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Wealth Effects, Moral Hazard and Adverse Selection in a Principal-Agent Model

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Wealth effects, moral hazard and adverse selection in a principal-agent model.

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Abstract

This paper introduces hidden action in an adverse selection model, assuming that a risk neutral principal faces a risk averse agent. The optimal contract is compared against the two benchmark situations where either only hidden information or only hidden action arises. Most properties of incentive contracts pointed out by the literature still hold. Dealing with the two sources of asymmetric information only modifies the optimal effort, wealth effects playing an important role to determine this optimal action.

Key words : adverse selection, moral hazard, wealth effect.

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1 Introduction

In organizations agents take decisions and search for information necessary to act, so agency relationships characterized by hidden action and hidden information have a great empirical relevance. Most of the incentive theory literature separates the two issues, one part dealing with adverse selection alone, and the other with hidden action. On one hand you meet trade-off between rent and efficiency, on the other hand, conflicts between insurance and incentive. It is well known, that, surprisingly, the optimal contracts in these two settings exhibit some very different properties. Depending on the available information, asymmetric information then creates two different issues, even if it comes from the same economic setting: the principal ignores something known by the agent.

However, the frontier between these two kinds of problems sometimes seems fragile. For instance, imagine an agent whose utility function $U(t, a, \theta)$ depends on money t, action a, and private information θ. Assuming that the principal observes the action but not the type reduces the problem to an adverse selection one. Conversely, when the principal observes the type and can learn some information about the decision through a random signal, she then faces moral hazard. These two situations are now well known, but for our purpose, modify a little this standard information structure, the principal ignoring now both type and action. When a signal about action is available, then the relationship is affected by both hidden information and hidden action which occur together. But, surprisingly, when such a signal does not exist, the situation is adverse selection once again.[1]

What are the lessons of this example? Because the optimal scheme depends on available information, adverse selection and moral hazard situations are in reality more linked than incentive theory literature suggests. Moreover, adverse selection may reflect a situation in which the principal is less informed than she would be under both hidden action and hidden

[1] To see why, note that in this case the agent's utility can be rewritten as a function of money and information $u(t, \theta) = Max_a U(t, a, \theta)$, which makes the action to disappear.

information. Finally, the differences observed between the two benchmark models, in the spirit of Baron and Myerson (1982) on the adverse selection side, and following Holmström (1979) as far as moral hazard is concerned, could be due to the fact that they stand as limit cases of a unique unified model, in which both information problems exist. Perhaps in such a situation, and this would be good news for incentive theory, the two families of properties would hold together, so that the conclusions of moral hazard models and those of adverse selection could be extended to the mixed case. The principal's trade-off would then consist in comparing rent, insurance, and efficiency. This intuition motivates this paper.

Our object is to answer the following questions: what properties has the optimal contract under hidden action and hidden information? How does the presence of both asymmetric information sources alter incentive contracts? What do the two benchmark trade-offs between risk and insurance, rent and efficiency become? In order to answer, we have to compare three models of asymmetric information. So we need to adopt a basic and simple framework. In such an attention, we introduce a hidden action element in the standard model of adverse selection due to Baron and Myerson (1982). As seen in the presentation of the model, this modification is exactly equivalent to introduce hidden information in Homlström's (1979) basic hidden action model. In other words, we gather all the assumptions made in these two benchmark models.

In such a context, and on condition that the first order approach holds, we show here that the optimal schemes keep all usual properties. First, if the principal wants to implement the same action profile under adverse selection taken isolated than under both adverse selection and moral hazard, she will pay the same rent to the agent in those two settings. Second, the optimal transfer under both adverse selection and moral hazard has all the features pointed out by Holmström (1979). Finally, mixing the two information sources only changes optimal actions.

So most results concerning optimal contracts can be extended to the case where both information sources arise together. However, as far as optimal actions are concerned, the principal objective seems deeply affected by wealth effects, the key point being how wealth and risk interact to modify the agent's behavior. Because the principal pays a rent to the agent, she modifies how the agent perceives the risk he bears, which is due to the presence of hidden action. So the convenient trade-off of our mixed model will be made in terms of insurance, rent and efficiency.

