bkC-bZ+2bCY+2bBZ+kZ-2YZ\right)\gamma+\left(Z-bC+4bCZ-2C^2-2Z^2\right)\varepsilon=0$$

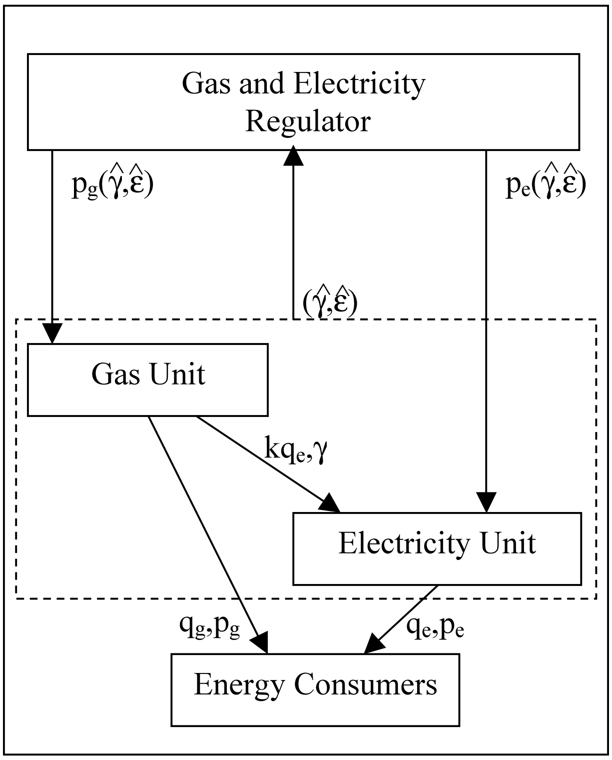


Figure 6. One regulator, one firm.

We know that, for the contracts to induce truth-telling, these equalities must hold for every possible pair of values of γ and ε. So, by equating the coefficients of these variables in both polynomials to zero, we solved for {A,B,C,X,Y,Z}, and obtained the following 7 relevant solutions (see Appendix C for details). For notational simplicity, we define, as earlier, $t=1+k^2-2bk$.

1. <u>Fixed price pairs of contracts</u>

This solution corresponds to taking B = Y = C = Z = 0 and setting p_g and p_e according to a "worst case scenario". Here, if the regulator can make production of both goods compulsory, she can induce cross-subsidies between gas and electricity while still respecting the global individual rationality constraint of the integrated firm. However, if she cannot, she must ensure that the unit margin for each of the two goods is positive when firms are most inefficient, that is, when both costs are as high as they can be, which implies $p_g = G$ and $p_e = E + k\,G$.

2. <u>Pairs of contracts depending only on γ</u>

These contracts correspond to those in which C = Z = 0. For the contracts to be incentive powered in $\hat{\gamma}$, we need the following conditions to be satisfied:

$A=\frac{a}{2}$, $B=\frac{1}{2}$, and $Y=\frac{b}{2}$.

The exact value of the free parameter X should be set by the regulator according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

3. Pairs of contracts depending only on ε

These contracts correspond to those in which B = Y = 0. For the contracts to be incentive powered in $\hat{\varepsilon}$, we need the following conditions to be satisfied:

$$C = -\frac{k-b}{2t},\ Z = \frac{1-bk}{2t}, \text{ and } X = kA - \frac{a(k-1)}{2}.$$

The exact value of the free parameter A should be set by the regulator according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

4. Pairs of contracts in which the price of gas is fixed

These contracts correspond to those in which B = C = 0. For the contracts to be incentive powered in $\hat{\varepsilon}$, we need the following conditions to be satisfied:

$$Y = \frac{k-b}{2},\ Z = \frac{1}{2}, \text{ and } X = bA + \frac{a(1-b)}{2}.$$

The exact value of the free parameter A should be set by the regulator according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

5. Pairs of contracts in which the price of electricity is fixed

These contracts correspond to those in which Y = Z = 0. For the contracts to be incentive powered in $\hat{\gamma}$, we need the following conditions to be satisfied:

$$B = \frac{1-bk}{2},\ C = \frac{-b}{2}, \text{ and } X = \frac{2A - a(1-b)}{2b}.$$

The exact value of the free parameter A should be set by the regulator according to a "worst case scenario" in order to respect the individual rationality constraint of both firms.

6. Pairs of "complete" contracts

These contracts correspond to those in which $BZ = CY \neq 0$. In these contracts, both prices depend on both cost announcements. For the contracts to be incentive powered in $\hat{\gamma}$ and $\hat{\varepsilon}$, we need the following conditions to be satisfied:

First define $d = \pm\sqrt{1-16C^2(1-b^2)}$ for notational simplicity, and then set

$$B = \frac{1-d}{4} + (k-b)C,\ X = a\frac{1+4C(1-b)-d}{8C} - A\frac{1-4bC-d}{4C},$$

$$Y = \frac{1+d}{4}\frac{B}{C} + bB, \text{ and } Z = \frac{1+d}{4} + bC. \text{ We also naturally need } |C| < \frac{1}{4\sqrt{1-b^2}}.$$

Notice that there are two families of contracts, depending on whether d is positive or negative, and that they both have two free parameters (A and C). These should be chosen by the regulator according to a "worst case scenario".

Observe that by taking $C = \frac{-b}{2}$ in the contract with a negative *d*, we obtain the fixed electricity prices mentioned in point 5 above.

7. <u>Fully specified pairs of contracts</u>

There are two fully specified contracts, of which the first corresponds to the following parameter values:

$A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = 0$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \frac{1}{2}$.

It is interesting to notice that this contract implements standard IC regulation for the price of gas, independent of the announced cost of electricity production, and a "corrected" form of standard regulation for electricity, in which the denominator corresponds to the total marginal cost of electricity production (i.e. $k\gamma + \varepsilon$).

The second fully specified contract is given by:

$A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = \frac{k-b}{2t}$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \frac{k(k-b)}{2t}$.

Notice that both the parameters for the fixed part and the part corresponding to the gas cost are the same as in the previous contract. Nevertheless, here the price of gas depends on the cost of producing electricity, whereas in the previous contract it did not.

Intuition tells us that, due to the presence of individual rationality (participation) constraints for both firms, the fully specified contracts should perform better, because they minimise informational rent extraction by the firms. However, incentive compatibility is achieved at a cost, and we will return to this problem in Section 5 when tuning contract coefficients and comparing the resulting outcomes, and show that in fact this is not generally the case.

Observe, as in Section 4.3, that there are no incentive powered contracts with "exclusive cross dependency", but we can regulate both prices and achieve incentive compatibility by using only one of the two cost announcements, by carefully tuning the coefficients of both contracts.

Lemma 2. *Incentive compatible regulation in the integrated monopoly case can be implemented by using either none, only one, or the two cost announcements, but careful tuning of both contracts is necessary in all cases. There are no yardstick type incentive compatible contracts.*

Lemmas 1 and 2 can be combined into the general result of the following proposition:

Proposition 4. *Regardless of the prevailing industry structure, incentive compatible regulation can be implemented by using either none, only one, or the two cost announcements, but careful tuning of both contracts is always necessary. There are no yardstick type incentive compatible contracts.*

4.5 Summary of Section Results

Table 1 summarizes the feasible IC contracts in each of the situations that we have considered. Settings with two regulators that share their information are analytically similar to those with only one integrated regulator, so the families of linear IC contracts are the same.

Table 1. Incentive compatible linear contracts for each industry structure

Setting		One firm				Two firms			
		2RNS		2RS	1R	2RNS		2RS	1R
Type of Contracts									
Gas	Electricity		DOF		DOF		DOF		DOF
price-cap	price-cap	✓	2	✓	2	✓	2	✓	2
price-cap	"cost plus"	×	-	×	-	✓	1	✓	1
"cost plus"	price-cap	×	-	×	-	✓	1	✓	1
price-cap	complete	×	-	✓	1	×	-	×	-
complete	price-cap	×	-	✓	1	×	-	×	-
"cost plus"	"yardstick"	×	-	✓	1	×	-	✓	2
"yardstick"	"cost plus"	×	-	✓	1	×	-	✓	2
complete	complete	×	-	✓✓✓	2,2,0	×	-	✓	0
"yardstick"	"yardstick"	×	-	×	-	×	-	×	-
Total number of linear incentive compatible contract subsets		1		7		3		4	

Legend : DOF Degrees of freedom in the pair of contracts
2RNS Two regulators not sharing the information
2RS Two regulators sharing the information
1R One regulator

The first important conclusion is that the types of contracts within the IC set depend on industry structure. For instance, a pair of price-cap and cost-plus contracts will never be IC with a dual-product monopoly, but *may* be IC with two regulated firms, if coefficients are correctly tuned. This follows the intuition that regulators are better off when they face two competing firms rather than one integrated firm. However, the "price-cap + complete" pairs of contracts provide a fair counterexample: they will never be IC if there are two firms, but *may* be IC if there is only one.

It is also interesting to compare the latitude available to regulators in all settings by looking at the total potential types of pairs of contracts with their degrees of freedom. For instance, two sector-specific regulators not sharing their information can only achieve IC with a pair of price-caps when there is only one firm, but they can also use a tuned pair of price-cap and cost-plus contracts when there are two. Again, this follows the intuition that regulators are relatively better off with two competing firms than with only one, but the sharing of information provides the counterexample. Whereas in the two firm context there are only four feasible IC regulatory contracts, in the one firm setting there are seven. Regulators have thus a wider range of IC combinations of contracts when they face only one firm.

A closer look at the IC contract families described in detail in Section 4 allows us to draw more elaborate conclusions. Even when the same pairs of contracts are IC in both industry structures, the constraints for the choice of coefficient values are different. For example, if we look at contracts contingent on the announced cost of gas only, we can see that in the duopoly case the regulator can freely choose two coefficients, while in the monopoly setting she has only one degree of freedom. Moreover, the contracts are different, and the former are not a subset of the latter. The same can be said about the contracts contingent on the announced cost of electricity only. The differences are even clearer when looking at the fully specified contracts across industry structures.

It is fairly simple to see that most of the IC contracts in each set are not robust to changes in industry structure, that is, contracts that are IC for one firm, are not IC for two firms, and vice-versa. However, there are some (very specific) IC contracts that are indeed robust to these changes.

In what follows, we analyse the implied coordination issues arising from Proposition 4 and compare the performances and outcomes of different IC contracts.

5. Coordination Issues and Optimal Regulation

In the previous section we have fully characterized the set of linear IC pairs of regulatory contracts for each possible industry structure and institutional arrangement. In this section we continue to assume that industry structure is exogenous and we determine the optimal regulatory contract within the IC set for each analytical setting. Thus, here we study what happens for a given institutional arrangement and industry organization, before moving on to normative recommendations and policy implications in Section 6, where we endogenize the organization of the industry.

5.1 Two independent sector specific regulators not sharing their information

5.1.1 Monopoly Case

In the previous section, we showed that two independent, sector specific regulators that do not share their information (or, equivalently, who cannot contract on the cost parameter of the other sector), are unable to achieve incentive compatibility with contracts other than pairs of price-caps in the one firm case.

If regulators could coordinate their actions, they would optimally implement cross subsidies between both goods and set the minimum prices that verify the IR constraint of the single firm, as in point 5.3.1 below. However, if they do not coordinate their actions, they will only ensure non negative productions and margins in their own respective sectors, which will result in the following optimal prices $p_g^{m,2RNS*} = G$ and $p_e^{m,2RNS*} = kG + E$.

Coordinated choices will be studied in Section 5.3.

5.1.2 Duopoly Case

Besides the fact that the two regulators must agree upon which *types* of contracts they will use (Proposition 2), in all pairs of contracts other than two price-caps, the fixed part of the "variable" contract in each pair depends on the price of the other good, necessitating coordination in the choice of contract coefficients as well. Since we see in Table 1 that these pairs of contracts have only one degree of freedom, this will naturally pose an important obstacle to two independent regulators with competing agendas.

If the regulators agree to use a pair of price-caps, ex ante individual rationality for each firm requires that each price covers at least the cost of production in the "worst case scenario", that is, when idiosyncratic costs take their maximum possible value, so the selected optimal pair of contracts will be the same, that is $p_g^{d,2RNS*} = G$ and $p_e^{d,2RNS*} = kG + E$.

5.2 Two regulators sharing their information

We have already shown that two regulators that do not share their information cannot achieve incentive compatibility with contracts different from price-caps unless they coordinate. Here, we show that regulators must also coordinate when they can use the same (shared) information.

5.2.1 Integrated Monopoly Case

Coordination issues also arise in this institutional arrangement, as implied by the following Lemma:

Lemma 3. *Two sector-specific regulators that share their information must not only agree on which of the different possible regulatory contracts they will use in order to achieve global incentive compatibility, but also on the choice of the contract coefficients.*

Proof

Observe in Table 1 that all families of IC contracts require the simultaneous implementation of two regulatory contracts of a special nature. That is, contracts are IC *by pairs*, and not individually.

For example, if the electricity regulator decides that she will put in place a contract that is independent from the cost announced by the gas firm, global incentive compatibility will be achieved only if the gas regulator chooses the same type of contract (also independent from $\hat{\gamma}$). Moreover, as all contracts (excepting fixed prices, of course) impose "crossed constraints" between the contract coefficients for both industries, it is not sufficient for her to choose a contract of the right type, because if contract coefficients do not verify the relationships described in point 4.4.3 (in the example), the contracts will still not be globally IC.

Observe furthermore that some of the IC contract sets in Table 1 have only one degree of freedom, which means that both regulators must agree upon which one of them is going to set that free coefficient, and who is the one that will have to follow suit, like in point 5.1.2 above. Both regulators cannot set contract coefficients simultaneously without losing incentive compatibility, unless they do so according to the relationships among coefficients described in Section 4.4. *Q.E.D.*

The only pair of contracts that allows full latitude to each regulator is the pairs of price-caps: in this case their best course of action will be the same than that of two regulators that do not share their information, because by implementing a pair of price-caps, even if they share the information, they decide not to use it.

Coordinated choices will be studied in Section 5.3.

5.2.2 Duopoly Case

The same coordination issues described above arise for two regulators when confronting two firms (see Table 1, the relationships between contract coefficients described in Section 4.3, and Appendix B). Lemma 3 applies fully in this setting, for exactly the same analogous reasons. The lemma can thus be extended to the following general proposition:

Proposition 5. *Regardless of the underlying industry structure, two sector specific regulators that share their information must fully coordinate their actions in order to achieve global incentive compatibility. This means they must take joint decisions and "negotiate" about the optimal choice of contract types and coefficients, and should not decide independently.*

However, due to the nature of the ex ante optimal choice of contract types and coefficients when there are two single product firms in the market, that will be discussed in point 5.3.2, this coordination might actually be achieved spontaneously. We will get back to this point later on.

5.3 One regulator

There are naturally no coordination issues in the single integrated regulator case. Here, the regulator must compare the expected outcome of the different possible contracts with regards to her welfare preferences, and pick the one that performs better.

Fully specified contracts ought to be intuitively preferred, since they achieve incentive compatibility and use all the available instruments. We will now see that this is not generally the case, and that in fact this pair of contracts is generally dominated by others.

5.3.1 Monopoly Case

Of the seven types of pairs of linear IC contracts shown in Table 1 and described in detail in Section 4.4 and Appendix C, only four are potentially optimal ex ante. The other three are strictly dominated ex ante (see Appendix D) as follows $7a \succ 7b$; $4 \underset{U}{\succ} 7a$; $4 \underset{U}{\succ} 5$; $3 \underset{U}{\succ} 2$, with $\succ$ representing ex post dominance and $\underset{U}{\succ}$ representing ex ante dominance with uniform distributions.

Dominance among the four remaining optimal pairs of contracts cannot be generally asserted for all possible parameter values. A fortiori, the choice of the best optimal pair of contracts by the regulator changes according to parameter values (see Appendix D). The graphic in Figure 7 illustrates this result, summarized in Lemma 4, by showing the optimal contract choice for fixed values of *a*, *G* and *E*, as a function of parameters *b* and *k*, for uniform cost distributions.

Lemma 4. *The optimal types of incentive compatible regulatory contracts that should be implemented to regulate an integrated dual-product firms change with substitutability and technical dependency among both products.*

Proof
See numerical example below and Appendix D. *Q.E.D.*

If substitutability b is sufficiently low (yellow and green regions), the ex ante socially optimal contract choice is a pair of fixed price contracts, in which prices are set so as to (ex ante) optimally subsidize electricity production with gas revenue. For intermediate values of b and values of k in the blue region, the ex ante socially optimal contract choice is a pair of contracts in which the price of gas is fixed at its expected cost level. Finally, for large values of b and values of k in the cyan region, the ex ante socially optimal contract choice is the ex ante optimal pair of ε contingent contracts. Observe that there is a yellow region between the blue and the cyan region in which fixed price contracts also dominate.

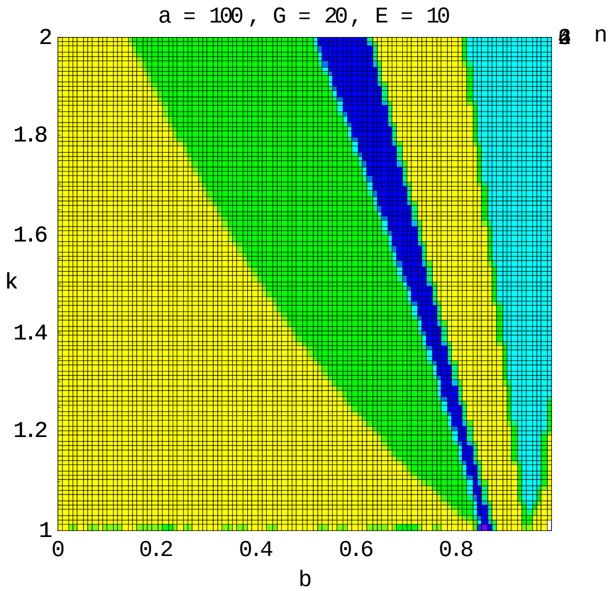


Figure 7. Ordering of IC contracts
as a function of substitutability and technical dependency
(: $1 \succ 3 \succ 4$; : $1 \succ 4 \succ 3$; : $4 \succ 1 \succ 3$; : $3 \succ 1 \succ 4$)

The shape and size of these regions change quantitatively with the remaining three parameters, but their qualitative positions remain the same. As total demand parameter a increases relative to G and E, the region in which a pair of price-caps dominates the other possible pairs of contracts grows larger; as E increases relative to G, the blue region shrinks.[13]

5.3.2 Duopoly Case

There are four types of pairs of linear IC contracts, described in detail in Section 4.3 and Appendix B. In order to obtain the ex ante optimal values of contract coefficients, the regulator must solve fifth degree equations, which lack any explicit algebraic solution. General comparisons are thus impossible, and we must restrict ourselves to the analysis of representative numerical examples to give us an idea of the qualitative comparisons at hand.

Dominance among the optimal pairs of contracts of the different types cannot be generally asserted for all possible parameter values. A fortiori, the choice of the best optimal pair of contracts by the regulator changes according to parameter values, as summarized in the following Lemma.

[13] See Appendix D for illustrations of these effects.

Lemma 5. *The optimal types of incentive compatible regulatory contracts that should be implemented to regulate two competing single product firms change with substitutability and technical dependency among both products.*

Proof

See numerical example below and Appendix D. *Q.E.D.*

The graphic in Figure 8 illustrates this result, by showing the optimal contract choice for fixed values of *a*, *G* and *E*, as a function of parameters *b* and *k*, for uniform cost distributions.

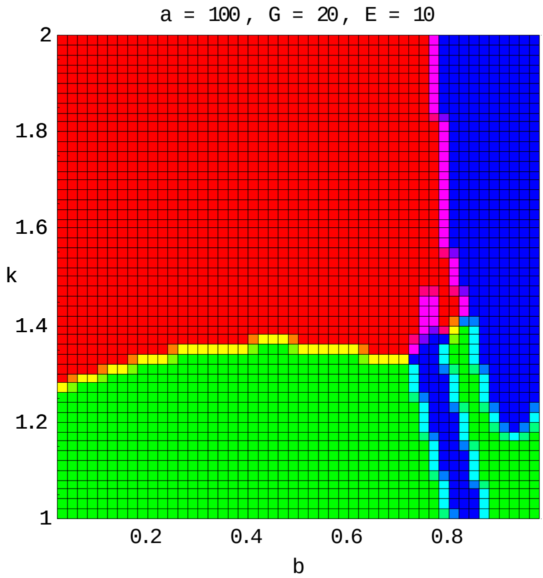


Figure 8. Optimal choice of IC contracts as a function of substitutability and technical dependency (: 1; : 2 ; : 3)

The green colour in the graphic corresponds to areas in which the regulator maximizes expected social welfare by choosing an optimally tuned pair of fixed price contracts; the blue zone corresponds to those where she should choose a pair of optimally tuned epsilon contingent contracts; and the red zone to those in which a pair of optimally tuned gamma contingent contracts performs best. If substitutability *b* is low, the regulator should prefer a pair of price-caps when technical dependency *k* is also low, and a pair of gamma contingent contracts when *k* is high. On the other hand, if substitutability *b* is high, a pair of epsilon contingent contracts might be better.

If total demand increases relative to cost uncertainties, the zone where price-caps dominate gets larger, and the zone in which gamma contingent contracts dominate decreases. On the other hand, if electricity cost uncertainty increases relative to gas cost uncertainty and total demand (or equivalently if gas cost uncertainty decreases relative to electricity cost uncertainty and total demand), the zones in which price-caps and epsilon contingent contracts dominate shrinks, and the zone in which gamma contingent contracts dominate expands.[14]

When cost uncertainties increase, so do information rents, so it becomes increasingly interesting and useful to extract that information instead of foregoing all the benefits of incentive compatibility. These results can summarized in the following Proposition:

[14] See Appendix D for illustrations of these effects.

Proposition 6. *Regardless of the underlying industry structure, the nature of the optimal pairs of incentive compatible regulatory contracts that should be implemented to regulate both goods change with substitutability and technical dependency among both products.*

Proof

Follows straightforwardly from Lemmas 4 and 5 above. *Q.E.D.*

6. Comparison of Structures and Gains to Regulation

We now consider the problem of a social planner who decides whether or not to allow gas and electricity firms to merge, and also whether the two industries will be regulated by one or two institutions.