Recently some authors (Chassagnon and Chiappori (1997), Jullien, Salanié and Salanié (2000), Chiappori, Jullien, Salanié and Salanié (2001)) analyze situations where hidden information and hidden action are present together, in an insurance context. To a certain extent, their analysis yields the same result we present here. Chassagnon and Chiappori (1997) are interested in competitive situations, in a model with two outcomes, two actions and two types. They use the framework of Rothschild and Stiglitz (1976) to analyze the equilibrium of the insurance market. Jullien, Salanié and Salanié (2000) assume that adverse selection concerns the agent's risk aversion, whereas we follow here Baron and Myerson (1982) with the fact that the principal ignores the agent's production cost. They show how the adverse selection mechanisms are still present in a mixed model.

Two papers are very close to ours. The first one (due to Baron and Besanko (1987)) studies a procurement relationship when a principal observes an imperfect monitor of the agent's production cost. This production cost depends on a productivity parameter and an effort chosen by the agent, and in addition may be random. More precisely, Baron and Besanko focus on two cases, either the agent's cost is random and the monitor certain or conversely the agent's cost is certain and the monitor random. This means that either only adverse selection exists or hidden action and hidden information both appear. Unusually in an adverse selection model à la Baron and Myerson, the principal faces an agent who bears some unobservable risk, without being able to provide him insurance, so that in the two cases, the principal faces a trade-off in terms of rent, insurance and efficiency. The situation in which both hidden action and hidden information are present cannot be simply derived from the adverse selection case by assuming that the principal cannot observe action. For this reason, no comparison is conducted in the sense we develop here, with a view to understanding what an additional asymmetric information element creates. As a consequence, our conclusions are rather different, even if some of their properties hold in our context. Similarities between our and their results are pointed in the body text.

The same remark can be made concerning the second paper, due to Faynzilberg and Kumar (1997). With close assumptions, the authors generalize Baron and Besanko results by giving the necessary optimality conditions leading to the optimal contract, but provide no comparison with the adverse selection case.

As a matter of fact, these two papers use very general or complex information structures. Simplifying this technology will allow to obtain a tractable model well related to the benchmark ones, which makes any comparison easier.

Section 2 presents the model and applies the first order approach to our context. In section 3 we develop the properties of the optimal schemes, then section 4 is devoted to an interpretation of these properties in terms of risk premium. Finally section 5 concludes.

2 Model and preliminary results

2.1 The setting

Consider the following basic agency model. As in Holmström (1979), an agent (he) can engage in effort, denoted by a and chosen from an interval A. This decision is unobservable by the principal (she), and, together with a random state of nature, creates an observable monetary outcome $x \in X$. Let $f(x, a)$ the density function of this signal when the agent has chosen the action a. The principal can pay the agent a monetary transfer, $t \in R$, depending on the observed outcome x.

As assumed in Baron and Myerson (1982), the principal ignores the value of a random variable, called the agent's type and noted θ. It belongs to an interval $\Theta = [\theta_0, \theta_1]$ and has a density function $g(\theta)$. The principal being risk neutral, her utility function is given by $S(a, \theta) - t$.

Effort and type affect the agent's production monetary cost $c(a, \theta)$, that increases with a and θ, is convex with respect to a, and satisfies the standard single crossing property $c_{\theta a}(a, \theta) \geq 0$. As assumed in Holmström (1979), the agent is risk averse, his utility function can be written as $U(t) - c(a, \theta)$ where $U(.)$ designs a Von Neumann Morgenstern (VNM hereafter) utility function, increasing and strictly concave, that is $U' > 0$ and $U'' < 0$. Its inverse function is noted φ.

At the time of contracting the agent has a private information θ about the preferences (his cost and the principal's surplus S). The principal has only prior beliefs represented by g. Ignoring θ, the principal offers a menu of contracts, $t(x, m), x \in X, m \in \Theta$ to the agent[2], who refuses or agrees to participate to the relationship, then selects one of them by reporting a message m. If the agent refuses, he obtains his reserve utility $\underline{U}$. If he agrees, after reporting his message, he decides to exert an effort a. Then the random state of nature x is realized leading to payment.

Both hidden information and hidden action are then present in this relationship. In what follows, hidden information (respectively hidden action) will state as a synonymous of adverse selection (moral hazard).

Note that our assumptions exactly introduce a moral hazard element in Baron and Myerson (1982) private information model. To see why, imagine that the signal perfectly reveals the action: the principal deduces from the observation of x what action a has be chosen. This new framework describes Baron and Myerson adverse selection problem, the only difference

[2] The revelation principle implies that we can restrict our attention to direct mechanisms.

being that the agent is risk averse. Moreover, our model exactly adds a private information element to Holmström's (1979) moral hazard model: if the principal's prior beliefs were perfect, the principal would know the true value of type and the model would be Holmström's one.

This feature is the source of our paper, because it allows to compare our model where hidden information and hidden action exist simultaneously against the two benchmark models, due to Baron and Myerson (1982) on one hand, and to Holmström (1979) on the other hand. In order to obtain such a property, a key assumption is that the density function of the signal does not depend on θ, which means that the principal can infer no information about the agent's type through the observation of output. Without such an assumption, a general writing of the density function would be $f(x, a, \theta)$. But in that case, if the principal could control action, such a signal would reveal some information about the agent's type, evidently used to build the optimal contract. Particularly if the agent is risk neutral and unlimited transfers can be payed, the principal would implement the first best allocation, which is very different from Baron and Myerson context. This remark is the heart of this paper, because our goal is to compare optimal contracts when both problems are mixed and when they are not.

Note that however some of our results can be generalized to the case where the density function of the signal depends on the type but has affiliated values (as in Faynzilberg and Kumar (1997)).[3]

[3] If the signal has affiliated values, there exist some functions h and l such that $f(x, a, \theta) = l(x, h(a, \theta)), \forall a \in A$ and $\forall \theta \in \Theta$. But choosing h or a is equivalent for the agent. Define $C(h, \theta)$ as the minimal feasible production cost given h for all the actions such that $h(a, \theta) = h$ that is $C(h, \theta) = Min_{a/h(a,\theta)=h} c(a, \theta)$ and imagine that the agent chooses h rather than a. The principal then faces an agent, whose cost function is $C(h, \theta)$ and whose activity generates a signal with a density function $l(x, h)$, which is exactly our model. What is different is the related relationship when only hidden knowledge exists.

2.2 The principal's program

Invoking the revelation principle (see Myerson 1982 for a general version including moral hazard), we can describe the incentive compatible allocations in each of the three situations, termed hidden information (HI), moral hazard (or hidden action HA), and mixed hidden information and hidden action (HIHA) as in the following definitions:

Definition 1: $\{t(x,\theta), a(\theta), \theta \in \Theta\}$ *is implementable under hidden information if the agent tells the truth, whatever his type, that means whatever* $\theta \in \Theta$ *we have*

$$\int_X U(t(x,\theta))f(x,a(\theta))dx - c(a(\theta),\theta) \geq \int_X U(t(x,m))f(x,a(m))dx - c(a(m),\theta), \forall m \in \Theta. \quad (1)$$

Definition 2: $\{t(x,\theta), a(\theta), \theta \in \Theta\}$ *is implementable under hidden action if the agent is obedient, so he chooses the recommended action whatever his type is, that means* $\forall \theta \in \Theta$,

$$\int_X U(t(x,\theta))f(x,a(\theta))dx - c(a(\theta),\theta) \geq \int_X U(t(x,\theta))f(x,a)dx - c(a,\theta), \forall a \in A. \quad (2)$$

Definition 3: $\{t(x,\theta), a(\theta), \theta \in \Theta\}$ *is implementable with hidden action and hidden information if the agent tells the truth and is obedient, he chooses the recommended action whatever his type, that means* $\forall \theta \in \Theta$,

$$\int_X U(t(x,\theta))f(x,a(\theta))dx - c(a(\theta),\theta) \geq \int_X U(t(x,m))f(x,a)dx - c(a,\theta) \forall m \in \Theta, \forall a \in A. \quad (3)$$