6.1. Number of Regulators

In our context, information sharing and joint welfare maximization improve the performance of regulatory contracts. Where regulatory regimes overlap, regulators are indirectly competing with each other. In other words, when there are two regulators, each maximises the welfare of its own sector and imposes an externality on the other. These externalities may be internalised through joint decision-making. These results are consistent with the general multi-principal economic literature that says that multiple principals with conflicting objectives lead to a reduction in overall welfare.

In our model, joint regulation is not only better, but is actually *necessary* to achieve IC for pairs of contracts other than two price-caps. Otherwise, incentive compatibility is lost, and the ensuing strategic manipulation of costs would strongly weaken regulatory control.

Thus, within a given industry structure one regulator always performs at least as well as two regulators sharing information. Therefore, in what follows we will compare settings with only one regulator.

6.2. Gains to Regulation

The analysis of the benchmark case in Section 3 allowed us to determine that perfect regulation permits a 33% welfare increase over the best attainable result with no regulation (i.e. unregulated monopoly). However, in practice informational asymmetries and incentive compatibility conflicts give rise to information rents and welfare losses under regulation. We now determine the feasible improvement in welfare in the presence of informational asymmetries.

6.2.a. One Firm

Figure 9 illustrates welfare gains to regulation in the monopoly case for the same parameter values as the previous graphics. These gains oscillate generally between 20% and 31%, and increase when G and E are small relative to a. This result supports the intuition that if cost uncertainty is low, then information extraction will be less costly, and gains to regulation higher. On the other hand, if cost uncertainty is high, then large rents must be sacrificed to extract information, and welfare gains are lower.

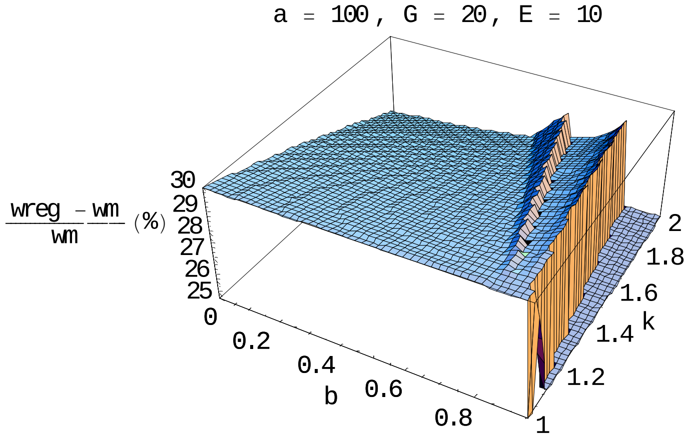


Figure 9. Welfare gains to imperfect regulation: monopoly.

Welfare gains tend to decrease with increases in vertical dependency, and to increase with substitutability, but this is not always the case, as can be seen in the previous Figure.

6.2.a. Two Firms

Figure 10 illustrates welfare gains to regulation in the duopoly case for the same parameter values as the previous graphics. These gains oscillate generally between 20% and 130%,[15] and increase when G and E are small relative to a. This result also supports the intuition we mentioned earlier.

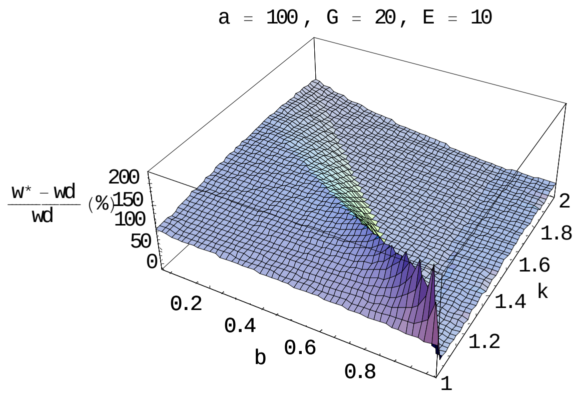


Figure 10. Welfare gains to imperfect regulation: duopoly.

Welfare gains tend to be relatively stable with respect to vertical dependency, and to decrease with substitutability, but this is not always the case, as can be seen in the previous Figure.

[15] This 130% is actually larger than the "perfect" 33% cited earlier because here we are comparing the regulated outcome with an unregulated duopoly, which is not the best possible unregulated option, as shown in Proposition 1 and its Corollary in Section 3.

6.3. Industry Structure

As we have just seen, there are welfare gains to regulation. If we admit that these gains outweigh regulatory costs, we can further compare which of the two regulated outcomes is to be preferred.

Proposition 7. *In the presence of informational asymmetries and optimal incentive compatible regulation, the integrated, dual-product firm, does not always perform better than two competing single-product firms.*

Proof

See numerical example in Figure 11 below. *Q.E.D.*

Figure 11 below illustrates, for the same parameter values as the previous graphics, which of the two regulated outcomes performs better from an expected welfare point of view, assuming that optimal incentive compatible regulation is implemented under each industry structure.

The red regions correspond to zones in which two optimally regulated firms produce a higher welfare than a regulated dual-product monopoly, and the cyan regions correspond to those in which the opposite is true. When technical dependency is low, the regulated monopoly outcome is to be preferred, unless substitutability is very high. On the contrary, when technical dependency is high, the regulated duopoly performs better, unless substitutability is in a specific, relatively high, range.

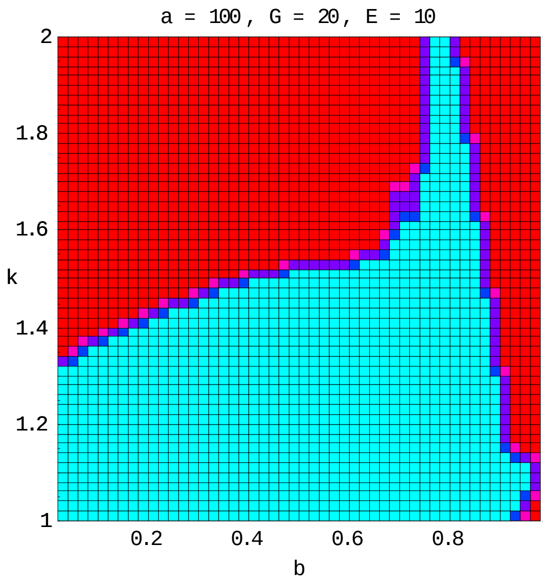


Figure 11. Optimal choice of industry structure.
(: duopoly ; : monopoly)

Thus, the general result in Proposition 1 stating that a single, dual-product firm, performs better in the unregulated perfect information setting, does no longer hold in presence of informational asymmetries. As shown in the previous figure, under some circumstances it might be better to regulate two competing firms than a single dual-product monopolist.

There are two competing effects at work that explain this qualitative difference: the informational advantages of the integrated firm, and the ex ante cross subsidization available to the regulator. When there is only one firm, the producer has the power to manipulate simultaneously *both* cost announcements, and is only confronted to what the regulator does. On the other hand, in this setting the regulator can implement ex ante cross subsidization between both lines of products to a certain extent, in such a way as to minimize the impact of the individual rationality constraint of the firm: she is then able to choose "better" prices for both commodities, while ensuring positive profits for the integrated firm, knowing that internal transfers will be efficiently made at marginal cost. When there are two firms, on the one hand the regulator must respect two tighter individual rationality constraints, but on the other each firm can manipulate only *one* cost announcement, and is confronted not only to regulatory discretion, but also to the behaviour of its competitor.

Overall, welfare gains to regulation as compared to the optimal unregulated situation (that is, with only one integrated firm) are of the same order of magnitude in both industry settings. In other words, relative differences among both outcomes are small in magnitude (most of the time in the 5% range). However, in any case the global welfare gains to regulation with respect to the unregulated outcome are important enough to justify regulatory action in both industry structures.

7. Shortcomings and Extensions

In this section we discuss in greater some implicit simplifying assumptions of our model.

1. Price setting central must be gas fired

This analysis applies only when the power plant that sets the spot price in the electric system (that is, the last generating unit that is "called in", merit-wise) is gas fired. This is the case in most electric systems around the globe, and should continue to be so in the foreseeable future, since combined cycle gas turbines (CCGT) is the current technology of choice in the electricity generation industry.[16]

Any increase in the price of gas is likely to have an immediate impact in the spot price of electricity because gas fired electricity producers will tend to offset such an increase in their costs by increasing their own sales price, the price of electricity.[17] However, this limits the "raise your rival's costs" strategy of the natural gas firm, because it cannot go higher than the cost of the next competing technology in the electric system. For example, if the first non-

[16] The Annual Energy Review published by the European Commission [2000] states that, worldwide, "since 1993 the contribution of thermal generation has accelerated, meeting more than 70% of incremental production. In 1997 it contributed as much as 96% of additional production due to the slowdown of nuclear output. […] Gas use [for electricity production] has more than doubled since 1980", and in the EU, "increases [of gas demand since 1990] were spectacular in the power sector (104%)". Joskow [2001] reports that "during the 90's natural gas was widely viewed as the fossil fuel of choice. It is relatively clean […], it was relatively cheap and it could fuel new efficient CCGT electric generating facilities. The CCGT facilities in turn were ideally suited for supporting the evolving competitive electricity markets since they could be built quickly, at relatively small minimum efficient scale, were less capital intensive, and could be more easily sited than conventional generating plants. […Total] natural gas consumption in the US increased by 22% between 1990 and 2000, and electricity produced with natural gas increased by 57% in the same period, and almost all new generating plants under construction at the end of the decade were fuelled by natural gas. Natural gas consumption is projected to continue to grow rapidly in the next two decades, [… increase] dominated by rapidly growing utilization of natural gas to produce electricity using CCGT generation technology".

[17] This was one of the many factors explaining the energy crisis in California.

gas out-of-merit unit is coal based, the gas firm will be able to increase the price of electricity only up to the marginal cost of that coal based unit by raising the price of gas. Any further increase in the price of gas will only reduce final consumer demand for gas without affecting the price of electricity.

2. Bundled supply and competitive segments in the energy sector

Even though traditionally the markets of gas and electricity have been served by a vertically integrated local monopoly each, and in some cases a single integrated dual provider, some stages in the production chain have recently been opened to competition. Technical progress has reduced the minimum efficient scale in production, so both electricity generation and natural gas extraction at the well level can be realized by decentralized, competing firms.

This means that both natural gas and electricity are, at the end consumer level, bundled composite products of (i) competitive components (the natural gas "molecule" itself in one case, or generated electricity in the other) and (ii) regulated components (pipeline carriage or electricity transmission, respectively).[18] The analysis presented here focuses on the interactions of price regulation and assumes that markets for the "competitive component" are perfectly competitive.[19] We thus abstract from "double marginalization" and "producers' market power" issues inside each of the two industries by considering vertically integrated suppliers.

3. "Base" power in electricity and fixed coefficient technology

Our model assumes that all power is generated from gas. In practice, power is generated from a portfolio of different primary energies. Some technologies less expensive than gas on the margin, so they are "called-in" to product before CCGT plants.[20]

In our model, we also assume that the production technology has fixed coefficients, i.e. electricity generators cannot substitute their gas input with other alternatives. This holds in practice as long as the price of gas does not move beyond the bounds defined by the prices of the immediately superior and inferior primary sources of energy, and as long as the installed capacity of gas generation is not exhausted. So in fact this hypothesis is equivalent to the issue discussed in point 1 above.

Adding "base" capacity to the model presented or introducing a production technology with discretely variable coefficients would not provide any further qualitative insights about the issues analysed herein. However, it is true that some of the effects that we identified are amplified by this simplification, because here an increase in the price of gas alters the cost of all the electricity produced. But this is less an important issue than it seems. In fact, as long as the price-setting central is gas fired, any increase in the price of gas would push up the spot

[18] For an analysis of the optimality of "bundled" or "unbundled" supply of complementary products, see Gilbert and Riordan [1995].

[19] This is, in fact, arguably not the case in these industries, where many reports show the existence of market power. However, the effects of producers market power on prices produce the same kind of interactions among both types of energies as those analysed here.

[20] Some of them, notably hydro and nuclear plants, are even "turned on" all the time, constituting what is called "base" power.

price in the whole electric system, which means that any excess of "base" power can be traded at this increased price level. In other words, any increase in the price of gas alters the marginal cost of the whole electric system, thus altering the value of electric energy, and thus its price.

These variations can of course be offset by long term contracting by electricity firms, but these dynamic considerations would add enormously in complexity to the model without bringing forward equivalent qualitative improvements to our results.

4. Limits to substitutability

Both gas and electricity have specific uses in which they are not perfectly substitutable. For instance, gas is best suited for heating applications, and electricity is more appropriate for lighting. This limited substitutability is partly captured by the substitution parameter *b*, but an alternative model with "captive" consumption of both forms of energy could be considered.

Switching costs associated with the passage from one energy source to the other are not considered in this model. A more elaborate version that considered this switching costs could be built, and, by reducing substitutability, it would probably weaken competition and increase firms' market power.

5. Cost of regulation

An interesting extension to the model would be one in which the institutional resources needed to regulate would be taken into account. In fact, the present model does not consider the difference in government spending associated to the existence of one or two regulators.

Cost-benefit analysis of regulatory authorities have only recently been undertaken, and there is still not enough evidence to calibrate such a model. One could alternatively defend either the view that two limited jurisdiction, sector specific regulators are less expensive than a single, broader, more complex and technically demanding integrated regulator, or the opposing view that there are also economies of scale in regulatory bodies. In any case, a model encompassing political economy aspects would be an interesting complement to the present analysis.

In such a context, a structure in which only one product is regulated (intuition says that, should we allow for only one sector specific regulator, we should prefer to regulate gas, but the alternative setting could also be considered), and the other price would be determined by market forces. For this analysis, we should compare the savings in terms of regulation and coordination costs, to the potential losses due to inefficient pricing in the electricity market.

One should also consider the issue whether one of the proposed regulatory structures favours one model of business (namely separated supply of gas and electricity, or integrated provision) over the other. That is, for a given regulatory structure and no merger control, firms might be induced to merge (or to separate) to increase their profits. Capture of institutional design by incumbent firms in this context is not necessarily a minor issue.

8. Conclusions

The double interaction between gas and electricity both as substitutes in the final energy market and in the input/output market for electricity generation raises many important questions, especially when taking into account the fact that both sectors are regulated.

The vertical relationship between electricity generation and gas production gives rise to market power concerns *vis à vis* the gas firm, that has incentives to increase its own price in order to raise its rival's production costs, an argument for integration of the two firms. However, integration also gives rise to market power concerns (in the final market), as we move from a situation of competing firms to monopoly. Practice has recently seen the convergence not only of energy firms, among which we find an increasing number of multi-product suppliers, but also of energy regulators, pioneered by the merger of the former OFGAS and OFFER into OFGEM, an integrated gas and electricity regulator.

In considering the desirability of mergers of gas and electricity monopolies, we must gauge the relative importance of the "increased competition" and the "double marginalization" effects. We have shown that in unregulated markets with two competing firms, one for gas and one for electricity, price competition with horizontally differentiated goods with linear demands leads to an outcome that is, from a global welfare point of view, worse than that of the single, integrated, dual product monopolist. Moreover, all members of society (firms and consumers) benefit from such a merger, so this kind of operation should not only be generally welcome and accepted, it should even be encouraged.

Once we introduce perfect regulation, however, firms' profits disappear and global welfare maximised together with consumer surplus. Mergers are socially neutral, and firms have no strong incentives to merge, given that their profits will be zero in all cases. In this paper, our main goal has been to explore what happens in intermediate situations where regulation is imperfect due to different kinds of informational asymmetries. The nature of such interactions call for an in-depth analysis of the potential interference among separate regulators and the role of the double link between the two markets into the design of performing regulation, which, if carried out in a rather simplistic way, might be very costly for society.

In fact, if regulation is costly, it may be better not to regulate at all than to try to implement IC regulation with two completely independent regulators. In the extreme case of a dual product monopoly regulated by two institutions, the regulated firm can exploit the pricing rules to achieve monopoly pricing, while in the duopoly case, incentive compatibility is not feasible.

One potential remedy is to implement regulation through two agencies that can contract in both firms' private information, but these schemes pose very serious coordination problems. It turns out that the sets of IC contracts available to two distinct regulators are the same as those available to a single regulator, but whereas a single regulator has all the incentives to choose contract coefficients harmoniously, two separate regulators must coordinate their action and tune the coefficients together or decide to sacrifice incentive compatibility.

The sets of IC contracts include price-caps, cost-plus, and some other more complex combinations of these, but no pure yardstick pairs of contracts, in which the price of the regulated good depends only on the announced production cost of the other. An interesting feature is that, regardless of the industry structure, incentive compatibility can be achieved by using one of the two announced costs only, which means that the double link between gas and electricity can "pass-through" regulation of one energy form to the other. This gives rise to an interesting extension in which only one of the two energies would be regulated, and the other left open to the competition of its substitute form of energy.

We have seen that these sets are quite different when there are two firms and when there is only one. This means that the regulators should actually care about industry structure when regulating both prices, because the performance of a given pair of regulatory contracts can vary a lot across industry structures: incentive compatibility of regulatory contracts is not robust to changes in industry structure.

Joint regulation not only performs better in general, but it is necessary when optimal contracts are not a pair of price-caps. When there are two regulators, each one of them exerts an externality on the other while maximising it own sector's welfare, and their distortionary effect is minimised only by joint decision-making. When miscoordination produces loss of incentive compatibility, regulators are even worse off and regulated outcomes can be worse than monopoly pricing. This provides strong justification to regulatory unification, beyond the standard argument of administrative cost reduction. In other words, because the benefits of incentive compatibility are important, and joint regulation is necessary to achieve it, sectoral regulators *should* merge.

The choice of the optimal type of contracts varies with substitutability and technical dependency, for all industry structures and institutional arrangements. Thus, regulators must pay careful attention to these parameters when choosing their instruments and the values of the coefficients that implement them. Welfare gains to regulation under incentive compatible contracts, with respect to the best unregulated outcome, are generally between 20% and 33% of global welfare. This means that incentive compatibility is not achieved at a great cost, and that on the contrary this kind of regulation allows for outcomes that are close to the welfare gains implied by perfect regulation.

As with regards to industry structure, the standard "double marginalization" arguments that plead in favour of firm integration in an unregulated setting with perfect information, no longer hold in the presence of informational asymmetries. We have shown that between the generally valid recommendation to allow and encourage firm mergers when there is no price regulation, and the indifference with regards to industry structure prevailing under perfect regulation, there is a whole range of different regulatory recommendations when it comes to merger desirability, that are closely related to the links existing between both industries and the relative importance of information uncertainties. Regulators and institutional planners should pay close attention to the nature and strength of these links, before being tempted to erroneously generalize the fairly intuitive full information results.

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Appendix A

Market Equilibrium in the Duopoly Case, with No Regulation

Equilibrium in a market with two separate, unregulated firms, is given by the intersection of the best response functions that are a solution of the individual maximization of each of the firms' profits, that is:

$$\max_{p_g} \Pi_g = (p_g - c_g)\cdot q_g + (p_g - c_g)\cdot q_t \qquad \text{(A.1)}$$

$$\max_{p_e} \Pi_e = (p_e - c_e)\cdot q_e \qquad \text{(A.2)}$$

By substituting demand functions (2) and cost functions into expressions (A.1) and (A.2), we can rewrite the profit functions as follows:

$$\Pi_g = \frac{\left[a(1-b)(1+k)-(k-b)p_e-(1-bk)p_g\right](p_g-\gamma)}{1-b^2} \qquad \text{(A.3)}$$

$$\Pi_e = \frac{\left[a(1-b)(1+k)-p_e+b\cdot p_g\right](p_e-k\cdot p_g-\varepsilon)}{1-b^2} \qquad \text{(A.4)}$$

Observe that, while the profits of the electricity firm are always concave in p_e, the profits of the gas firm will be concave in p_g only if $(1-b\,k)$ is positive. In fact, if electricity generation is heavily dependent on gas and substitution is easy, the gas firm will increase its price to the monopoly level, and this will result in no electricity production. Thus, we will restrict our analysis to cases in which:

$$(1-b\,k) > 0 \qquad \text{(A.5)}$$

If both profits are concave, equilibrium occurs at the intersection of the following best response functions:

$$p_g^*(p_e) = \frac{a+\gamma}{2} + \frac{(k-b)(a-p_e)}{2(1-b\cdot k)} \qquad \text{(A.6)}$$

$$p_e^*(p_g) = \frac{a+k\cdot p_g+\varepsilon}{2} - \frac{b(a-p_g)}{2} \qquad \text{(A.7)}$$

The best response functions are the sum of two terms: the "monopoly prices", similar to those found in expressions (9) and (10), and the "interaction effect" depending on the price chosen by the other firm. It is also interesting to notice that, due to the particular relationship existing between gas and electricity, the best response function of gas differs from the one found in standard analyses of duopolies in the literature[1], given the vertical dependence of

[1] Standard differentiated duopolist best response functions, with independent costs and linear demands, are given for comparison : $p_i^*(p_j) = \frac{a+c_i}{2} - \frac{b(a-p_j)}{2}$, $i \neq j$.

electricity generation on gas. The electricity firm, on the other hand, behaves in the standard way.

In fact, the best response function of the gas firm is decreasing in the price of electricity, whereas in the standard case both best response functions are always increasing.

Equilibrium prices are given by:

$$p_g^* = \frac{a(1-b)(2+b+k)+2(1-bk)\gamma-(k-b)\varepsilon}{k^2-b^2-4bk+4} \tag{A.8}$$

$$p_e^* = \frac{a(1-b)(2+b+k+k(k-b))+(1-bk)((b+k)\gamma+2\varepsilon)}{k^2-b^2-4bk+4} \tag{A.9}$$

And the corresponding equilibrium quantities are:

$$q_g^* = \frac{a(1-b)\left(k^2-k+2+\left(k^2-3k+1\right)b-kb^2\right)-(1-bk)\left(2-bk-k^2\right)\gamma+\left(b+k-2kb^2\right)\varepsilon}{(1-b)(1+b)\left(k^2-b^2-4bk+4\right)} \tag{A.10}$$

$$q_e^* = \frac{a(1-b)(2-k-2bk+b)-(1-bk)(k-b)\gamma+\left(b^2+bk-2\right)\varepsilon}{(1-b)(1+b)\left(k^2-b^2-4bk+4\right)} \tag{A.11}$$

It is straightforward to notice, given that electricity is, in total, costlier to produce than gas, that the most binding non-negativity constraint will be the one for the quantity of electricity. Sign analysis for this quantity is reported in Table A1.