The principal has to choose an incentive compatible mechanism, $t(x,\theta)$ and $a(\theta)$ in order to maximize her expected gain H :

$$H = \int_\Theta g(\theta)[S(a(\theta),\theta) - \int_X t(x,\theta)f(x,a(\theta))dx]d\theta \quad (4)$$

subject to the participation constraint

$$\int_X U(t(x,\theta))f(x,a(\theta))dx - c(a(\theta),\theta) \geq \underline{U},$$

and some incentive constraints depending on the informational context. We then have to take into account condition (1) when the principal knows action but ignores type, condition

(2) when on the opposite the principal knows type but does not observe action and condition (3) when she observes nothing.

Note that every menu of contracts that is incentive compatible under both hidden information and hidden action, is also incentive compatible under only adverse selection, or under only moral hazard. A simple argument of revealed preferences thus allows to conclude that the principal's expected utility will be lower when the two problems are mixed than when there exists only one source of asymmetric information.

As usual in adverse selection models, define $v(\theta)$ as the expected utility, associated with an action $a(\theta)$, of a type θ agent:

$$v(\theta) \equiv \int_X U(t(x,\theta))f(x,a(\theta))dx - c(a(\theta),\theta)$$

This definition simplifies the writing of incentive constraints, as shown in the following lemma.

Lemma 1 *Under hidden information, an implementable contract satisfies* $v'(\theta) = -c_\theta(a(\theta),\theta)$ *(RC) and* $c_{\theta a}(a(\theta),\theta)a'(\theta) \leq 0$ *(RC2). Under hidden action, it satisfies* $\int_X U(t(x,\theta))f_a(x,a(\theta))dx = c_a(a(\theta),\theta)$ *(IC) and* $\int_X U(t(x,\theta))f_{aa}(x,a(\theta))dx - c_{aa}(a(\theta),\theta) \leq 0$ *(IC2). Finally, under both hidden information and hidden action we have RC, IC and* $a'(\theta)\{\int U(t(x,\theta))f_{aa}(x,a(\theta))dx - c_{aa}(a(\theta),\theta)\} \geq c_{\theta a}(a(\theta),\theta)$.

The proof of lemma 1 is standard and is reported in appendix as all other omitted proofs.[4]

In a aim of simplicity, we ignore in this paper second order conditions (RC_2) and (IR_2), that is we assume that the so called "first order approach" is valid.[5] We can replace the incentive compatibility constraint (given by (1), (2) or (3) depending on the informational

[4] See also Laffont and Tirole (1986) and Baron and Besanko (1987) for more details.

[5] The terminology "first order approach" comes from moral hazard theory. Main papers assume that the agent's incentive constraint (2) can be replaced by (IC) in the principal's program. Even if the term "first order approach" is not used in adverse selection models, the second order condition (RC2) is often ignored.

context) by the condition (RC) in the adverse selection situation, by (IC) in the moral hazard one and by these two conditions when both problems are mixed. In this case, one can conclude that the set of implementable allocations when both problems are present is exactly the intersection of the two sets of implementable allocations under hidden action alone and hidden information alone.

Finally the principal has to choose $t(x,\theta), v(\theta)$ and $a(\theta)$ with a view to maximizing her expected gain H given in (4) subject to the incentive compatible constraints, that we rewrite below:

$$\int_X f(x,a(\theta))U(t(x,\theta))dx - c(a(\theta),\theta) = v(\theta) \tag{UC}$$

$$v'(\theta) = -c_\theta(a(\theta),\theta) \tag{RC}$$

$$\int_X f_a(x,a(\theta))U(t(x,\theta))dx = c_a(a(\theta),\theta) \tag{IC}$$

$$v(\theta) \geq \underline{U} \tag{PC}$$

The utility constraint UC defines the state variable $v(\theta)$ as the expected utility (rent) of an agent whose type is θ. RC (revelation constraint) and IC (incentive constraint) are the first order conditions implied by the fact that an agent θ has to reveal the truth and to choose the action $a(\theta)$, and PC is a standard participation constraint. Let P_{HIHA} denote this program. Note that if the principal observes action, the program is unchanged except IC disappears (let P_{HI} this adverse selection program), and in like manner, if the principal knows the agent's type, she ignores RC, P_{HA} is the hidden action program where RC is omitted.