Table A1. Sign study for the equilibrium quantities of the unregulated duopoly

Value of b in terms of k		0 $\frac{2-k}{2k-1}$	$\frac{1}{k}$	$\frac{\sqrt{k^2-8}-k}{2}$	$\frac{1+\sqrt{8k^2+1}}{4k}$	$\sqrt{5k^2+4}-2k$	$\frac{k^2-3k+1+\sqrt{\left(k^2-3k+1\right)^2+4k\left(k^2-k+2\right)}}{2k}$	1
q_g	Fixed coef in numerator	+	+	+	+	+	+	- (*)
	Coef of γ in numerator	-	-	+	-	-	-	-
	Coef of ε in numerator	+	+	+	+	-	-	-
	Denominator	+	+	+	+	+	- (***)	- (***)
q_e	Fixed coef in numerator	+ (**)	-	-	-	-	-	-
	Coef of γ in numerator	-	-	+	+	+	+	+
	Coef of ε in numerator	-	-	-	+	+	+	+
	Denominator	+	+	+	+	+	- (***)	- (***)

(*) this interval is empty if k>3/2.
(**) this interval is empty if k>2.
(***) this interval is empty if k>3.

The table shows that for the fixed part of this quantity (that is, the part not depending on γ or ε) to be positive, we need that $b<\frac{2-k}{2k-1}$.[2] If this were not the case, the fixed

[2] Another possibility is to have $-2k+\sqrt{5k^2+4}<b<1$, and big values of γ and ε relative to a, but this would imply that q_g would be negative, and it would mean that Π_g is convex in p_g.

coefficient would be negative, and unless a was small relative to γ and ε, q_e would be negative in equilibrium.

This means that in the two firm case there is a critical value of b, b_0^c, above which electricity would never be produced and the equilibrium prices and quantities would correspond to corner solutions (that is, monopoly pricing in gas and no electricity). We should keep this condition in mind when comparing results.

$$b_0^c = \frac{2-k}{2k-1} \tag{A.12}$$

Notice also that this condition imposes an upper bound k^{max} for k, because, as can be seen from Table A1, if $k > 2$ and a is large enough, q_e will always be negative. The larger the technical dependency, the larger the gas producer's advantage, and the lower the maximum b admitting non-negative electricity production.

$$k^{max} = 2 \tag{A.13}$$

Within these intervals for b and k, there is also a critical value of a, relative to γ and ε, for q_e to be positive. By rearranging expression (A.11) we find this critical value to be:

$$a_{\min}^{q_e>0} = \frac{(1-bk)(k-b)\gamma + ((1-bk)+(1-b^2))\varepsilon}{(1-b)(2(1-bk)-(k-b))} \tag{A.14}$$

Detailed Proof of Proposition 1.
("Consumer surplus is higher in the monopoly case than in the duopoly case.")
This proof will be carried out in two steps:

First, we will show that the difference of consumer surpluses can be written as a parabola that is convex in demand coefficient a. Therefore, the largest root of this parabola corresponds to the minimum value of a such that the difference of consumer surpluses is positive. We will calculate this root, that we call $a_{\min}^{\Delta S>0}$.

Second, we show that $a_{\min}^{\Delta S>0} < a_{\min}^{q_e>0}$, where $a_{\min}^{q_e>0}$ is the minimum value of a such that q_e will be positive. Therefore, the condition on a for q_e to be positive is sufficient for the difference of consumer surpluses to be positive.

Step 1. *Convexity of the difference of consumer surpluses in parameter a.*
Compute equilibrium consumer surplus in the monopoly case by using expressions (9) to (12) and in the duopoly case by using expressions (16) to (19) in the main text. Subtract the latter from the former, and observe that this expression ΔS can be written as a parabola in a, in which the coefficient of a^2 is equal to :

$$\frac{2(1-b)(k-b)(16+12b-b^3-4k-28bk-13b^2k+4k^2+7bk^2+10b^2k^2-k^3-2bk^3)}{8(1-b^2)(4-b^2-4bk+k^2)^2}, \tag{A.15}$$

which has the same sign than :

$$\left(16+12b-b^3-4k-28bk-13b^2k+4k^2+7bk^2+10b^2k^2-k^3-2bk^3\right), \tag{A.16}$$

because both *(1-b)* and *(k-b)* are positive, and so is the denominator.

We will now show that this expression is also positive in order to show that the coefficient of a^2 is positive.

We begin by rewriting expression (A.16) as a polynomial in b:

$$\left(\left(16-4k+4k^2-k^3\right)-b\left(k^3-7k^2+28k-12\right)+b^2(10k-13)k-b^3\right) \tag{A.17}$$

Observe that the coefficient of b^3 is negative, and so is the coefficient of b when $1 < k < 2$, so we can find a lower bound for expression (A.16) by replacing b by its maximum value, namely $\frac{2-k}{2k-1}$:

$$\left(\left(16-4k+4k^2-k^3\right)-\frac{2-k}{2k-1}\left(k^3-7k^2+28k-12\right)+b^2(10k-13)k-\left(\frac{2-k}{2k-1}\right)^3\right), \tag{A.18}$$

which can be rewritten :

$$\frac{-k\left[\left(1-b^2\right)(10k-13)(2k-1)^3-(k-1)\left(72(k-1)^3+192(k-1)^2+111(k-1)+30\right)\right]}{(2k-1)^3}. \tag{A.19}$$

In expression (A.19), the denominator is positive and the numerator has the opposite sign of the expression between brackets. We end this part of the proof by showing that this last expression is always negative.

If $k < 1.3$, then both terms are negative, so the whole expression is negative.

If, on the other hand, $k > 1.3$, then the whole expression is smaller than:

$$(10k-13)(2k-1)^3-(k-1)\left(72(k-1)^3+192(k-1)^2+111(k-1)+30\right), \tag{A.20}$$

which is equal to:

$$8k^4-128k^3+249k^2-184k+52, \tag{A.21}$$

so therefore always negative when $1.3 < k < 2$.

Thus, within the boundaries of existence of interior duopoly equilibria, the difference in consumer surpluses between the monopoly and duopoly cases is convex in demand parameter *a*.

Therefore, by finding the largest of the two roots of this parabola, we find a lower bound for a in order for ΔS to be positive. This root is equal to:

$$
\begin{aligned}
a_{\min}^{\Delta S>0} = &\left[\left(16+12b-b^3+4k-24bk-13b^2k-b^3k+8k^2-3bk^2+7b^2k^2+k^3-7bk^3+k^4\right)\gamma+\right.\\
&\left(16+12b-b^3+4k-20bk-13b^2k+4k^2-3bk^2+k^3\right)\varepsilon+\\
&\left.\left(k^2-b^2-4bk+4\right)\left((k-1)\gamma+\varepsilon\right)\sqrt{\left(1-b^2\right)\left(16-b^2-18bk+3k^2\right)}\right]/\\
&\left[2(1-b)\left(16+12b-b^3-4k-28bk-13b^2k+4k^2+7bk^2+10b^2k^2-k^3-2bk^3\right)\right] \qquad \text{(A.22)}
\end{aligned}
$$

Step 2. *Comparison of* $a_{\min}^{\Delta S>0}$ *and* $a_{\min}^{q_e>0}$.

Define $\tau=\sqrt{\left(1-b^2\right)\left(16-b^2-18bk+3k^2\right)}$, and calculate $a_{\min}^{q_e>0}-a_{\min}^{\Delta S>0}$: (A.23)

$$
-\frac{\left(k^2-b^2-4bk+4\right)\left((k-1)\gamma+\varepsilon\right)\left(b^3(2k+1)-b^2\left(2k^2-10k+1\right)-b\left(3k^2+2k(\tau-5)+10-\tau\right)-\left(k^2-2k+8+(k-2)\tau\right)\right)}{2(1-b)\left(2-k-b(2k-1)\right)\left(16+12b-b^3-4k-28bk-13b^2k+4k^2+7bk^2+10b^2k^2-k^3-2bk^3\right)}
$$

Observe that all factors in the denominator are positive, because $b<\dfrac{2-k}{2k-1}<1$, and because the last term is the same as in expression (A.16) that we have already shown to be positive.

Therefore, expression (A.23) has the opposite sign of the following expression:

$$
b^3(2k+1)-b^2\left(2k^2-10k+1\right)-b\left(3k^2-10(k-1)-\tau(2k-1)\right)-\left(k^2-2k+8+(k-2)\tau\right) \qquad \text{(A.24)}
$$

which is always negative, as can be seen in the following figure:

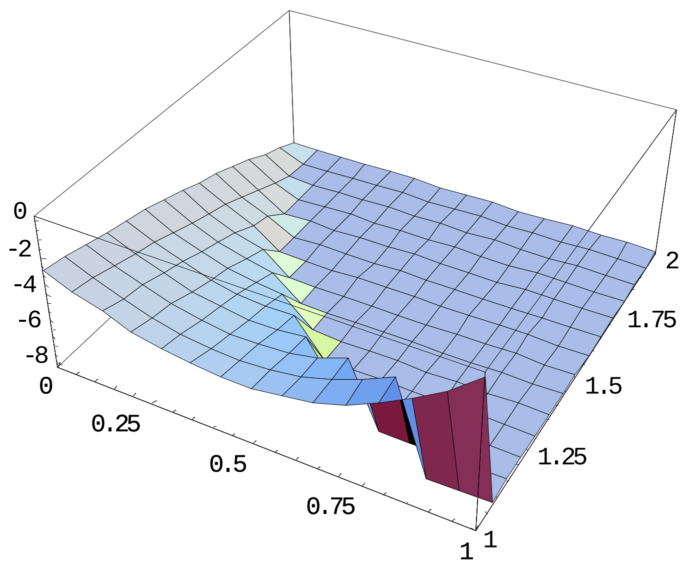


Figure A1. Numerical evaluation of expression (A.24)

We conclude that $a_{\min}^{q_e>0}-a_{\min}^{\Delta S>0}>0$, and thus if a is large enough for q_e to be positive, then ΔS will necessarily be positive as well. *Q.E.D.*

Figure A2 below shows an example of the difference in consumer surplus for given values of b and k within the boundaries of existence of interior duopoly equilibria, as a function of cost parameters γ and ε.

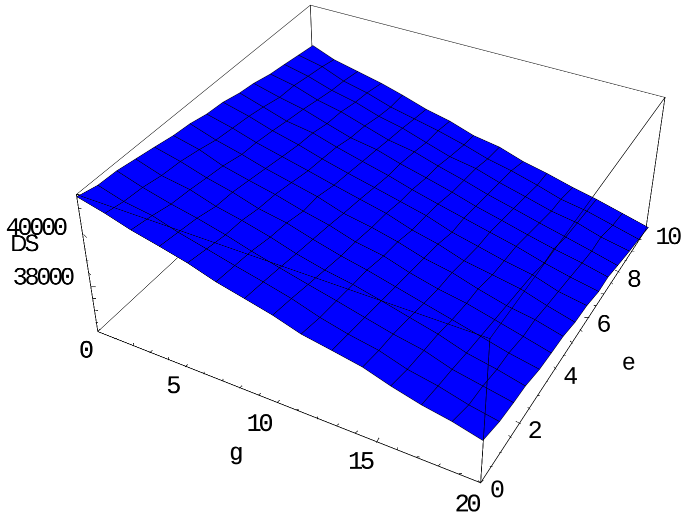


Figure A2. Difference in consumer surplus between the two firm and the one firm equilibria. Case within the boundary of existence of duopoly equilibrium: k = 1.3, b = 0.4, a = 600, $\gamma \in [0,20]$, $\varepsilon \in [0,10]$

Appendix B: Incentive Compatibility with Two Firms

I. **Private cost announcements** (Figure 3A)

If $p_g(\hat{\gamma})$ and $p_e(\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator(s) for gas and electricity as a function of the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the payoff functions of the gas and electricity firms are respectively given by:

$$\Pi_g(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}), \gamma) = \left[q_g(p_g(\hat{\gamma}), p_e(\hat{\varepsilon})) + k \cdot q_e(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}))\right](p_g(\hat{\gamma}) - \gamma) \qquad \text{(B.I.1)}$$

$$\Pi_e(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}), \varepsilon) = q_e(p_g(\hat{\gamma}), p_e(\hat{\varepsilon})) \cdot (p_e(\hat{\varepsilon}) - k \cdot p_g(\hat{\gamma}) - \varepsilon) \qquad \text{(B.I.2)}$$

Each firm will try to maximize its own expected profit, that is:

$$E_\varepsilon[\Pi_g] = \int_0^E \left[q_g\left(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}(\varepsilon))\right) + k \cdot q_e\left(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}(\varepsilon))\right)\right] \cdot \left(p_g(\hat{\gamma}) - \gamma\right) f(\varepsilon) d\varepsilon \qquad \text{(B.I.3)}$$

$$E_\gamma[\Pi_e] = \int_0^G q_e\left(p_g(\hat{\gamma}(\gamma)), p_e(\hat{\varepsilon})\right) \cdot \left(p_e(\hat{\varepsilon}) - k \cdot p_g(\hat{\gamma}(\gamma)) - \varepsilon\right) g(\gamma) d\gamma \qquad \text{(B.I.4)}$$

We begin our search by imposing incentive compatibility constraints and thus looking for truthful equilibria.

By definition, truthful equilibria exist when it is optimal for each firm to announce its true type, given that the competing firm announces its own true type. So, in order to find a truthful equilibrium, we can impose $\hat{\varepsilon} = \varepsilon$ when solving for the gas firm, and $\hat{\gamma} = \gamma$ when solving for the electricity firm. Expected payoffs (B.I.3) and (B.I.4) become respectively:

$$E_\varepsilon[\Pi_g] = \int_0^E \left[q_g(p_g(\hat{\gamma}), p_e(\varepsilon)) + k \cdot q_e(p_g(\hat{\gamma}), p_e(\varepsilon))\right] \cdot (p_g(\hat{\gamma}) - \gamma) f(\varepsilon) d\varepsilon \qquad \text{(B.I.5)}$$

$$E_\gamma[\Pi_e] = \int_0^G q_e(p_g(\gamma), p_e(\hat{\varepsilon})) \cdot (p_e(\hat{\varepsilon}) - k \cdot p_g(\gamma) - \varepsilon) g(\gamma) d\gamma \qquad \text{(B.I.6)}$$

Incentive compatibility requires that announcing the true value of the private cost parameter be an optimal choice, which means that the first partial derivatives of these expressions with respect to the announced parameter must be equal to zero when the announcement is equal to the true value of the parameter. For this to be true, we need that this equality holds for all values of the integration variable, and thus we obtain:

$$\tfrac{\partial p_g}{\partial \hat{\gamma}}\left[a(1-b)(1+k) + (b-k)p_e + (b \cdot k - 1)(2p_g - \gamma)\right] \Big/_{\hat{\gamma} = \gamma} = 0, \forall \varepsilon \qquad (\text{IC}_g)$$

$$\tfrac{\partial p_e}{\partial \hat{\varepsilon}}\left[2p_e - \varepsilon - (b+k)p_g - a(1-b)\right] \Big/_{\hat{\varepsilon} = \varepsilon} = 0, \forall \gamma \qquad (\text{IC}_e)$$

In a truthful equilibrium, both incentive compatibility constraints must be simultaneously satisfied, and this will happen in any of the four following combinations:

(a) $\frac{\partial p_g}{\partial \hat{\gamma}} = 0$ (which means constant gas prices) and $\frac{\partial p_e}{\partial \hat{\varepsilon}} = 0$ (which means constant electricity prices), so a pair of fixed prices contracts; or

(b) $\frac{\partial p_g}{\partial \hat{\gamma}} = 0$ (which means constant gas prices) and $p_e(\hat{\varepsilon}) = \dfrac{a(1-b) + p_g(k+b) + \hat{\varepsilon}}{2}$, which is a linear regulatory contract independent of γ, with standard coefficient ½ for $\hat{\varepsilon}$; or

(c) $\frac{\partial p_e}{\partial \hat{\varepsilon}} = 0$ (which means constant electricity prices) and $p_g(\hat{\gamma}) = \dfrac{a(1-b)(k+1) - p_e(k-b) + (1-bk)\hat{\gamma}}{2(1-bk)}$, which is a linear regulatory contract independent of ε, with standard coefficient ½ for $\hat{\gamma}$; or

(d) $\frac{\partial p_g}{\partial \hat{\gamma}} \neq 0$, $\frac{\partial p_e}{\partial \hat{\varepsilon}} \neq 0$, and $\begin{cases} a(1-b)(1+k) + (b-k)p_e(\varepsilon) + (b \cdot k - 1)(2p_g(\gamma) - \gamma) = 0 \\ 2p_e(\varepsilon) - \varepsilon - (b+k)p_g(\gamma) - a(1-b) = 0 \end{cases}$.

The second equation in this system allows to easily state p_e in terms of p_g, so by replacing this expression into the first equation we obtain $p_g(\gamma) = \dfrac{2(bk-1)\gamma - a(1-b)(2+k+b) + (k-b)\varepsilon}{(b+k)(b-k) + 4(bk-1)}$, which cannot be simultaneously verified for all values of ε. A similar analysis shows naturally that in this case p_e cannot be independent from $\hat{\gamma}$.

So the only way to achieve incentive compatibility in this context is by choosing a fixed price for one of the goods, and a linear incentive compatible regulatory contract for the other (or choosing two fixed prices).

If they both try to implement regulatory contracts (other than fixed prices) simultaneously, incentive compatibility will be lost. The regulators must coordinate their action and agree upon which of the two goods is going to have a fixed price, and which will have a linear regulatory contract.

II. <u>**Public cost announcements** (Figure 4A)</u>

If $p_g(\hat{\gamma}, \hat{\varepsilon})$ and $p_e(\hat{\gamma}, \hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator(s) for gas and electricity as a function of the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the profits of the gas and electricity firms are respectively given by:

$$\Pi_g(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon}), \gamma) = \lfloor q_g(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon})) + k \cdot q_e(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon})) \rfloor (p_g(\hat{\gamma},\hat{\varepsilon}) - \gamma) \quad \text{(i)}$$

$$\Pi_e(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon}), \varepsilon) = q_e(p_g(\hat{\gamma},\hat{\varepsilon}), p_e(\hat{\gamma},\hat{\varepsilon})) \cdot (p_e(\hat{\gamma},\hat{\varepsilon}) - k \cdot p_g(\hat{\gamma},\hat{\varepsilon}) - \varepsilon) \quad \text{(ii)}$$

Each firm will try to maximize its own expected payoff, that is:

$$E_\varepsilon[\Pi_g] = \int_0^E [q_g(p_g(\hat{\gamma},\hat{\varepsilon}(\varepsilon)), p_e(\hat{\gamma},\hat{\varepsilon}(\varepsilon))) + k\cdot q_e(p_g(\hat{\gamma},\hat{\varepsilon}(\varepsilon)), p_e(\hat{\gamma},\hat{\varepsilon}(\varepsilon)))]\cdot(p_g(\hat{\gamma},\hat{\varepsilon}(\varepsilon)) - \gamma) f(\varepsilon) d\varepsilon \quad \text{(iii)}$$

$$E_\gamma[\Pi_e] = \int_0^G q_e(p_g(\hat{\gamma}(\gamma),\hat{\varepsilon}), p_e(\hat{\gamma}(\gamma),\hat{\varepsilon}))\cdot(p_e(\hat{\gamma}(\gamma),\hat{\varepsilon}) - k\cdot p_g(\hat{\gamma}(\gamma),\hat{\varepsilon}) - \varepsilon) g(\gamma) d\gamma \quad \text{(iv)}$$

We begin our search by imposing incentive compatibility constraints and thus looking for truthful equilibria.

By definition, truthful equilibria exist when it is optimal for each firm to announce its true type, given that the competing firm announces its true type. So, in order to find a truthful equilibrium, we can impose $\hat{\varepsilon} = \varepsilon$ when solving for the gas firm, and $\hat{\gamma} = \gamma$ when solving for the electricity firm. Expected payoffs (iii) and (iv) become respectively:

$$E_\varepsilon[\Pi_g] = \int_0^E [q_g(p_g(\hat{\gamma},\varepsilon), p_e(\hat{\gamma},\varepsilon)) + k\cdot q_e(p_g(\hat{\gamma},\varepsilon), p_e(\hat{\gamma},\varepsilon))]\cdot(p_g(\hat{\gamma},\varepsilon) - \gamma) f(\varepsilon) d\varepsilon \quad \text{(v)}$$

$$E_\gamma[\Pi_e] = \int_0^G q_e(p_g(\gamma,\hat{\varepsilon}), p_e(\gamma,\hat{\varepsilon}))\cdot(p_e(\gamma,\hat{\varepsilon}) - k\cdot p_g(\gamma,\hat{\varepsilon}) - \varepsilon) g(\gamma) d\gamma \quad \text{(vi)}$$

Incentive compatibility requires that announcing the true value of the private cost parameter be an optimal choice, which means that the first partial derivatives of these expressions with respect to the announced parameter must be equal to zero when the announce is equal to the true value of the parameter. For this to be true, we need that this equality holds for all values of the integration variable, and thus, by replacing the demand functions, we obtain:

$$[a(1-b)(1+k) + (b-k)p_e + (b\cdot k - 1)p_g]\tfrac{\partial p_g}{\partial \gamma} + (p_g - \gamma)\left[(b-k)\tfrac{\partial p_e}{\partial \gamma} + (b\cdot k - 1)\tfrac{\partial p_g}{\partial \gamma}\right] = 0 \quad (\mathrm{IC_g})$$

$$[p_e - \varepsilon - k\cdot p_g]\left(\tfrac{\partial p_e}{\partial \varepsilon} - b\tfrac{\partial p_g}{\partial \varepsilon}\right) + [p_e - a(1-b) - b\cdot p_g]\left(\tfrac{\partial p_e}{\partial \varepsilon} - k\tfrac{\partial p_g}{\partial \varepsilon}\right) = 0 \quad (\mathrm{IC_e})$$

And by restricting our attention to linear contract we get:

$$[a(1-b)(1+k) + (b-k)(X + Y\hat{\gamma} + Z\hat{\varepsilon}) + (b\cdot k - 1)(A + B\hat{\gamma} + C\hat{\varepsilon})]B + (A + (B-1)\hat{\gamma} + C\hat{\varepsilon})[(b-k)Y + (b\cdot k - 1)B] = 0$$
$$[X + Y\hat{\gamma} + (Z-1)\hat{\varepsilon} - k\cdot(A + B\hat{\gamma} + C\hat{\varepsilon})](Z - bC) + [X + Y\hat{\gamma} + Z\hat{\varepsilon} - a(1-b) - b\cdot(A + B\hat{\gamma} + C\hat{\varepsilon})](Z - kC) = 0$$

As we know that these equalities must hold for every possible pair of values of γ and ε, by equating the coefficients of these variables in both polynomials we obtain the following system of equations (for notational simplicity, we take $d = a(1-b)(1+k)$):

$$\begin{cases} dB + (b-k)(BX + AY) + 2AB(bk-1) = 0 \\ [(b-k)Y + (bk-1)B](2B-1) = 0 \\ (b-k)(BZ + CY) + 2CB(bk-1) = 0 \\ (X - kA)(Z - bC) + (X - a(1-b) - bA)(Z - kC) = 0 \\ (Y - kB)(Z - bC) + (Y - bB)(Z - kC) = 0 \\ [2(Z - kC) - 1](Z - bC) = 0 \end{cases}$$

This system has four "families" of generic solutions, three of which have two degrees of freedom, as one would expect from classic regulation models, that can be used by the regulator to restrain firms' profits. The last "family" of solutions is fully specified.

We will now study them one by one.

B.II.1 Fixed price contracts

This solution corresponds to taking $B = Y = C = Z = 0$, which naturally verifies (IC_g) and (IC_e) because all partial derivatives are equal to zero. Contract coefficients A and X can be freely chosen by the regulator, and should be set so as to maximize global welfare while respecting the individual rationality and non negativity constraints.

Obviously this solution lacks any truth-inducing power, because the price outcome is arbitrarily set by the regulator, independently of firms' cost announcements.

The regulator should decide whether to ensure production of both goods in the "worst case scenario", that is, to guarantee the satisfaction of both individual rationality constraints. If she decided to do so, she should set p_g and p_e equal to marginal prices when firms are most inefficient, which means $p_g = G$ and $p_e = E + k\,G$. If E and G are large (relative to a), these prices may be above monopoly prices (so very inefficient). On the other hand, if E and G are relatively small, "worst case" fixed pricing might still be a good proxy to marginal cost pricing, so actually quite efficient.

If a is large enough for both equilibrium quantities to be positive, profits will also be positive (because prices were set in order to ensure non-negative margins).

B.II.2 Contracts independent of γ

These contracts correspond to those in which $B = Y = 0$. The announcement made by the gas firm is completely irrelevant for the pricing decisions, so (IC_g) is trivially satisfied. There are no incentives for the gas firm to announce its true type, nor to lie, because its behaviour will not influence the outcome.

This leaves us with only two equations in the system and the two corresponding "subfamilies" of solutions, each with two degrees of freedom. The regulator can freely choose contract coefficients A and C, that is, the fixed and variable coefficients of the gas contract, but is then forced to set the coefficients of the electricity contract according to one of the following rules:

(B.II.1) $\begin{cases} X = a(1-b) + bA \\ Z = bC \end{cases}$

It can easily be seen that this pricing rule is equivalent to choosing an arbitrary gas contract and then setting the electricity price to $p_e^r = a(1-b) + bp_g^r$. Thus, we can compute the resulting demand for electricity and see that it is always nil:

$$q_e^r = \frac{a}{1+b} - \frac{1}{1-b^2} p_e^r + b\frac{1}{1-b^2} p_g^r = \frac{a(1-b) - a(1-b) - bp_g^r + bp_g^r}{1-b^2} = 0$$

We can then disregard this solution to the incentive compatibility constraints, as it will always result in no electricity production. This form of regulation is equivalent to the standard regulation model with linear demand, for in fact, demand for gas is equal to $q_g^r = a - p_g^r$.

Notice that the electricity firm has no incentives for truth-telling in this setting. As noticed by Martimort and Stole (2001), the "truthful when indifferent" hypothesis in this context might be too strong, because the cost announced by the electricity firm will determine the profits of the gas firm. Chances are that the strategic value of the cost announcement would be exploited (either to achieve monopoly pricing if side payments or collusion are possible, or to reduce the rival's profits if competition is fierce).

(B.II.2.1) $$\begin{cases} X = \dfrac{a(1-b) + A(b+k+2k(k-b)C)}{2(1+(k-b)C)}, C \neq \dfrac{1}{b-k} \\ Z = kC + \frac{1}{2} \end{cases}$$

In the formula for computing *Z*, we can easily recognize the ½ found in standard regulation models with linear demands, corrected exactly by the impact that a variation in the electricity cost parameter would have on the price of gas.

We can compute the profit of the electricity firm of type ε announcing $\hat{\varepsilon}$ under this contract scheme:

$$\pi_e(\varepsilon;\hat{\varepsilon}) = \frac{[1+2C(k-b)][A(k-b)-a(1-b)+(1+C(k-b))(2\varepsilon-\hat{\varepsilon})][A(k-b)-a(1-b)+(1+C(k-b))\hat{\varepsilon}]}{4(1-b^2)(1+C(k-b))^2}$$

For the contract to be incentive compatible, the second order condition for the profit of the electricity firm (with respect to the announced cost parameter) should be satisfied, which means

$$\frac{\partial^2 \pi_e(\varepsilon;\hat{\varepsilon})}{\partial \hat{\varepsilon}^2} = -\frac{1+2C(k-b)}{2(1-b^2)} < 0 \Leftrightarrow C > -\frac{1}{2(k-b)}. \qquad \text{(B.II.1)}$$

Notice that this is exactly the same condition needed for $\pi_e(\varepsilon;\varepsilon)$ to be positive, so it should seem natural that the regulator should restrict herself to values of *C* greater than $-\dfrac{1}{2(k-b)}$ only.

Observe that if inequality (B.II.1) holds and contract coefficient *A* is positive, then *X* is also always positive.

The equilibrium demanded quantity of electricity is given by:

$$q_e = \frac{[1+2C(k-b)][a(1-b)-A(k-b)-(1+C(k-b))\varepsilon]}{2(1-b^2)(1+C(k-b))}$$

Notice that when (B.II.1) holds, both the denominator and the first term in the numerator are positive, so the whole expression will be positive iff $a(1-b)-A(k-b)-(1+C(k-b))\varepsilon > 0$. Given that $1+C(k-b) > 0$ if (B.II.1) holds, this expression is decreasing in ε, so the inequality must be satisfied for the largest possible ε, that is, *E*. Thus we obtain an upper bound for *C*, which is decreasing in *A*:

$$C \leq \frac{a(1-b)-(k-b)A-E}{(k-b)E} \tag{B.II.2}$$

It is straightforward to check that condition (B.II.2) is also a necessary and sufficient condition for the unit margin of electricity to be positive.

So now we have both upper and lower bounds for C as a function of A and all model parameters:

$$-\frac{1}{2(k-b)} \leq C \leq \frac{a(1-b)-(k-b)A-E}{(k-b)E} \tag{B.II.3}$$

For this interval not to be empty, we need:

$$A \leq a\frac{1-b}{k-b} - \frac{1}{2(k-b)}E \tag{B.II.4}$$

Let's take a look at the gas firm's profits:

$$\pi_g(\hat{\varepsilon}) = \frac{(A+C\hat{\varepsilon}-\gamma)}{2(1-b^2)(1+C(k-b))}[a(1-b)(b+k+2+2C(k-b)(k+1)) - A[1-b^2+(1-2bk+k^2)(1+C(k-b))] +$$
$$+[k-b+C[(k-b)^2+2(1-2bk+k^2)(1+C(k-b))]]\hat{\varepsilon}]$$

The denominator of this expression is always positive if condition (B.II.1) holds, so the sign of $\pi_g(\hat{\varepsilon})$ is given by the sign of the numerator.

For the unit margin not to be negative for all possible values of γ and ε, we need $A \geq G$ if C is positive, or $A \geq G - CE$ if C is negative.

Non-negativity of the produced quantity of gas imposes binding constraints on contract coefficients only if $1-bk>0$, and $C > \frac{b}{2(1-bk)}$.

The regulator should choose contract coefficients A and C ex ante so as to maximize expected global welfare subject to the individual rationality constraints for both firms and non negativity constraints for produced quantities, that is,

$$\underset{A,C}{Max} \quad V_2(A,C) = E[W_2(A,C,\gamma,\varepsilon)] = \int_0^E \int_0^G W_2(A,C,\gamma,\varepsilon)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$
$$s.t. \begin{cases} \pi_g(A,C,\gamma,\varepsilon) \geq 0, \forall\gamma, \forall\varepsilon \\ \pi_e(A,C,\varepsilon) \geq 0, \forall\varepsilon \\ q_g(A,C,\varepsilon) \geq 0, \forall\varepsilon \\ q_e(A,C,\varepsilon) \geq 0, \forall\varepsilon \end{cases}$$

Unfortunately, this constrained optimisation problem is rather complicated, because of the complex form taken by the objective function $W_2(A,C,\gamma,\varepsilon)$ and all four constraints, so no general solution can be given in the general case, even with uniform cost distribution functions. In fact, the solution to this constrained maximisation problem requires solving a fifth degree equation for one of the contract coefficients. There is thus no explicit algebraic form for the ex ante optimal values, and we are left with numerical comparison only.

The following figure illustrates expected global welfare for optimally tuned contracts of this nature (i.e. contracts that solve the maximization problem above) as a function of *b* and *k* with uniform cost distributions, for selected values of *a*, *G* and *E* that we use for benchmarks. The "hole" in expected welfare corresponds to areas where the solution set is empty (that is, where all ex ante individual rationality constraints cannot be simultaneously verified).

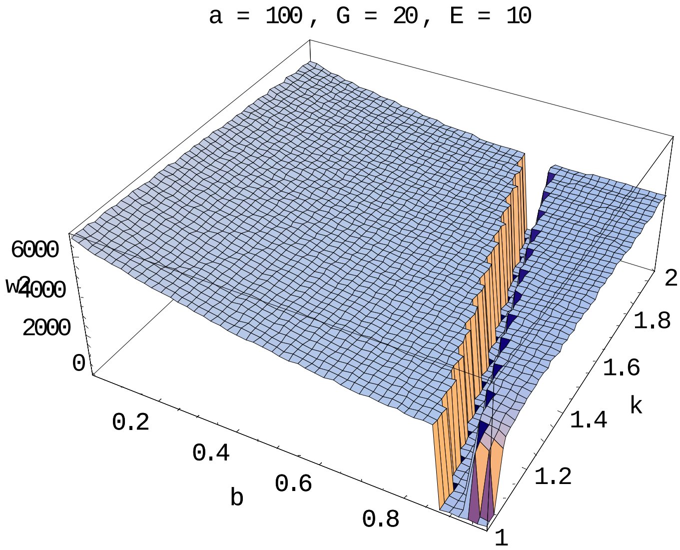


Figure B2. Expected welfare with optimal epsilon contingent contracts

(B.II.2.2) $$\begin{cases} C = -\dfrac{1}{k-b} \\ A = \dfrac{a(1-b)}{k-b} \\ Z = -\dfrac{k+b}{2(k-b)} \end{cases}$$

This solution corresponds to the degenerate case of solution **(B.II.1)**, that is, the case when the value of *X* in the previous solution is undetermined. There is only one degree of freedom in this solution, given by the arbitrary value of *X*.

Observe that both prices are decreasing on $\hat{\varepsilon}$, and notice that in this case the second derivative of the electricity firm's profits is equal to $\dfrac{\partial^2 \pi_e(\varepsilon;\hat{\varepsilon})}{\partial \hat{\varepsilon}^2} = \dfrac{1}{2(1-b^2)}$, which is always positive (as one should expect from the condition on *C* derived in the preceding section), so this solution corresponds in fact to a minimum in the profits, and the firm has no interest in revealing its true cost.

As this contract is not incentive compatible, the electricity firm will always report either the lowest or the highest possible cost, which means that this contract has no more incentive power than fixed price contracts described in Section B.II.1.

B.II.3 Contracts independent of ε

These contracts correspond to those in which $C = Z = 0$. Here, it is the announcement made by the electricity firm that is completely irrelevant for the pricing decisions, so (IC_e) is trivially satisfied. There are no incentives for the electricity firm to announce its true type, nor to lie, because its behaviour will not influence the outcome.

We are also left with only two equations in the system and their two corresponding "subfamilies" of solutions, each with two degrees of freedom.

The regulator can always freely choose the contract coefficient A, that is, the fixed component of the gas contract, but is then forced to set the other coefficients of the electricity contract according to one of the following set of rules:

(B.II.3.1) $$\begin{cases} B \neq 0; B \neq \frac{1}{2} \\ Y = \dfrac{1-b\cdot k}{b-k}B \\ X = \dfrac{A(1-b\cdot k)-d}{b-k} \end{cases}$$

It can easily be seen that this pricing rule is equivalent to choosing an arbitrary gas contract (but with $B \neq 0$ and $B \neq \frac{1}{2}$), and then setting the electricity price to $p_e^r = \dfrac{d}{k-b} + \dfrac{bk-1}{k-b}p_g^r$. Thus, we can compute the resulting demands for electricity and gas, and see that they are differently signed, which is incompatible with simultaneous production of both goods:

$$q_g = \frac{a(1-b) - p_g^r + b\left(\dfrac{d}{k-b} + \dfrac{bk-1}{k-b}p_g^r\right)}{1-b^2} = \frac{a(1-b)(k-b) - p_g^r(k-b) + bd + b(bk-1)p_g^r}{(1-b^2)(k-b)} = \frac{k(a-p_g^r)}{k-b}$$

$$q_e = \frac{a(1-b) - \left(\dfrac{d}{k-b} + \dfrac{bk-1}{k-b}p_g^r\right) + bp_g^r}{1-b^2} = \frac{a(1-b)(k-b) - (bk-1)p_g^r - d + b(k-b)p_g^r}{(1-b^2)(k-b)} = \frac{p_g^r - a}{k-b}$$

We can then disregard this solution to the incentive compatibility constraints, as it will always result in consumption of only one of the two products.

(B.II.3.2) $\begin{cases} B = \frac{1}{2} \\ X = \dfrac{2A(b \cdot k - 1 - (k-b)Y) + d}{k-b} \end{cases}$

The value of coefficient B is the familiar ½ found in standard regulation models with linear demands. The price of gas should increase only by half the amount of the announcement made by the gas firm.

Equilibrium quantities and profits are given by:

$$q_g = \frac{2a(1-b^2)k - 2A((1-b^2)k + (1-bk)b + 2Y(k-b)b) + (k-b)(2Yb-1)\hat{\gamma}}{2(1-b^2)(k-b)}$$

$$q_e = \frac{-2a(1-b^2) + 2A((1-b^2) + (1-bk) + 2Y(k-b)) - (k-b)(2Y-b)\hat{\gamma}}{2(1-b^2)(k-b)}$$

$$\pi_g = \frac{(1-bk+2Y(k-b))(2A-\hat{\gamma})(2A-2\gamma+\hat{\gamma})}{4(1-b^2)}$$

$$\pi_e = -q_e \cdot \left(q_e + \frac{(k-b)\left[-2a(1-b) + 2A(k-b) + (k-b)\gamma + 2\varepsilon\right]}{2(1-b^2)(k-b)} \right)$$

In order to provide the right truth-telling incentives, the regulatory contract must satisfy the second order optimality condition for the gas firm, that is

$$\frac{\partial^2 \pi_g(\hat{\gamma},\gamma)}{\partial \hat{\gamma}^2} \leq 0 \Leftrightarrow \frac{bk-1-2Y(k-b)}{2(1-b^2)} \leq 0 \Leftrightarrow Y \geq \frac{bk-1}{2(k-b)}$$

Notice that this condition also ensures that the regulated equilibrium profit of the gas firm will be non-negative.

In order to ensure that the unit margin of the gas producer is non-negative, we require that $A + \frac{\hat{\gamma}}{2} - \gamma \geq 0$, which is equivalent to $A \geq \frac{\gamma}{2}, \forall \gamma$ if the mechanism induces truth-telling. So we actually require $A \geq \frac{G}{2}$.

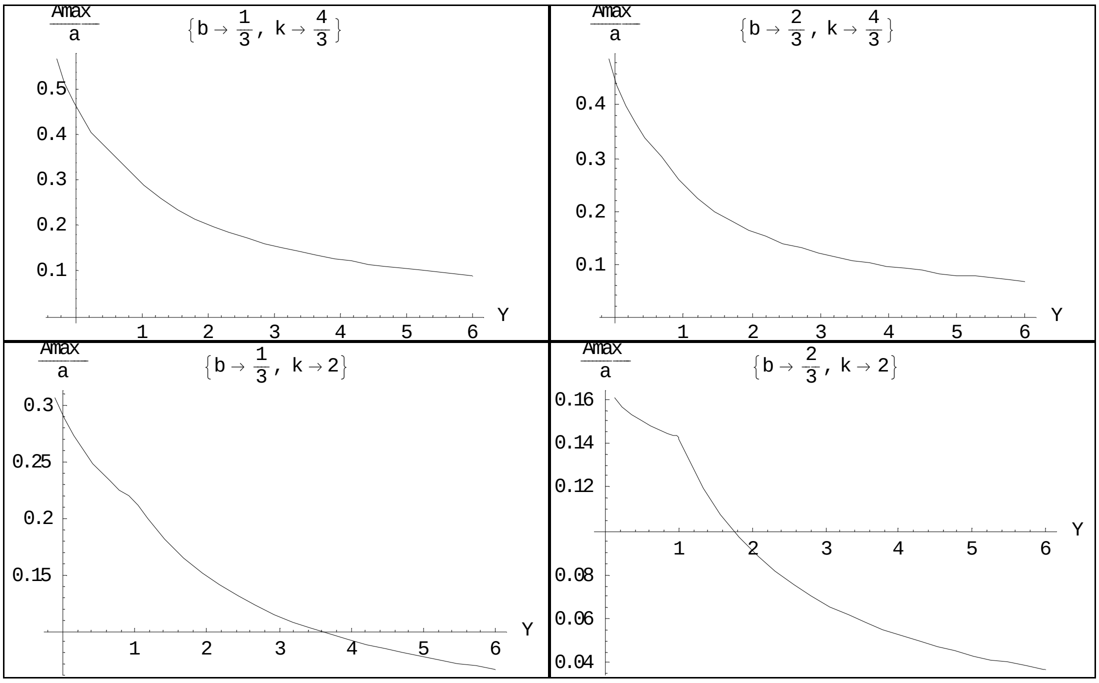
Likewise, for the unit margin of electricity not to become negative, the regulator must ensure that $(X - kA) + (Y - \frac{k}{2})\hat{\gamma} - \varepsilon \geq 0, \forall \hat{\gamma}, \forall \varepsilon$, which in the « worst case scenario » means that this constraint must be then satisfied by $\varepsilon = E$ in all cases, and by $\gamma = G$ if $k - 2Y > 0$, or $\gamma = 0$ if not. We thus obtain the following additional constraints for A:

$$A \leq \frac{2d - 2(k-b)E - (k-b)(k-2Y)G}{2\left[(1-b^2) + (1-bk) + 2(k-b)Y + (k-b)^2\right]}, \text{ if } Y < \frac{k}{2}, \text{ or}$$

$$A \leq \frac{2d - 2(k-b)E}{2\left[(1-b^2) + (1-bk) + 2(k-b)Y + (k-b)^2\right]}, \text{ if } \frac{k}{2} \leq Y.$$

Observe that the numerators in both expressions are always positive in the relevant intervals for Y, and that the condition $A \le \frac{d}{\left[(1-b^2)+(1-bk)+2(k-b)Y+(k-b)^2\right]}$ is necessary for $\pi_e \ge 0$.

The upper bound for A as a fraction of a and a function of Y, for specific values of b and k is illustrated in the following graphics.



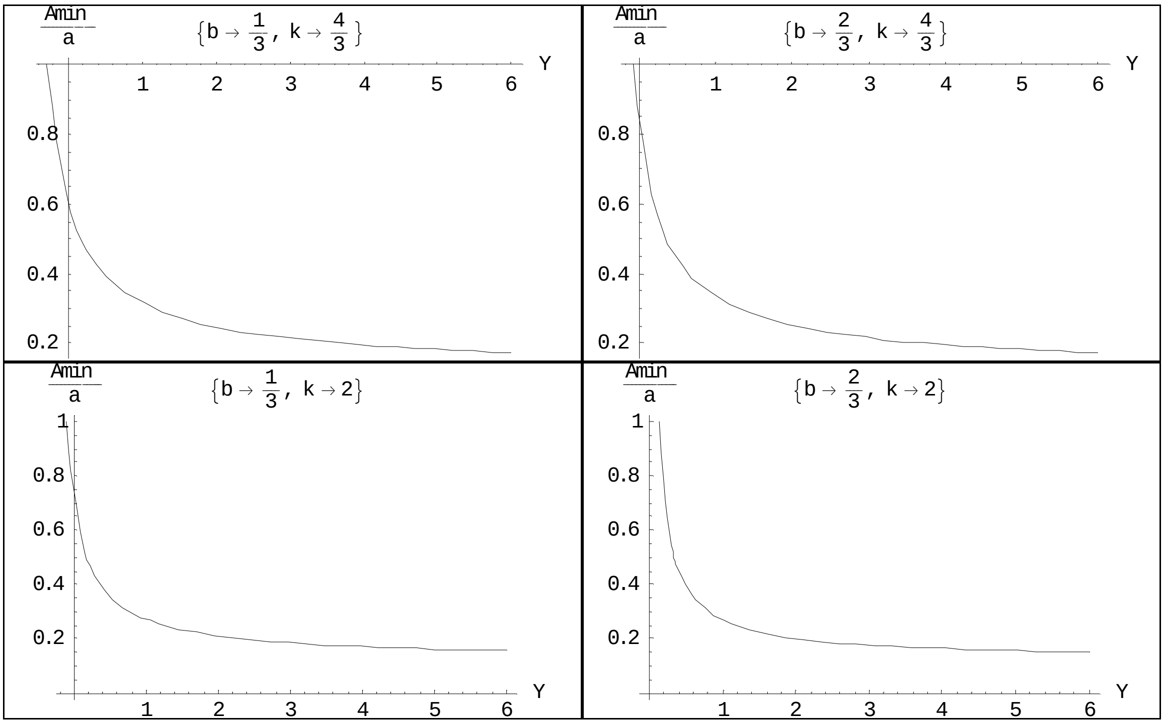
The non-negativity constraint for the demanded quantity of electricity yields:

$$A \ge \frac{a(1-b^2)}{\left[(1-b^2)+(1-bk)+2(k-b)Y\right]} \text{ if } \frac{bk-1}{2(k-b)} \le Y \le \frac{b}{2} \text{ , or}$$

$$A \ge \frac{a(1-b^2)}{\left[(1-b^2)+(1-bk)+2(k-b)Y\right]} + \frac{(k-b)(2Y-b)G}{2\left[(1-b^2)+(1-bk)+2(k-b)Y\right]} \text{ if } \frac{b}{2} < Y .$$

Observe that the numerators in both expressions are always positive in the relevant intervals for Y, and that the condition $A \ge \frac{a(1-b^2)}{\left[(1-b^2)+(1-bk)+2(k-b)Y\right]}$ is necessary for $q_e \ge 0$. Observe also that the lower bound for A decreases from a to $\frac{a}{2}$ when Y increases from $\frac{bk-1}{2(k-b)}$ to $\frac{b}{2}$, and then goes on decreasing more slowly.

The lower bound for A as a fraction of a and a function of Y, for specific values of b and k is illustrated in the following graphics.



Proposition B.II.3.b.

If $Y < \frac{1}{2} - \frac{1-bk}{k-b}$, then there are no values for A such that both $q_e \geq 0$ and $\pi_e \geq 0$.

Proof

By combining the two previously determined necessary conditions for A we get:

$$\frac{a(1-b^2)}{\left[(1-b^2)+(1-bk)+2(k-b)Y\right]} \leq A \leq \frac{d}{\left[(1-b^2)+(1-bk)+2(k-b)Y+(k-b)^2\right]}$$

So for the interval not to be empty, that is, for the lower bound of A to be inferior to its upper bound, we need:

$$\frac{a(1-b^2)}{\left[(1-b^2)+(1-bk)+2(k-b)Y\right]} \leq \frac{d}{\left[(1-b^2)+(1-bk)+2(k-b)Y+(k-b)^2\right]}$$

$$\Leftrightarrow (1+b)\left[(1-b^2)+(1-bk)+2(k-b)Y+(k-b)^2\right] \leq (1+k)\left[(1-b^2)+(1-bk)+2(k-b)Y\right]$$

$$\Leftrightarrow (1+b)\left[(1-b^2)+(1-bk)+2(k-b)Y\right] + (1+b)(k-b)^2 \leq (1+k)\left[(1-b^2)+(1-bk)+2(k-b)Y\right]$$

$$\Leftrightarrow (1+b)(k-b)^2 \leq (k-b)\left[(1-b^2)+(1-bk)+2(k-b)Y\right] \Leftrightarrow (1+b)(k-b) \leq \left[(1-b^2)+(1-bk)+2(k-b)Y\right]$$

$$\Leftrightarrow (1-2Y)(k-b) \leq 2(1-bk) \Leftrightarrow Y \geq \frac{(k-b)-2(1-bk)}{2(k-b)} = \frac{1}{2} - \frac{1-bk}{k-b}.\ Q.E.D.$$

Notice that this does still not mean that there will be solutions for values of Y greater than $\frac{1}{2} - \frac{1-bk}{k-b}$, but we can further restrain the analysis to only these values of Y. Observe that $\frac{bk-1}{2(k-b)} \leq \frac{1}{2} - \frac{1-bk}{k-b}$.

By looking at the non negativity constraint for the final demanded quantity of gas we obtain:

$$A \leq \frac{ak(1-b^2)}{\left[k(1-b^2)+b(1-bk)+2bY(k-b)\right]}+\frac{(k-b)(2bY-1)G}{2\left[k(1-b^2)+b(1-bk)+2bY(k-b)\right]} \leq a \quad \text{if}$$

$\frac{bk-1}{2(k-b)} \leq Y \leq \frac{1}{2b}$, or

$$A \leq \frac{ak(1-b^2)}{\left[k(1-b^2)+b(1-bk)+2bY(k-b)\right]} \leq \frac{a}{2} \text{ if } \frac{1}{2b} \leq Y .$$

The regulator should choose contract coefficients A and Y ex ante so as to maximize expected global welfare subject to the individual rationality constraints for both firms and non negativity constraints for produced quantities, that is,

$$\underset{A,Y}{Max} \quad V_3(A,Y)=E\left[W_3(A,Y,\gamma,\varepsilon)\right]=\int_0^E\int_0^G W_3(A,Y,\gamma,\varepsilon)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$s.t. \quad \begin{cases} \pi_g(A,Y,\gamma)\geq 0, \forall\gamma \\ \pi_e(A,Y,\gamma,\varepsilon)\geq 0, \forall\gamma, \forall\varepsilon \\ q_g(A,Y,\gamma)\geq 0, \forall\gamma \\ q_e(A,Y,\gamma)\geq 0, \forall\gamma \end{cases}$$

As $W_3(A,Y,\gamma,\varepsilon)$ is a rather complicated function, the solution to this constrained maximisation problem requires solving a fifth degree equation for one of the contract coefficients, even with uniform cost distribution functions. There is thus no explicit algebraic form for the ex ante optimal values of A and Y, and we are left with numerical comparisons only.

The following figure illustrates expected global welfare for optimally tuned contracts of this nature (i.e. contracts that solve the maximization problem above) as a function of b and k with uniform cost distributions, for selected values of a, G and E that we use for benchmarks.

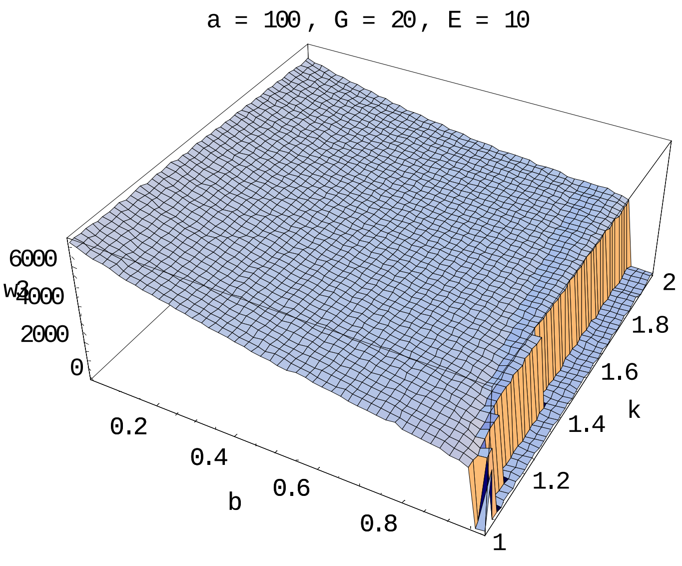


Figure B3. Expected welfare with optimal gamma contingent contracts

B.II.4 Fully specified contracts

There are two fully specified, fixed coefficient contracts, for both of which B = ½. As was said earlier, this value is the familiar value of the variable part of standard regulation contracts with linear demands.

The two contracts will be indexed by "+" and "-" superscripts, and the rules for setting the other five coefficients are as follows:

Define $t = 1 + k^2 - 2bk$ and observe that t is always positive, because it can be rewritten as $t = (k-b)^2 + (1-b^2)$, which is the sum of two positive quantities.

Compute $C^- = \dfrac{-t - \sqrt{(1-b^2)t}}{2(k-b)t}$ and $C^+ = \dfrac{-t + \sqrt{(1-b^2)t}}{2(k-b)t}$. Notice that they are both negative ($\sqrt{(1-b^2)t} < t \Leftrightarrow \sqrt{(1-b^2)} < \sqrt{t} \Leftrightarrow (1-b^2) < t \Leftrightarrow 0 < (k-b)^2$), which means that the regulated gas price is decreasing in the announced electricity cost parameter.

Set $Y^- = \dfrac{bk - 1 - \sqrt{(1-b^2)t}}{2(k-b)}$ and $Y^+ = \dfrac{bk - 1 + \sqrt{(1-b^2)t}}{2(k-b)}$. The interested reader should wonder if these two coefficients are also (symmetrically) negative, meaning that the regulated electricity price is decreasing in the announced gas cost parameter. However, it can easily be seen that this is not always the case. In fact, Y^+ will always be positive, and Y^- will be negative iff $b < \frac{1+\sqrt{8k^2+1}}{4k}$.

Proof

$$Y^+ > 0 \Leftrightarrow bk - 1 + \sqrt{(1-b^2)t} > 0 \Leftrightarrow 1 - bk < \sqrt{(1-b^2)t} \Leftrightarrow \begin{cases} 1 - bk < 0 \Leftrightarrow b > \frac{1}{k}, \textit{or} \\ b \le \frac{1}{k} \textit{ and } (1-bk)^2 < (1-b^2)t \end{cases}$$

$$(1-bk)^2 < (1-b^2)t \Leftrightarrow b^2k^2 < k^2 - b^2 - b^2k^2 + 2b^3k^2 \Leftrightarrow 2b^2k(k-b) < k^2 - b^2 \Leftrightarrow 2b^2k < k + b$$

$$\Leftrightarrow 2kb^2 - b - k < 0 \Leftrightarrow b \in \left]\frac{1-\sqrt{8k^2+1}}{4k}; \frac{1+\sqrt{8k^2+1}}{4k}\right[.$$

By observing that the interval $\left[0; \dfrac{1}{k}\right]$ is contained in the preceding interval when $k > 1$, we can replace the second case conditions by $b \le \frac{1}{k}$, and thus we obtain that $Y^+ > 0$ iff $b > \frac{1}{k}$ or $b \le \frac{1}{k}$. *Q.E.D.*

$$Y^- < 0 \Leftrightarrow bk - 1 - \sqrt{(1-b^2)t} < 0 \Leftrightarrow bk - 1 < \sqrt{(1-b^2)t} \Leftrightarrow \begin{cases} bk - 1 < 0 \Leftrightarrow b < \frac{1}{k}, \textit{or} \\ b \ge \frac{1}{k} \textit{ and } (bk-1)^2 < (1-b^2)t \end{cases}$$

$$(bk-1)^2 < (1-b^2)t \Leftrightarrow b^2k^2 < k^2 - b^2 - b^2k^2 + 2b^3k^2 \Leftrightarrow 2b^2k(k-b) < k^2 - b^2 \Leftrightarrow 2b^2k < k + b$$

$$\Leftrightarrow 2kb^2 - b - k < 0 \Leftrightarrow b \in \left]\frac{1-\sqrt{8k^2+1}}{4k}; \frac{1+\sqrt{8k^2+1}}{4k}\right[.$$

Observe now that the lower bound in this interval is negative, and that the upper bound is always between 0 and 1 when $k > 1$, so the second condition in the bracket can be replaced by $\frac{1}{k} \le b < \frac{1+\sqrt{8k^2+1}}{4k}$. Summing up, we find that $Y^- < 0$ iff $b < \frac{1}{k}$ or $\frac{1}{k} \le b < \frac{1+\sqrt{8k^2+1}}{4k}$, that is, iff $b < \frac{1+\sqrt{8k^2+1}}{4k}$. *Q.E.D.*

Pick $Z^- = -\frac{bt + k\sqrt{(1-b^2)t}}{2(k-b)t}$ and $Z^+ = -\frac{bt - k\sqrt{(1-b^2)t}}{2(k-b)t}$. Observe that Z^- is always negative, because all the terms in the expression are positive, and that Z^+ will be negative iff $b > \frac{1+\sqrt{8k^2+1}}{4k}$.

This result means that when there is high substitutability or when the regulator chooses the "-" contract, the regulated electricity price is decreasing in the announced electricity cost. Note that we showed earlier that the regulated gas price is also decreasing in $\hat{\varepsilon}$ (because both C^- and C^+ are always negative). Using this pricing rule, then, implies that a raise in the idiosyncratic cost of producing electricity lowers both regulated prices.

Proof

$$Z^+ < 0 \Leftrightarrow bt - k\sqrt{(1-b^2)t} > 0 \Leftrightarrow b\sqrt{t} > k\sqrt{(1-b^2)} \Leftrightarrow b^2 t > k^2(1-b^2)$$
$$\Leftrightarrow b^2\left[(k-b)^2 + (1-b^2)\right] > k^2(1-b^2) \Leftrightarrow b^2(k-b)^2 > (k^2-b^2)(1-b^2) \Leftrightarrow b^2(k-b) > (k+b)(1-b^2)$$
$$\Leftrightarrow 2kb^2 > k + b \Leftrightarrow 2kb^2 - b - k > 0 \Leftrightarrow b \notin \left[\frac{1-\sqrt{8k^2+1}}{4k}; \frac{1+\sqrt{8k^2+1}}{4k}\right].$$

As we noted earlier, the lower bound in this interval is negative, and that the upper bound is always between 0 and 1 when $k > 1$, so Z^+ is negative iff $b > \frac{1+\sqrt{8k^2+1}}{4k}$. *Q.E.D.*

Choose $A^- = \frac{a(t(1-b) - (k-1)\sqrt{(1-b^2)t})}{2t(k-b)}$ and $A^+ = \frac{a(t(1-b) + (k-1)\sqrt{(1-b^2)t})}{2t(k-b)}$, and notice that A^+ is always positive, because all of its terms are positive, and that A^- is positive if $1 - bk > 0$, that is, if $b < \frac{1}{k}$.

Finally compute $X^- = \frac{a\left[(1-b)(1+2k)t - (1+k-2bk)\sqrt{(1-b^2)t}\right]}{2t(k-b)}$ and $X^+ = \frac{a\left[(1-b)(1+2k)t + (1+k-2bk)\sqrt{(1-b^2)t}\right]}{2t(k-b)}$.

It is then possible to explicitly calculate the values of the prices, quantities, profits and surplus as functions of the parameters *a*, *b*, *k* and the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$. Only one of these two solutions will satisfy the second order condition, and it turns out to be the "+" solution.

In fact, for the general linear regulatory contracts, the second order conditions are:

$$\frac{\partial^2 \pi_g(\gamma;\hat{\gamma})}{\partial \hat{\gamma}^2} = -\frac{2B((1-bk)B+(k-b)Y)}{1-b^2} < 0$$

$$\frac{\partial^2 \pi_e(\varepsilon;\hat{\varepsilon})}{\partial \hat{\varepsilon}^2} = -\frac{2(Z-bC)(Z-kC)}{1-b^2} < 0$$

And in this case we additionally have $B = Z - kC = \frac{1}{2}$, so these conditions translate into:

$$\frac{1}{2}(1-bk)+(k-b)Y > 0 \Leftrightarrow Y > \frac{bk-1}{2(k-b)}$$

$$Z - bC > 0 \Leftrightarrow \tfrac{1}{2} + (k-b)C > 0 \Leftrightarrow C > -\frac{1}{2(k-b)}$$

It is very easy to check that Y^+ and C^+ satisfy these conditions, but that Y^- and C^- don't. This means that the "-" solution has no incentive power (because it corresponds to a minimum in the profit functions) and that the induced behaviour of the firms would be extreme values of the cost parameters instead of truth telling. The result of this type contract would be no different from that of the fixed price contract described in Section B.II.1.

Thus, we conclude that the only relevant fully specified contract is the "+" solution, given by:

$A^+ = \dfrac{a(t(1-b)+(k-1)\sqrt{(1-b^2)t})}{2t(k-b)}$, always positive.

$B^+ = ½$

$C^+ = \dfrac{-t+\sqrt{(1-b^2)t}}{2(k-b)t}$, always negative.

$X^+ = \dfrac{a\left[(1-b)(1+2k)t+(1+k-2bk)\sqrt{(1-b^2)t}\right]}{2t(k-b)}$

$Y^+ = \dfrac{bk-1+\sqrt{(1-b^2)t}}{2(k-b)}$, always positive.

$Z^+ = -\dfrac{bt-k\sqrt{(1-b^2)t}}{2(k-b)t}$, negative iff $b > \dfrac{1+\sqrt{8k^2+1}}{4k}$.

Quantity outcomes under this contract are:

$$q_g = \frac{a(1-b)\left[(2k-1)(1+b)t-(k-1+2bk)\sqrt{(1-b^2)t}\right]-\left[k(1-b^2)-b\sqrt{(1-b^2)t}\right]\gamma+\left[t(1-b^2)-(1-bk)\sqrt{(1-b^2)t}\right]\varepsilon}{2(1-b^2)(k-b)t}$$

$$q_e = \frac{a(1-b)\left[(k-1)\sqrt{(1-b^2)t}-(1+b)t\right]+\left[(1-b^2)-\sqrt{(1-b^2)t}\right]\gamma-(k-b)\sqrt{(1-b^2)t}\,\varepsilon}{2(1-b^2)(k-b)t}$$

As the coefficients for γ and ε are negative in the expression for q_e, it can be seen that the predicted q_e will always be negative if $1-bk<0$, so we must restrain the analysis to cases in which $1-bk\geq 0$.

If this condition holds, then the coefficient for γ in the expression for q_g is negative, and the one for ε, on the other hand, is positive. It can be seen that the term in brackets multiplying a is always positive by rewriting it as the sum of two positive terms in the following way:

$$(2k-1)(1+b)t-(k-1+2bk)\sqrt{(1-b^2)t}=(2k-1)(1+b)\left(t-\sqrt{(1-b^2)t}\right)+(k-b)\sqrt{(1-b^2)t}\geq 0$$

Appendix C: Incentive Compatibility with One Firm

III. <u>**Private cost announcements**</u> <u>(Figure 3B)</u>

If $p_g(\hat{\gamma})$ and $p_e(\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator(s) for gas and electricity as a function of the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the profit of the dual product firm is given by:

$$\Pi_d(p_g(\hat{\gamma}), p_e(\hat{\varepsilon}), \gamma, \varepsilon) = q_g(p_g(\hat{\gamma}), p_e(\hat{\varepsilon})) \cdot (p_g(\hat{\gamma}) - \gamma) + q_e(p_g(\hat{\gamma}), p_e(\hat{\varepsilon})) \cdot (p_e(\hat{\varepsilon}) - \varepsilon - k\gamma) \qquad \text{(C.I.1)}$$

The firm will maximize its profits by announcing an optimal pair of values $\hat{\gamma}$ and $\hat{\varepsilon}$, that will determine the prices, and thus its own profits. By replacing the demand functions into expression (C.I.1) we obtain:

$$\Pi_d = (a(1-b) - p_g(\hat{\gamma}) + b \cdot p_e(\hat{\varepsilon})) \cdot (p_g(\hat{\gamma}) - \gamma) + (a(1-b) - p_e(\hat{\varepsilon}) + b \cdot p_g(\hat{\gamma})) \cdot (p_e(\hat{\varepsilon}) - \varepsilon - k\gamma) \qquad \text{(C.I.2)}$$

So the first order conditions for the maximization problem of the firm are:

$$\begin{cases} \frac{\partial \Pi_d}{\partial \hat{\gamma}} = 0 \Leftrightarrow \frac{\partial p_g}{\partial \hat{\gamma}} \left[a(1-b) + (1-bk)\gamma - b\varepsilon - 2p_g(\hat{\gamma}) + 2b \cdot p_e(\hat{\varepsilon}) \right] = 0 \\ \frac{\partial \Pi_d}{\partial \hat{\varepsilon}} = 0 \Leftrightarrow \frac{\partial p_e}{\partial \hat{\varepsilon}} \left[a(1-b) + (k-b)\gamma + \varepsilon - 2p_e(\hat{\varepsilon}) + 2b \cdot p_g(\hat{\gamma}) \right] = 0 \end{cases} \qquad \text{(C.I.3)}$$

Incentive compatibility requires that these two first order conditions be satisfied when $\hat{\gamma} = \gamma$ and $\hat{\varepsilon} = \varepsilon$. The four possible solutions of this system of equations are:

(a) $\frac{\partial p_g}{\partial \hat{\gamma}} = 0$ (which means constant gas prices) and $\frac{\partial p_e}{\partial \hat{\varepsilon}} = 0$ (which means constant electricity prices), so a pair of fixed price contracts; or

(b) $\frac{\partial p_g}{\partial \hat{\gamma}} = 0$ (which means constant gas prices) and $p_e(\hat{\varepsilon}) = \dfrac{a(1-b) + (k-b)\gamma + \varepsilon + 2b \cdot p_g}{2}$, which cannot be simultaneously verified for all values of γ; or

(c) $\frac{\partial p_e}{\partial \hat{\varepsilon}} = 0$ (which means constant electricity prices) and $p_g(\hat{\gamma}) = \dfrac{a(1-b) + (1-bk)\gamma - b\varepsilon + 2b \cdot p_e}{2}$, which cannot be simultaneously verified for all values of ε; or

(d) $\frac{\partial p_g}{\partial \hat{\gamma}} \neq 0$, $\frac{\partial p_e}{\partial \hat{\varepsilon}} \neq 0$, which imply $p_g(\hat{\gamma}) = \dfrac{a+\gamma}{2}$ and $p_e(\hat{\varepsilon}) = \dfrac{a + k\gamma + \varepsilon}{2}$. This last expression also cannot be simultaneously verified for all values of γ.

So there is no pair of independent contracts that induce truth-telling.

IV. **Public cost announcements** (Figure 4B)

If $p_g(\hat{\gamma},\hat{\varepsilon})$ and $p_e(\hat{\gamma},\hat{\varepsilon})$ are respectively the pricing rules proposed by the regulator(s) for gas and electricity as a function of the announced costs $\hat{\gamma}$ and $\hat{\varepsilon}$, then the profit of the dual product firm is given by:

$$\begin{aligned}\Pi_d(p_g(\hat{\gamma},\hat{\varepsilon}),p_e(\hat{\gamma},\hat{\varepsilon}),\gamma,\varepsilon) = \; & q_g(p_g(\hat{\gamma},\hat{\varepsilon}),p_e(\hat{\gamma},\hat{\varepsilon}))\cdot(p_g(\hat{\gamma},\hat{\varepsilon})-\gamma) \\ & + q_e(p_g(\hat{\gamma},\hat{\varepsilon}),p_e(\hat{\gamma},\hat{\varepsilon}))\cdot(p_e(\hat{\gamma},\hat{\varepsilon})-\varepsilon-k\gamma)\end{aligned} \quad \text{(i)}$$

The firm will maximize its profits by announcing an optimal pair of values $\hat{\gamma}$ and $\hat{\varepsilon}$, that will determine the prices, and thus its own profits.

By replacing demand profiles (2) we can write the incentive compatibility constraints for both contracts:

$$\left[a(1-b)+(1-b\cdot k)\gamma-b\cdot\varepsilon-2p_g+2b\cdot p_e\right]\tfrac{\partial p_g}{\partial\hat{\gamma}}+\left[a(1-b)+(k-b)\gamma+\varepsilon-2p_e+2b\cdot p_g\right]\tfrac{\partial p_e}{\partial\hat{\gamma}}=0 \quad (\text{IC}_g)$$

$$\left[a(1-b)+(1-b\cdot k)\gamma-b\cdot\varepsilon-2p_g+2b\cdot p_e\right]\tfrac{\partial p_g}{\partial\hat{\varepsilon}}+\left[a(1-b)+(k-b)\gamma+\varepsilon-2p_e+2b\cdot p_g\right]\tfrac{\partial p_e}{\partial\hat{\varepsilon}}=0 \quad (\text{IC}_e)$$

Which, when restricting to linear contracts under multidimensional incitation, can be rewritten:

$$\begin{aligned}&(aB-2AB-abB+2bBX+aY-abY+2bAY-2XY)\\ &\quad+\left(B-2B^2-bkB-bY+4bBY+kY-2Y^2\right)\gamma+(Y-bB+2bBZ+2bCY-2BC-2YZ)\varepsilon=0\end{aligned}$$

$$\begin{aligned}&(aC-2AC-abC+2bCX+aZ-abZ+2bAZ-2XZ)\\ &\quad+(C-2BC-bkC-bZ+2bCY+2bBZ+kZ-2YZ)\gamma+\left(Z-bC+4bCZ-2C^2-2Z^2\right)\varepsilon=0\end{aligned}$$

Expressions above correspond to (ICg) and (ICe) evaluated in $\{\hat{\gamma}=\gamma,\hat{\varepsilon}=\varepsilon\}$, that is, when both cost announcements are simultaneously true. Thus, we do not look for contracts that are individually incentive compatible, that is, contracts that would induce truth-telling in $\hat{\gamma}$ even if $\hat{\varepsilon}\neq\varepsilon$, and *vice versa*, which would of course limit regulatory latitude. These are, then, the most general forms of incentive compatible contracts. [3]

When looking for truth-inducing contracts in a multidimensional context, both equalities must simultaneously hold for every possible pair of values of γ and ε, so, by equating the coefficients of these variables in both polynomials to zero, we obtain the following system of equations:

[3] See the Technical Note for an in depth discussion of this point.

$$\begin{cases} B\left(a(1-b)-2A+2bX\right)+Y\left(a(1-b)-2X+2bA\right)=0 \\ B\left(1-bk+2bY-2B\right)+Y\left(k-b+2bB-2Y\right)=0 \\ Y-bB-2Z(Y-bB)+2C(bY-B)=0 \\ C\left(a(1-b)-2A+2bX\right)+Z\left(a(1-b)-2X+2bA\right)=0 \\ C\left(1-bk+2bY-2B\right)+Z\left(k-b+2bB-2Y\right)=0 \\ 2Z^2-(1+4bC)Z+2C^2+bC=0 \end{cases} \qquad \text{(ICCs)}$$

This system has seven "families" of generic solutions, two of which have two degrees of freedom, four that have one degree of freedom, and one that is fully specified.

We will now study them one by one. For convenience, we define:

$\phi=\phi(\gamma,\varepsilon)=2a(1-b)(a-(k+1)\gamma-\varepsilon)+t\cdot\gamma^2+2(k-b)\gamma\varepsilon+\varepsilon^2$, and
$\phi_{GE}=\phi(\gamma=G,\varepsilon=E)=2a(1-b)(a-(k+1)G-E)+t\cdot G^2+2(k-b)GE+E^2$

C.II.1 Fixed price contracts

This solution corresponds to taking $B = Y = C = Z = 0$, which naturally verifies (IC_g) and (IC_e) because all partial derivatives are equal to zero. Contract coefficients A and X can be freely chosen by the regulator, and should be set so as to maximize global welfare while respecting the individual rationality constraint. Obviously, this solution lacks any truth-inducing power, because the price outcome is arbitrarily set by the regulator, independently of firms' cost announcements.

Nevertheless, the regulator can (ex ante) optimally set prices to implement cross-subsidization between gas and electricity production, thus raising global welfare. Assuming that maximum demand parameter a is large enough relative to G and E (i.e. $a>\frac{((k-b)+(1-b))G+E}{2(1-b)}$), the pair of prices that maximizes expected global welfare subject to individual rationality constraint and non negativity of quantities is:

$$\begin{cases} A_1^*=\dfrac{a+G}{2}-\dfrac{\sqrt{2}\sqrt{1-b}\sqrt{\phi_{GE}}}{4(1-b)} \\ X_1^*=\dfrac{a+k\cdot G+E}{2}-\dfrac{\sqrt{2}\sqrt{1-b}\sqrt{\phi_{GE}}}{4(1-b)} \end{cases} \qquad \text{(C.II.1)}$$

Observe that both prices are strictly lower than the "saturated" monopoly prices.

If production of both goods cannot be made compulsory by the regulator, she should decide whether to ensure production of both goods by guaranteeing the satisfaction of individual rationality constraints for each of the two energy goods in the "worst case scenario". If she decided to do so, she should set p_g and p_e equal to marginal costs when firms are most inefficient, which means $p_g = G$ and $p_e = E + k\,G$, just like in the duopoly case. If E and G are large (relative to a), these prices may be above monopoly prices (so very inefficient).

C.II.2 Contracts independent of ε

These contracts correspond to those in which $C = Z = 0$. The electricity cost parameter announcement made by the firm is completely irrelevant for the pricing decisions, so (IC_e) is trivially satisfied. There are no incentives for the firm to reveal its true type (at least with regards to the value of ε), nor to lie, because its behaviour will not influence the outcome.

Only the first three equations in the system (ICCs) remain, and their solution is:

$A = \frac{a}{2}$, $B = \frac{1}{2}$, and $Y = \frac{b}{2}$.

The regulator can freely choose the contract coefficient X, that is, the fixed component of the electricity contract.

The second order condition of the problem is verified, for $\frac{\partial^2}{\partial \hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon}) = -\frac{1}{2} < 0$.

Observe that these contracts correspond to prices of the form $p_g = \dfrac{a+\hat{\gamma}}{2}$, that is, standard regulation for gas; and $p_e = X + \dfrac{b\hat{\gamma}}{2}$, which can be interpreted as some kind of "corrected regulation" for electricity. In fact, in this context the price of electricity increases more slowly than the price of gas with the announced parameter $\hat{\gamma}$, and it does so by a factor of b, i.e. the substitutability factor.

Equilibrium quantity outcomes under this contract are $q_g = \dfrac{a(1-2b)+2bX}{2(1-b^2)} - \dfrac{\gamma}{2}$, and $q_e = \dfrac{a(2-b)-2X}{2(1-b^2)}$, so the amount of electricity produced is totally independent of the cost parameters of the firm, and can be chosen by the regulator when setting the free contract coefficient X.

In order to ensure non-negativity of both quantities for every possible value of γ, the regulator should set X in the following interval (which is nonempty if $G \le a$):

$$\frac{G(1-b^2)-a(1-2b)}{2b} \le X \le \frac{a(2-b)}{2}. \qquad \text{(C.II.2)}$$

It is straightforward to show that profits can be written both as a concave parabola in contract coefficient X, and as a decreasing function (in the relevant region) of both cost parameters, so, in order to fulfil the individual rationality constraint of the firm, we need X to be between the roots of the parabola when $\gamma = G$ and $\varepsilon = E$, which are :

$$X_{\Pi} = \frac{a+(k-b)G+E \pm \sqrt{\phi_{GE}}}{2}$$

Notice that the largest of these two roots is always larger than the upper bound for X previously defined in (C.II.2), so only the smallest root may bind.

The regulator should choose contract coefficient X ex ante so as to maximize expected global welfare subject to the individual rationality constraint of the firm and non negativity of quantities, that is,

$$\underset{X}{Max} \quad V_2(X) = E[W_2(X,\gamma,\varepsilon)] = \int_0^E \int_0^G W_2(X,\gamma,\varepsilon) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$s.t. \quad \begin{cases} \pi_i(X,\gamma,\varepsilon) \geq 0, \forall \gamma, \forall \varepsilon \\ q_g(X,\gamma) \geq 0, \forall \gamma \\ q_e(X) \geq 0 \end{cases}$$

By rewriting $V_2(X)$ as a function of X and the parameter values we get an expression that allows us to obtain the interior solution of this problem:

$$V_2(X) = \int_0^E \int_0^G \left(\frac{a^2(7-8b) + 3(1-b^2)\gamma^2 - a(2-b)(k\gamma+\varepsilon) - 2a(3-b^2)\gamma - 4X^2 + 4X(ab - 2(k-b)\gamma - 2\varepsilon)}{8(1-b^2)} \right) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$\frac{\partial}{\partial X} V_2(X) = \int_0^E \int_0^G \frac{\partial}{\partial X} \left(\frac{a^2(7-8b) + 3(1-b^2)\gamma^2 - a(2-b)(k\gamma+\varepsilon) - 2a(3-b^2)\gamma - 4X^2 + 4X(ab - 2(k-b)\gamma - 2\varepsilon)}{8(1-b^2)} \right) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$\frac{\partial}{\partial X} V_2(X) = \int_0^E \int_0^G \left(\frac{4(ab - 2(k-b)\gamma - 2\varepsilon) - 8X + 0}{8(1-b^2)} \right) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$\frac{\partial}{\partial X} V_2(X) = 0 \Leftrightarrow X_2^* = \frac{ab + 2E[(k-b)\gamma + \varepsilon]}{2}$$

And if γ and ε are uniformly distributed,

$$X_{2U}^* = \frac{ab + (k-b)G + E}{2}$$

If the regulator decided to guarantee the satisfaction of individual rationality constraints for each of the two energy goods, she should also check that individual prices are not inferior to individual costs, which means $(k - \frac{b}{2})G + E \leq X$.

C.II.3 Contracts independent of γ

These contracts correspond to those in which $B = Y = 0$. Here, it is the gas cost parameter announcement made by the firm that is completely irrelevant for the pricing decisions, so (IC_g) is trivially satisfied. There are no incentives for the firm to announce its true type (at least with regards to the value of γ), nor to lie, because its behaviour will not influence the outcome.

This leaves us with only the last three equations in the system (ICCs) and its unique nontrivial solution, with one degree of freedom (as earlier, we set $t = k^2 - 2bk + 1$):

$$C = -\frac{k-b}{2t}, \text{ and } Z = \frac{1-bk}{2t}.$$

The regulator can freely choose either contract coefficient *A*, or contract coefficient *X*, that is, she can arbitrarily pick the fixed part of one of the contracts, but is then forced to set the other according to the following rule:

$$X = kA - \frac{a(k-1)}{2}$$

The second order condition of the problem is always verified, because $\frac{\partial^2}{\partial \hat{\varepsilon}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon}) = -\frac{1}{2t} < 0$.

Equilibrium quantity outcomes under this contract are $q_g = \frac{a(1-b)+(a-2A)(1-bk)}{2(1-b^2)} + k\frac{\varepsilon}{2t}$, and $q_e = \frac{a(1-b)+(a-2A)(k-b)}{2(1-b^2)} - \frac{\varepsilon}{2t}$. Observe that both quantities vary in opposite directions when ε increases, and that the ratio of the magnitude of these variations is equal to the technical transformation parameter *k*.

In order to ensure non-negativity of both quantities for every possible value of ε, the regulator should set *A* such that $A \le \frac{a}{2}\left(1+\frac{1-b}{k-b}\right) - \frac{1-b^2}{2(k-b)t}E$. Additionally, only if $1-bk<0$, $\frac{a}{2}\left(1-\frac{1-b}{bk-1}\right) \le A$. This interval is nonempty iff $E \le \frac{k-1}{bk-1}at$.

It is straightforward to show that profits can be written both as a concave parabola in contract coefficient *A*, and as a decreasing function of both cost parameters, so, in order to fulfil the individual rationality constraint of the firm, we need *A* to be between the roots of the parabola when $\gamma = G$ and $\varepsilon = E$, which are :

$$A_\Pi = \frac{a+G}{2} + \frac{(k-b)E \pm \sqrt{t\phi_{GE}}}{2t}$$

The regulator should choose contract coefficient *A* ex ante so as to maximize expected global welfare subject to the individual rationality constraint of the firm and non negativity of quantities, that is,

$$\underset{A}{Max} \quad V_3(A) = E[W_3(A,\gamma,\varepsilon)] = \int_0^E \int_0^G W_3(A,\gamma,\varepsilon)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$s.t. \quad \begin{cases} \pi_i(A,\gamma,\varepsilon) \ge 0, \forall\gamma, \forall\varepsilon \\ q_g(A,\varepsilon) \ge 0, \forall\varepsilon \Leftrightarrow q_g(A,\varepsilon=0) \ge 0 \\ q_e(A,\varepsilon) \ge 0, \forall\varepsilon \Leftrightarrow q_e(A,\varepsilon=E) \ge 0 \end{cases}$$

By rewriting $V_3(A)$ as a function of *A* and the parameter values we get an expression that allows us to obtain the interior solution of this problem:

$$\frac{\partial}{\partial A}V_3(A)=\int_0^E\int_0^G\frac{\partial}{\partial A}\left(\frac{-4A^2t^2+8At(t\gamma+(k-b)\varepsilon)+4Aa(k-1)(k-b)t+\psi_3(\gamma,\varepsilon)}{8(1-b^2)t}\right)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

, where $\psi_3(\gamma,\varepsilon)$ is a function independent of A.

$$\frac{\partial}{\partial A}V_3(A)=\int_0^E\int_0^G\left(\frac{-8At^2+8t(t\gamma+(k-b)\varepsilon)+4a(k-1)(k-b)t+0}{8(1-b^2)t}\right)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$\frac{\partial}{\partial A}V_3(A)=0\Leftrightarrow A_3^*=\frac{a(k-1)(k-b)+2E[t\gamma+(k-b)\varepsilon]}{2t}$$

And if γ and ε are uniformly distributed,

$$A_{3U}^*=\frac{a(k-1)(k-b)+tG+(k-b)E}{2t}$$

C.II.4 Contracts with fixed gas price

These contracts correspond to those in which $B = C = 0$. In this case, both cost announcements made by the firm influence only the price of electricity. Proposed contracts are of the form:

$$X=bA+\frac{a(1-b)}{2},\ Y=\frac{k-b}{2},\ \text{and}\ Z=\frac{1}{2}.$$

The regulator can freely choose either contract coefficient A, or contract coefficient X. That is, she can arbitrarily pick either the price of gas and then set the fixed part of the electricity contract accordingly; or choose the fixed part of the electricity price and then adjust the gas price by the rule $A=\frac{2X-a(1-b)}{2b}$.

Notice that Z, the coefficient for $\hat{\varepsilon}$ in the electricity contract, is the standard ½ found in linear incentive powered regulatory contracts.

The second order condition of the problem is that the matrix of second order derivatives of the profits with respect to the announced costs must be negative semidefinite, that is $\begin{bmatrix}\frac{\partial^2}{\partial\hat{\gamma}^2} & \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}}\\ \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}} & \frac{\partial^2}{\partial\hat{\varepsilon}^2}\end{bmatrix}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is negative semidefinite, i.e. $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})<0$, and $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\partial\hat{\varepsilon}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})-\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\geq 0$.

In this case these two expressions correspond respectively to $-\frac{(k-b)^2}{2(1-b^2)}<0$ and $\left(-\frac{(k-b)^2}{2(1-b^2)}\right)\left(-\frac{1}{2(1-b^2)}\right)-\left(\frac{(k-b)}{2(1-b^2)}\right)^2=0\geq 0$. By noting that $\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is a second order polynomial in the announced costs, or by looking at the matrix of second order derivatives and noticing that all terms are independent of the cost announcements, it is easy to see that third derivatives are all zero, so the point is a relative maximum.

Equilibrium quantity outcomes under this contract are $q_g=\frac{a(1-b)+b[(k-b)\gamma+\varepsilon]}{2(1-b^2)}+\frac{(a-2A)}{2}$, and $q_e=\frac{a(1-b)-[(k-b)\gamma+\varepsilon]}{2(1-b^2)}$. Observe that both quantities vary in opposite directions when costs increase, and that the magnitude of these variations are proportional to the substitution parameter *b*. Notice also that the quantity of electricity produced is independent of the contract coefficient *A*.

In order to ensure non-negativity of gas production for every possible value of the cost parameters γ and ε, the regulator should set *A* such that $A\leq\frac{a}{2}\frac{2+b}{1+b}$. Non-negativity of electricity production is feasible iff $a\geq\frac{(k-b)G+E}{1-b}$.

It is straightforward to show that profits can be written both as a concave parabola in contract coefficient *A*, and as a decreasing function of both cost parameters, so, in order to fulfil the individual rationality constraint of the firm, we need *A* to be between the roots of the parabola when $\gamma = G$ and $\varepsilon = E$, which are :

$$A_{\Pi}=\frac{a+G}{2}\pm\frac{\sqrt{(1-b^2)\phi_{GE}}}{2(1-b^2)}$$

The regulator should choose contract coefficient *A* ex ante so as to maximize expected global welfare subject to the individual rationality constraint of the firm and non negativity of quantities, that is,

$$\underset{A}{Max}\quad V_4(A)=E[W_4(A,\gamma,\varepsilon)]=\int_0^E\int_0^G W_4(A,\gamma,\varepsilon)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$s.t.\quad\begin{cases}\pi_i(A,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\\ q_g(A,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\Leftrightarrow q_g(A,\gamma=0,\varepsilon=0)\geq 0\\ q_e(A,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\Leftrightarrow q_e(A,\gamma=G,\varepsilon=E)\geq 0\end{cases}$$

By rewriting $V_4(A)$ as a function of *A* and the parameter values we get an expression that allows us to obtain the interior solution of this problem:

$$\frac{\partial}{\partial A}V_4(A)=\int_0^E\int_0^G\frac{\partial}{\partial A}\left(\frac{-4A^2(1-b^2)+8A(1-b^2)\gamma+\psi_4(\gamma,\varepsilon)}{8(1-b^2)}\right)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

, where $\psi_4(\gamma,\varepsilon)$ is a function independent of A.

$$\frac{\partial}{\partial A}V_4(A)=\int_0^E\int_0^G\left(\frac{-8A\left(1-b^2\right)+8\left(1-b^2\right)\gamma+0}{8\left(1-b^2\right)}\right)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$\frac{\partial}{\partial A}V_4(A)=0\Leftrightarrow A_4^*=E[\gamma]$$

And if γ is uniformly distributed,

$$A_{4U}^*=\frac{G}{2}$$

Notice that if the regulator decided to guarantee the satisfaction of individual rationality constraints for each of the two energy goods, she would be unable to set the price of gas at this level, which will be inferior to γ in some cases.

C.II.5 Contracts with fixed electricity price

These contracts correspond to those in which $Y = Z = 0$. In this case, both cost announcements made by the firm influence only the price of electricity. Proposed contracts are of the form:

$$B=\frac{1-bk}{2},\ C=\frac{-b}{2},\ \text{and}\ X=\frac{2A-a(1-b)}{2b}.$$

The regulator can freely choose either contract coefficient *A*, or contract coefficient *X*. That is, she can arbitrarily pick either the price of electricity and then set the fixed part of the gas contract by the rule $A=bX+\frac{a(1-b)}{2}$; or choose the fixed part of the gas price and then adjust the electricity price accordingly. Notice that these adjustment rules are symmetric to those found in point C.II.4 described above.

The second order condition of the problem is that the matrix of second order derivatives of the profits with respect to the announced costs must be negative semidefinite, that is $\begin{bmatrix}\frac{\partial^2}{\partial\hat{\gamma}^2} & \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}}\\ \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}} & \frac{\partial^2}{\partial\hat{\varepsilon}^2}\end{bmatrix}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is negative semidefinite, i.e. $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})<0$, and $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\partial\hat{\varepsilon}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})-\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\geq 0$.

In this case these two expressions correspond respectively to $-\frac{(1-bk)^2}{2\left(1-b^2\right)}<0$ and $\left(-\frac{(1-bk)^2}{2\left(1-b^2\right)}\right)\left(-\frac{b^2}{2\left(1-b^2\right)}\right)-\left(\frac{b(1-bk)}{2\left(1-b^2\right)}\right)^2=0\geq 0$. By noting that $\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is a second order polynomial in the announced costs, or by looking at the matrix of second order derivatives and noticing that all terms are independent of the cost announcements, it is easy to see that third derivatives are all zero, so the point is a relative maximum.

Equilibrium quantity outcomes under this contract are $q_g = \dfrac{a(1-b)-[(1-bk)\gamma - b\varepsilon]}{2(1-b^2)}$, and $q_e = (a-2A)+\dfrac{a(1-b)+b[(1-bk)\gamma - b\varepsilon]}{2(1-b^2)}$. Observe that both quantities vary in opposite directions when costs increase, and that the magnitude of these variations are proportional to the substitution parameter *b*. Notice also that the quantity of gas produced is independent of the contract coefficient *A*.

In order to ensure non-negativity of electricity production for every possible value of the cost parameters γ and ε, the regulator should set *A* such that $A \le \dfrac{a}{2}\dfrac{1+2b}{1+b} - \dfrac{b^3}{2(1-b^2)}E - \dfrac{b^2(bk-1)}{2(1-b^2)}G$. The last term of the expression should be included only if $bk-1>0$.

Non-negativity of gas production is feasible iff $a \ge \dfrac{1-bk}{1-b}G$.

It is straightforward to show that profits can be written both as a concave parabola in contract coefficient *A*, and as a decreasing function of both cost parameters, so, in order to fulfil the individual rationality constraint of the firm, we need *A* to be between the roots of the parabola when $\gamma = G$ and $\varepsilon = E$, which are :

$$A_\Pi = \frac{a+b(kG+E)}{2} \pm \frac{b\sqrt{(1-b^2)\phi_{GE}}}{2(1-b^2)}$$

The regulator should choose contract coefficient *A* ex ante so as to maximize expected global welfare subject to the individual rationality constraint of the firm and non negativity of quantities, that is,

$$\underset{A}{Max} \quad V_5(A) = E[W_5(A,\gamma,\varepsilon)] = \int_0^E \int_0^G W_5(A,\gamma,\varepsilon) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$s.t. \quad \begin{cases} \pi_i(A,\gamma,\varepsilon) \ge 0, \forall \gamma, \forall \varepsilon \\ q_g(A,\gamma,\varepsilon) \ge 0, \forall \gamma, \forall \varepsilon \Leftrightarrow q_g(A,\gamma,\varepsilon=0) \ge 0, \forall \gamma \\ q_e(A,\gamma,\varepsilon) \ge 0, \forall \gamma, \forall \varepsilon \Leftrightarrow q_e(A,\gamma,\varepsilon=E) \ge 0, \forall \gamma \end{cases}$$

By rewriting $V_5(A)$ as a function of *A* and the parameter values we get an expression that allows us to obtain the interior solution of this problem:

$$\frac{\partial}{\partial A}V_5(A) = \int_0^E \int_0^G \frac{\partial}{\partial A}\left(\frac{-4A^2(1-b^2)+8Ab(1-b^2)(k\gamma+\varepsilon)-4Aa(1-b)(1-b^2)+\psi_5(\gamma,\varepsilon)}{8b^2(1-b^2)} \right) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

, where $\psi_5(\gamma,\varepsilon)$ is a function independent of A.

$$\frac{\partial}{\partial A}V_5(A) = \int_0^E \int_0^G \left(\frac{-8A(1-b^2)+8b(1-b^2)(k\gamma+\varepsilon)+4a(1-b)(1-b^2)+0}{8(1-b^2)} \right) g(\gamma) f(\varepsilon) d\gamma d\varepsilon$$

$$\frac{\partial}{\partial A}V_5(A)=0 \Leftrightarrow A_5^*=\frac{a(1-b)+2bE[k\gamma+\varepsilon]}{2}$$

And if γ is uniformly distributed,

$$A_{5U}^*=\frac{a(1-b)+b(kG+E)}{2}$$

C.II.6 "Full" contracts

These contracts correspond to those in which $BZ=CY\neq 0$. This equation implies the equivalence of the incentive compatibility constraints with respect to $\hat{\gamma}$ and the ones with respect to $\hat{\varepsilon}$. In other words, of the six equations in the system (ICCs), the first three and the last three are equivalent. The system can thus be rewritten as follows :

$$\begin{cases} B(a(1-b)-2A+2bX)+Y(a(1-b)-2X+2bA)=0 \\ B(1-bk-2B+2bY)+Y(k-b-2Y+2bB)=0 \\ 2Z^2-(1+4bC)Z+2C^2+bC=0 \\ BZ=CY\neq 0 \end{cases}$$

First define $d=\pm\sqrt{1-16C^2(1-b^2)}$ for notational simplicity, and then set: $B=\frac{1-d}{4}+(k-b)C$, $X=a\frac{1+4C(1-b)-d}{8C}-A\frac{1-4bC-d}{4C}$, $Y=\frac{1+d}{4}\frac{B}{C}+bB$, and $Z=\frac{1+d}{4}+bC$. We also naturally need $|C|<\frac{1}{4\sqrt{1-b^2}}$.

Given that the third equation is quadratic in *Z* and *C*, there are two solutions to this system, depending on whether *d* is positive or negative. There are two free coefficients (*A* and *C* in this case).

Observe that by taking $C=\frac{-b}{2}$ in the contract with a negative *d*, we obtain the fixed electricity prices mentioned in point 5 above.

The second order condition of the problem is that the matrix of second order derivatives of the profits with respect to the announced costs must be negative semidefinite, that is $\begin{bmatrix} \frac{\partial^2}{\partial\hat{\gamma}^2} & \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}} \\ \frac{\partial^2}{\partial\hat{\gamma}\partial\hat{\varepsilon}} & \frac{\partial^2}{\partial\hat{\varepsilon}^2} \end{bmatrix}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is negative semidefinite, i.e. $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})<0$, and $\frac{\partial^2}{\partial\hat{\gamma}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\partial\hat{\varepsilon}^2}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})-\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\cdot\frac{\partial^2}{\hat{\gamma}\partial\hat{\varepsilon}}\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})\geq 0$.

In this case these two expressions correspond respectively to $-\frac{[(1-d)+4C(k-b)]^2\left[16\left(1-b^2\right)C^2+(1+d)^2\right]}{128\left(1-b^2\right)C^2}\leq 0$ and

$$\left(-\frac{[(1-d)+4C(k-b)]^2\left[16\left(1-b^2\right)C^2+(1+d)^2\right]}{128\left(1-b^2\right)C^2}\right)\cdot\left(-\frac{16\left(1-b^2\right)C^2+(1+d)^2}{8\left(1-b^2\right)}\right)-\left(-\frac{[(1-d)+4C(k-b)]\left[16\left(1-b^2\right)C^2+(1+d)^2\right]}{32\left(1-b^2\right)C^2}\right)^2=0\geq 0$$

By noting that $\Pi_d(\gamma,\varepsilon,\hat{\gamma},\hat{\varepsilon})$ is a second order polynomial in the announced costs, or by looking at the matrix of second order derivatives and noticing that all terms are independent of the cost announcements, it is easy to see that third derivatives are all zero, so the point is a relative maximum.

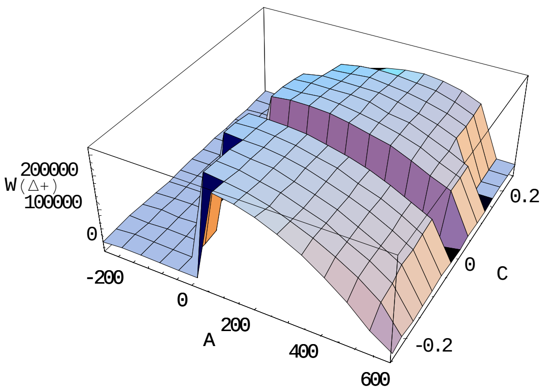
The regulator should choose contract coefficients A and C ex ante so as to maximize expected global welfare subject to the individual rationality constraint of the firm and non negativity of quantities, that is,

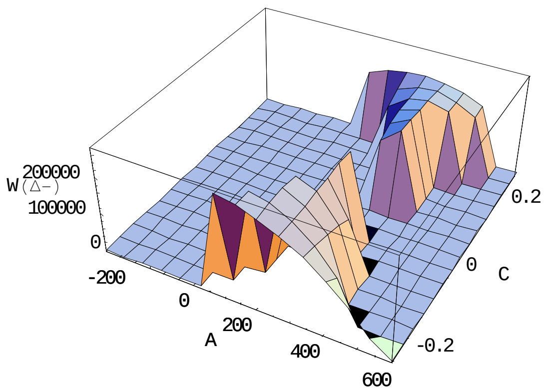
$$\underset{A,C}{Max}\quad V_6(A,C)=E[W_6(A,C,\gamma,\varepsilon)]=\int_0^E\int_0^G W_6(A,C,\gamma,\varepsilon)g(\gamma)f(\varepsilon)d\gamma d\varepsilon$$

$$s.t.\quad\begin{cases}\pi_i(A,C,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\\ q_g(A,C,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\\ q_e(A,C,\gamma,\varepsilon)\geq 0,\forall\gamma,\forall\varepsilon\end{cases}$$

The constrained problem cannot be analytically solved, for the associated welfare function is a fraction of fifth degree polynomials in the optimisation variables.

Here's how the problem looks:





C.II.7 Fully specified contracts

There are finally two fully specified, fixed coefficients pairs of contracts, that correspond to the non-zero solutions to the equations.

C.II.7.a

The first one corresponds to the following coefficient values:

$A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = 0$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \frac{1}{2}$.

Notice that both contracts have the same fixed part, namely $\frac{a}{2}$, and that both the coefficient for the gas cost in the gas contract and the coefficient for the electricity cost in the electricity contract are equal to ½. If we add the effect of coefficient *Y*, we can then rewrite both contracts as follows:

$p_g = \frac{a}{2} + \frac{1}{2}\hat{\gamma}$ and $p_e = \frac{a}{2} + \frac{1}{2}(k\hat{\gamma} + \hat{\varepsilon})$.

Observe that these expressions correspond to traditional independent regulation, in which the regulator sets each price equal to one half of the maximum demand parameter plus one half of the reported total marginal production cost in each industry. These contracts are incentive powered by construction, so these reported costs correspond to the real cost parameters.

The second order condition of the problem is that the matrix of second order derivatives of the profits with respect to the announced costs must be negative semidefinite, that is $\begin{bmatrix} \frac{\partial^2}{\partial \hat{\gamma}^2} & \frac{\partial^2}{\partial \hat{\gamma}\partial \hat{\varepsilon}} \\ \frac{\partial^2}{\partial \hat{\gamma}\partial \hat{\varepsilon}} & \frac{\partial^2}{\partial \hat{\varepsilon}^2} \end{bmatrix} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon})$ is negative semidefinite, i.e. $\frac{\partial^2}{\partial \hat{\gamma}^2}\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) < 0$, and $\frac{\partial^2}{\partial \hat{\gamma}^2}\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \cdot \frac{\partial^2}{\partial \hat{\varepsilon}^2}\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) - \frac{\partial^2}{\hat{\gamma}\partial \hat{\varepsilon}}\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \cdot \frac{\partial^2}{\hat{\gamma}\partial \hat{\varepsilon}}\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \geq 0$.

In this case these two expressions correspond respectively to $-\frac{t}{2(1-b^2)} < 0$ and $\left(-\frac{t}{2(1-b^2)}\right)\left(-\frac{1}{2(1-b^2)}\right) - \left(-\frac{(k-b)}{2(1-b^2)}\right)^2 = \frac{1}{4(1-b^2)} > 0$.

Equilibrium quantity outcomes under this contract are $q_g = \frac{a(1-b) - (1-bk)\gamma + b\varepsilon}{2(1-b^2)}$, and $q_e = \frac{a(1-b) - (k-b)\gamma - \varepsilon}{2(1-b^2)}$.

Non-negativity of electricity production for every possible value of the cost parameters γ and ε is feasible iff $a \geq \frac{(k-b)G + E}{1-b}$.

It is straightforward to show that profits are positive if *a* is large enough relative to *G* and *E* (for instance $a \geq (k+1)E + G$ is a sufficient condition).

C.II.7.b

The second fully specified pair of contracts is:

$A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = \dfrac{k-b}{2t}$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \dfrac{k(k-b)}{2t}$.

Notice that this pair of contracts is similar to the previous one, excepting the values of coefficients C and Z, that is, the coefficients that denote the impact that a variation in the report of the electricity cost has on each price. Also notice that, whenever $\hat{\varepsilon}$ is positive, the resulting gas price under this scheme will be higher than the one induced by the previous pair of contracts, because $C > 0$. The resulting electricity price will be higher iff $Z > ½$, that is, iff $1 - bk < 0$.

The second order condition of the problem is that the matrix of second order derivatives of the profits with respect to the announced costs must be negative semidefinite, that is $\begin{bmatrix} \frac{\partial^2}{\partial \hat{\gamma}^2} & \frac{\partial^2}{\partial \hat{\gamma} \partial \hat{\varepsilon}} \\ \frac{\partial^2}{\partial \hat{\gamma} \partial \hat{\varepsilon}} & \frac{\partial^2}{\partial \hat{\varepsilon}^2} \end{bmatrix} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon})$ is negative semidefinite, i.e. $\frac{\partial^2}{\partial \hat{\gamma}^2} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) < 0$, and $\frac{\partial^2}{\partial \hat{\gamma}^2} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \cdot \frac{\partial^2}{\partial \hat{\varepsilon}^2} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) - \frac{\partial^2}{\hat{\gamma} \partial \hat{\varepsilon}} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \cdot \frac{\partial^2}{\hat{\gamma} \partial \hat{\varepsilon}} \Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon}) \geq 0$.

In this case these two expressions correspond respectively to $-\dfrac{t}{2(1-b^2)} < 0$ and $\left(-\dfrac{t}{2(1-b^2)}\right)\left(-\dfrac{(k-b)^2}{2(1-b^2)t}\right) - \left(-\dfrac{(k-b)}{2(1-b^2)}\right)^2 = 0 \geq 0$. By noting that $\Pi_d(\gamma, \varepsilon, \hat{\gamma}, \hat{\varepsilon})$ is a second order polynomial in the announced costs, or by looking at the matrix of second order derivatives and noticing that all terms are independent of the cost announcements, it is easy to see that third derivatives are all zero, so the point is a relative maximum.

Equilibrium quantity outcomes under this contract are $q_g = \dfrac{a(1-b)t - (1-bk)[t\gamma + (k-b)\varepsilon]}{2(1-b^2)t}$, and $q_e = \dfrac{a(1-b)t - (k-b)[t\gamma + (k-b)\varepsilon]}{2(1-b^2)t}$.

Non-negativity of electricity production for every possible value of the cost parameters γ and ε is feasible iff $a \geq \dfrac{k-b}{1-b} \dfrac{tG + (k-b)E}{t}$.

It is straightforward to show that profits are positive if a is large enough relative to G and E (for instance $a \geq (k+1)E + G$ is a sufficient condition).

Appendix D

In this appendix we prove dominance, either ex post or ex ante, between some of the optimal incentive compatible contracts described in the preceding appendices.

D.1 One Firm

Lemma D.2.1. *Among both fully specified pairs of contracts, the first dominates (ex post) the second.*

Proof

Contract coefficients for both pairs of contracts are as follows:

C.7.a) $A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = 0$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \frac{1}{2}$.

C.7.b) $A = \frac{a}{2}$, $B = \frac{1}{2}$, $C = \dfrac{k-b}{2t}$; $X = \frac{a}{2}$, $Y = \frac{k}{2}$, and $Z = \dfrac{k(k-b)}{2t}$.

While it can be seen that the price of gas will be higher with the second pair of contracts than with the first, we cannot assert the same for the price of electricity: it will be higher only if $1 - b\,k < 0$. Nevertheless, when computing global welfare as a function of parameter values γ and ε, we get the following expression, in which all terms are positive:

$$W_{7a}(\gamma,\varepsilon) - W_{7b}(\gamma,\varepsilon) = \frac{(2a(k-1)+3\varepsilon)\varepsilon}{8t} \geq 0 .$$

This means that the welfare loss due to the increase in the price of gas is never offset by a welfare gain associated to a potential decrease in the price of electricity. *Q.E.D.*

This result of course also holds when taking the expected value over γ and ε. In fact, it is easy to see that ex ante dominance will be strict if $Pr(\varepsilon > 0) > 0$.

Lemma D.2.2. *Optimal pairs of contracts in which the price of gas is fixed dominate(ex post) pairs of contracts in which all coefficients are fully specified unless maximum demand parameter a is small relative to G. Ex ante dominance is always verified for uniform distributions.*

Proof

We have previously shown that the first of the two pairs of fully specified contracts performs better, so it is sufficient to prove that the optimal pair of contracts with fixed gas price performs even better.

This is shown by the following inequalities:

$$W_4(\gamma,\varepsilon) - W_{7a}(\gamma,\varepsilon) = \frac{(a+2E[\gamma]-3\gamma)(a-2E[\gamma]+\gamma)}{8} \geq 0$$

$$E_U[W_4(\gamma,\varepsilon)] - E_U[W_{7a}(\gamma,\varepsilon)] = \frac{a(a-G)}{8} \geq 0 \qquad \textit{Q.E.D.}$$

Lemma D.2.3. *Optimal pairs of contracts in which the price of gas is fixed dominate (ex ante) optimal pairs of contracts in which the price of electricity is fixed for uniform cost distributions.*

Proof

This can be seen by looking at the following inequality:

$$E_U[W_4(\gamma,\varepsilon)]-E_U[W_5(\gamma,\varepsilon)]=\frac{k\cdot G(2a-E)+2a(E-G)}{16}\geq 0 \Leftrightarrow k\geq 1\geq 1-\frac{E(2a-G)}{G(2a-E)}$$

Q.E.D.

Lemma D.2.4. *Optimal pairs of contracts in which both prices depend only on ε dominate (ex ante) optimal pairs of contracts in which both prices depend only on γ, for uniform cost distributions, if maximum demand parameter a is large enough relative to G and E.*

Proof

This can be seen by looking at the following inequality:

$$E_U[W_3(\gamma,\varepsilon)]-E_U[W_2(\gamma,\varepsilon)]=\frac{a(2a(1-b)k-t\cdot G-(k-1)E)}{8t}\geq 0 \Leftrightarrow a\geq\frac{t\cdot G+(k-1)E}{2k(1-b)}$$

Q.E.D.

Notice that this means that (couterintuitively) it is better to regulate via ε than via γ, that is, by using the downstream cost rather than the upstream cost!

Lemma D.2.5. *Dominance among the four remaining optimal pairs of contracts cannot be generally asserted for all possible parameter values.*

Proof

This can be seen by looking at the expressions of expected welfare for each pair of optimal contracts, and finding parameter values that change their order.

$$E_U[W_1^*]=\frac{a(2a-(k+1)G-E)}{4(1-b)}+\frac{a\sqrt{2(1-b)\phi}}{4(1-b^2)}$$

$$E_U[W_3^*]=\frac{a(2a-(k+1)G-E)}{4(1-b)}+\frac{a(1-b)(a(1-b)(k+1)^2-(1+b)(k-1)E)}{8(1-b^2)t}+\frac{\phi}{8(1-b^2)}$$

$$E_U[W_4^*]=\frac{a(2a-(k+1)G-E)}{4(1-b)}+\frac{2a(1-b)(a(1+b)+(k-b)G+E)-(k-b)GE}{16(1-b^2)}+\frac{\phi}{8(1-b^2)}$$

, where $\phi=2a(1-b)(a-(k+1)G-E)+tG^2+2(k-b)GE+E^2$ is a positive quantity equal to the denominator of the first-best welfare W^{FB}, evaluated in $\gamma=G$ and $\varepsilon=E$ (that is, in the worst possible cost scenario).

Contract 6 is too complicated to be solved analytically, but comparison of these three previous contracts is sufficient to prove our point. The *Mathematica* software application package was used to build numerical solutions to the constrained optimisation problem of the regulator in each IC contract set, and comparing the outcomes. The graphic in Figure D.1 illustrates this result, by showing the optimal contract choice for fixed values of *a*, *G* and *E*, as a function of parameters *b* and *k*.

If substitutability *b* is sufficiently low (yellow and green regions), the ex ante socially optimal contract choice is a pair of fixed price contracts, in which prices are set so as to (ex ante) optimally subsidize electricity production with gas revenue. For intermediate values of *b* and values of *k* in the blue region, the ex ante socially optimal contract choice is a pair of contracts in which the price of gas is fixed at its expected cost level. Finally, for large values of *b* and values of *k* in the cyan region, the ex ante socially optimal contract choice is the ex ante optimal pair of ε contingent contracts. Observe that there is a yellow region between the blue and the cyan region in which fixed price contracts also dominate.

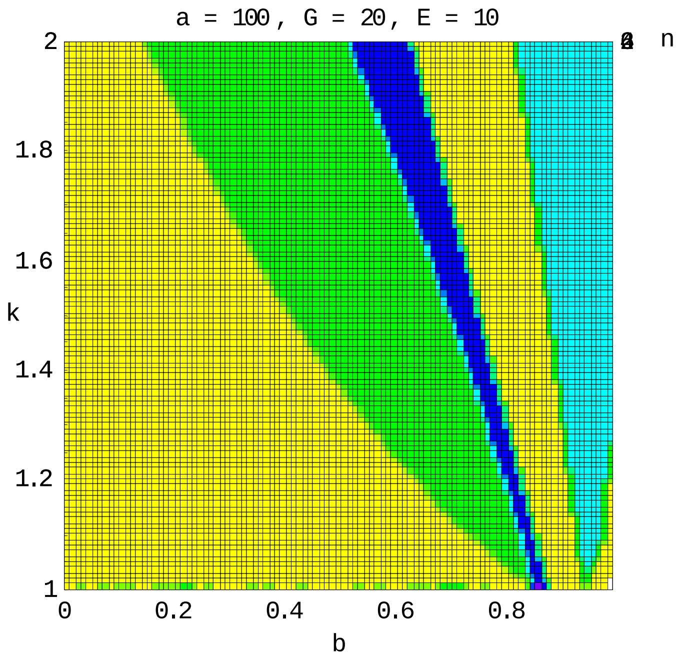


Figure D.1. Ordering of IC contracts
as a function of substitutability and technical dependency
(: $1 \succ 3 \succ 4$; : $1 \succ 4 \succ 3$; : $4 \succ 1 \succ 3$; : $3 \succ 1 \succ 4$)

The shape and size of these regions change quantitatively with the remaining three parameters, but their qualitative positions remain the same. As total demand parameter *a* increases relative to *G* and *E*, the region in which a pair of price-caps dominates the other possible pairs of contracts grows larger; as *E* increases relative to *G*, the blue region shrinks. These tendencies are shown in the following figures.

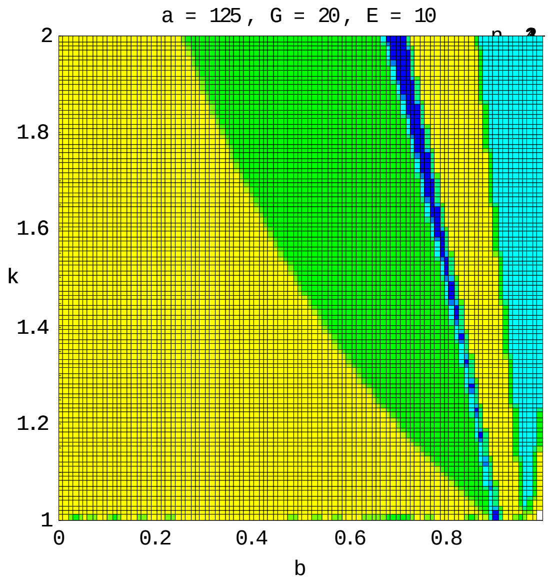


Figure D.2.

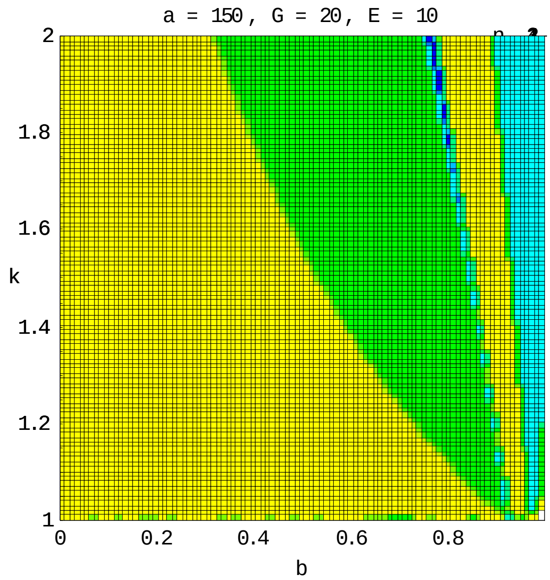


Figure D.3.

(: $1 \succ 3 \succ 4$; : $1 \succ 4 \succ 3$; : $4 \succ 1 \succ 3$; : $3 \succ 1 \succ 4$)

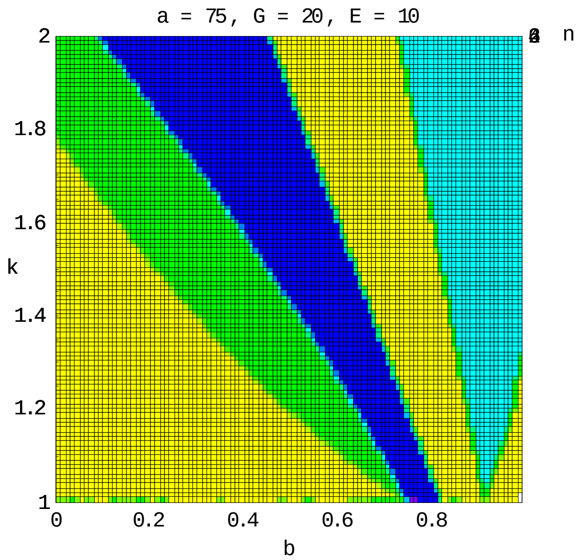


Figure D.4.

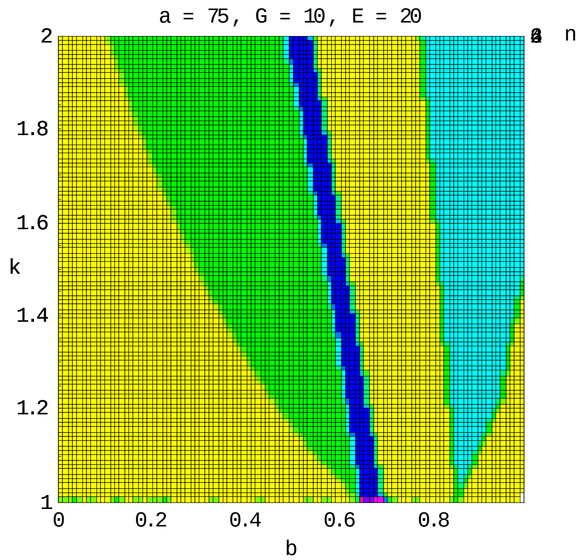


Figure D.5.

(: $1 \succ 3 \succ 4$; : $1 \succ 4 \succ 3$; : $4 \succ 1 \succ 3$; : $3 \succ 1 \succ 4$)

D.2 Two Firms

As was seen in Appendix B, the solution to the ex ante maximization problem of the regulator requires in some cases solving fifth degree equations in the contract coefficients, which have no explicit algebraic solutions. Thus, it is impossible to assert generally the dominance of one set of optimal contracts over the others. However, it is possible to fully implement numerical algorithms for solving each of the optimisation problems for a given set of parameters, and thus, for each given parameter set, contract performances can be compared. We solved some representative numerical examples to get an idea of the qualitative comparisons at hand.

The *Mathematica* software application package was used to build numerical solutions to the constrained optimisation problem of the regulator in each IC contract set, and comparing the outcomes. The following figures show which type of optimally tuned pair of contracts performs the best as a function of parameters b and k, for uniform cost distributions and specific values of a, G and E that we use for benchmarks. We also include the corresponding function of (optimal) expected welfare in each case.

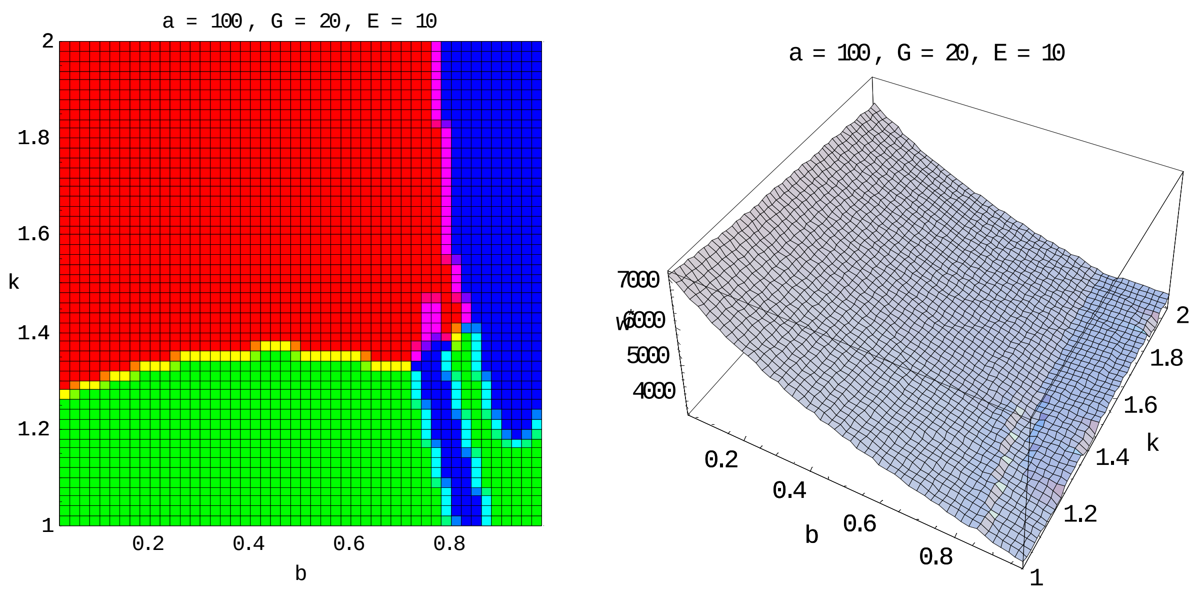


Figure D.6. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency
(: 1; : 2 ; : 3)

The green colour in the graphic corresponds to areas in which the regulator maximizes expected social welfare by choosing an optimally tuned pair of fixed price contracts; the blue zone corresponds to those where she should choose a pair of optimally tuned epsilon contingent contracts; and the red zone to those in which a pair of optimally tuned gamma contingent contracts performs best. If substitutability b is low, the regulator should prefer a pair of price-caps when technical dependency k is also low, and a pair of gamma contingent contracts when k is high. On the other hand, if substitutability b is high, a pair of epsilon contingent contracts might be better.

The following figures show how these regions change with parameters a, G and E.

If total demand increases relative to cost uncertainty, the zone where price-caps dominate gets larger, and the zone in which gamma contingent contracts dominate decreases.

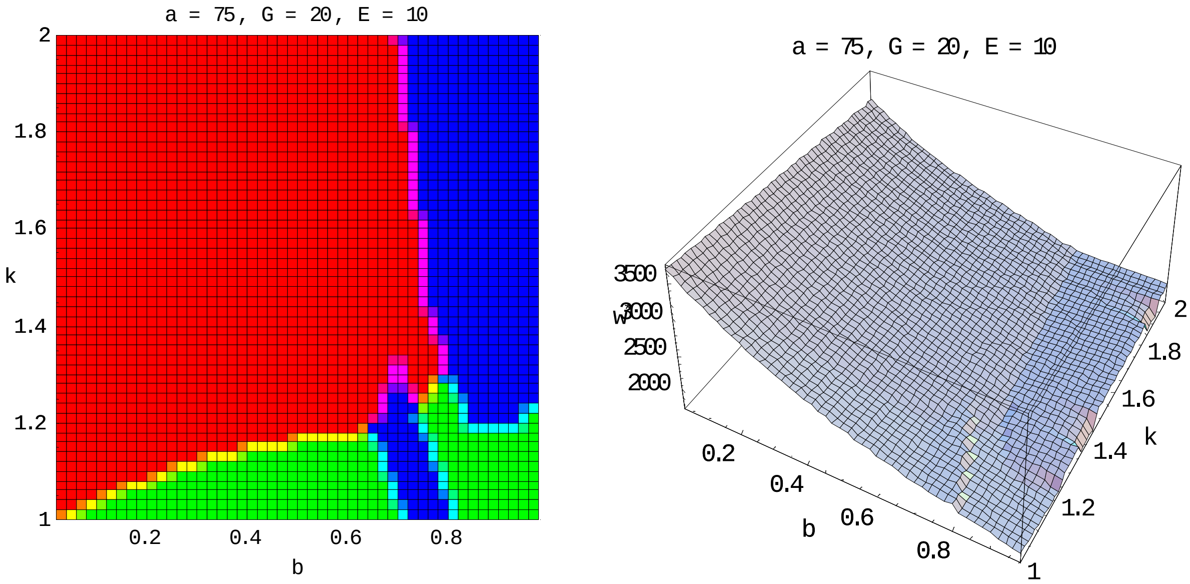


Figure D.7. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency

(: 1; : 2 ; : 3)

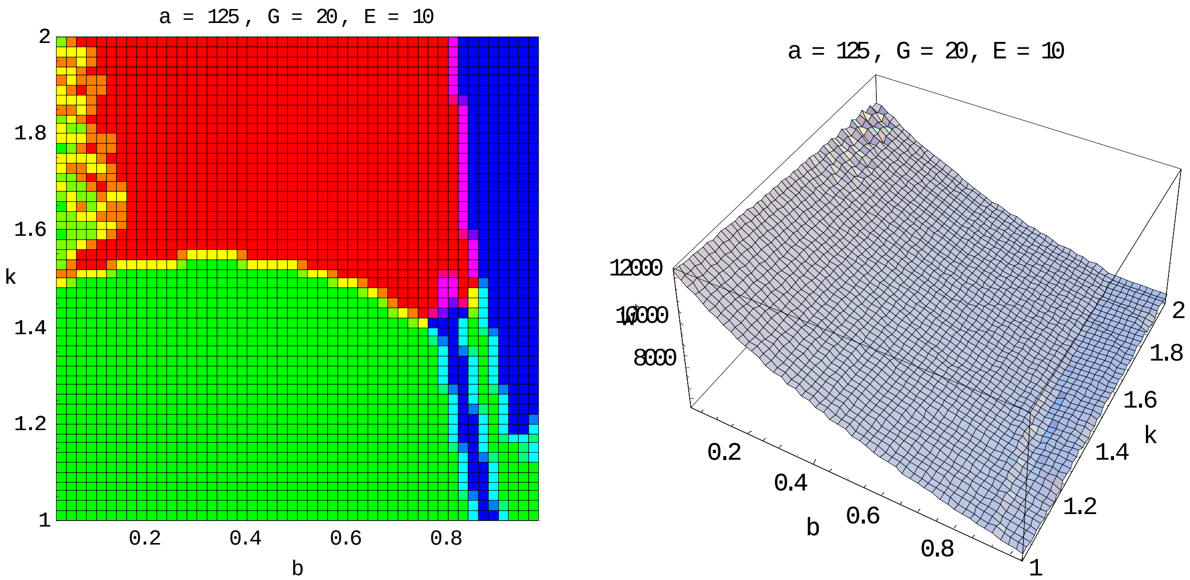


Figure D.8. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency

(: 1; : 2 ; : 3)

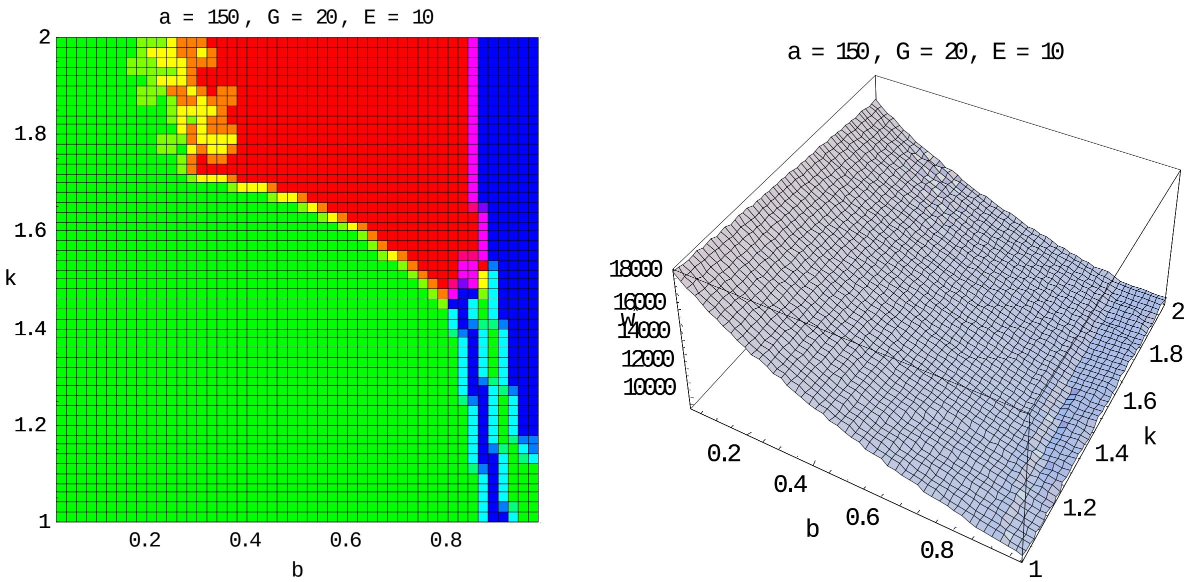


Figure D.9. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency
(: 1; : 2 ; : 3)

If electricity cost uncertainty increases relative to gas cost uncertainty and total demand (or equivalently if gas cost uncertainty decreases relative to electricity cost uncertainty and total demand), the zones in which price-caps and epsilon contingent contracts dominate shrinks, and the zone in which gamma contingent contracts dominate expands.

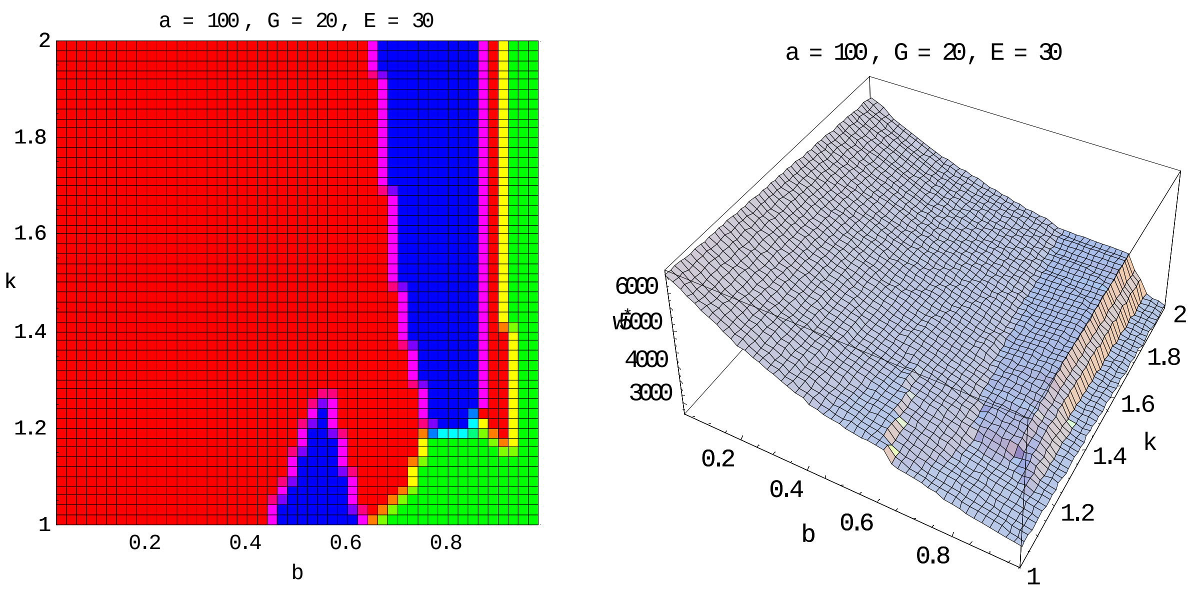


Figure D.10. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency
(: 1; : 2 ; : 3)

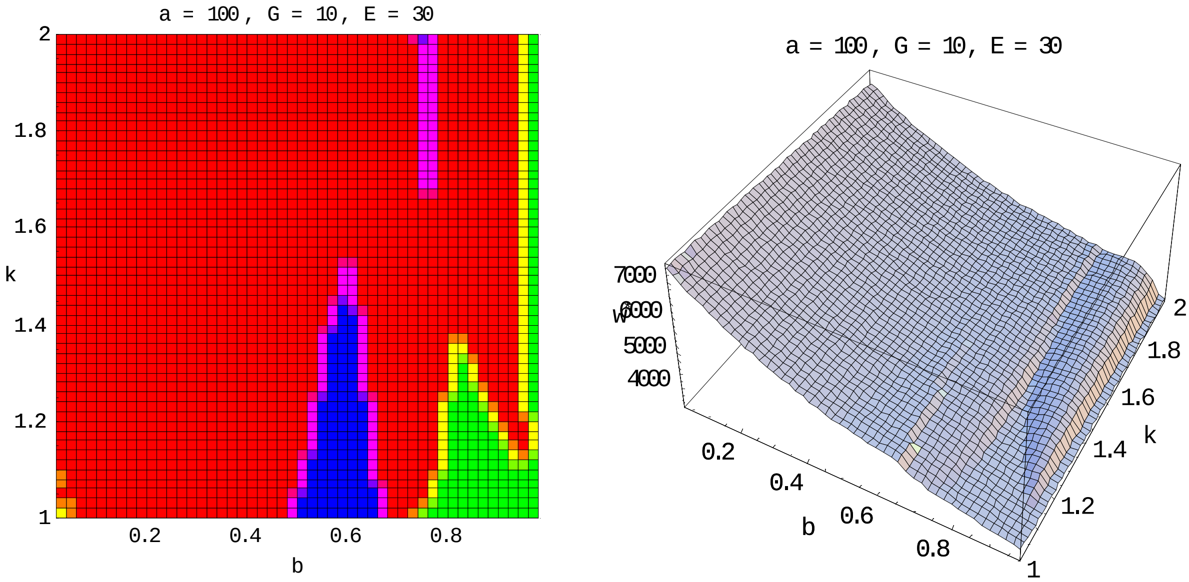


Figure D.11. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency

(: 1; : 2 ; : 3)

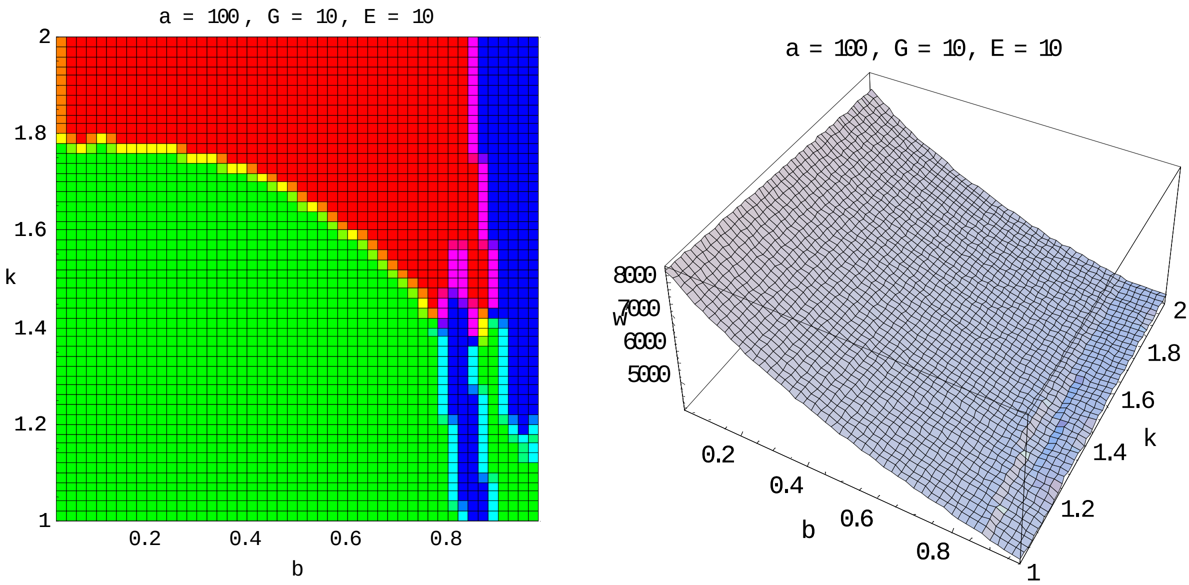


Figure D.12. Optimal choice of IC contracts and expected welfare as a function of substitutability and technical dependency

(: 1; : 2 ; : 3)