2.3 The two benchmark models

Before presenting our results concerning hidden action and hidden information taken together, we devote this subsection to present in an unified context some standard results concerning adverse selection and moral hazard, taken isolated. What follows can be found in the papers of Holmström (1979) and of Baron and Myerson (1982).

Adverse selection contract

Assume first that the principal can observe the action. An optimal contract maximizes H, subject to RC, UC, and PC. We present in lemma 2 its properties.

Lemma 2 *The optimal adverse selection contract entails perfect insurance, that is $t(x,\theta) = t(\theta)$, for every $x \in X$. Moreover, the participation constraint is binding $v(\theta_1) = \underline{U}$ and if the optimal action is interior, it solves equation (5) given below.*

$$\{S_a(a,\theta) - \varphi'(v+c)c_a\}g(\theta) - c_{\theta a}\int_{\theta_0}^{\theta} \varphi'(v(y) + c(a(y),y))g(y)dy = 0$$

Note that lemma 2 states that a stochastic contract will never be used to alleviate revelation problem in this model. Under perfect insurance, the expected payment to a $\theta-$agent can be straightforwardly derived from UC since it verifies $U(t(\theta)) - c(a(\theta),\theta) = v(\theta)$. As φ denotes the inverse function of $U(.)$, this expected transfer equates $\varphi(v + c(a,\theta))$, which represents what the principal has to pay for providing the agent an expected rent v under perfect insurance.[6] Taking into account this feature allows to rewrite the principal's program in a very usual way, in which the optimal action and the rent maximize the principal's objective $\int_{\Theta}\{S(a,\theta) - \varphi(v + c(a,\theta))\}g(\theta)d\theta$ subject only to RC and PC. As $\varphi(v + c(a,\theta))$ increases with v, the participation constraint of the highest type is binding, that is $v(\theta_1) = \underline{U}$, and, thanks to the monotony assumption under which $c_\theta(a,\theta) > 0$, slack for the other types. Solving this problem needs standard methods of dynamic optimization which give equation (5).

As usual in adverse selection problems, the first term of equation (5) comes from the surplus of the relation, benefit S minus cost φ. The second term reveals the rent mechanism: increasing the action for a type θ rises the expected transfer payed to all types below θ. This

[6]Note that in the adverse selection Baron and Myerson model, φ' is constant, the agent being risk neutral.

trade-off between rent and efficiency creates underproduction (relative to the perfect information action defined by $S_a(a,\theta) = \varphi'(\underline{U} + c)c_a)$, and the more important is the probability assigned to the interval of types $[\theta_0, \theta]$, the smaller will be the optimal action.

Hidden action

Assume now that the principal knows the agent's type but cannot observe his decision. For a given action profile $a(.)$, the optimal reward scheme minimizes the expected transfer, $\int_\Theta \{\int_X t(x,\theta) f(x, a(\theta))dx\} g(\theta)d\theta$ subject to UC, IC and PC. As proved in Holmström (1979), the participation constraint is binding whatever θ and there exist two (strictly) positive multipliers, $\lambda(\theta)$ and $\mu(\theta)$, such that:

$$\frac{1}{U'(t(x,\theta))} = \lambda(\theta) + \mu(\theta)\frac{f_a(x,a(\theta))}{f(x,a(\theta))} \tag{6}$$

Replacing this expression of the optimal transfer in UC, PC and IC leads to the values of $\lambda(\theta)$ and $\mu(\theta)$.

For our purpose, define $\Psi(\underline{U}, a(\theta), \theta)$ this optimized expected transfer, that is what the principal has to pay for providing the agent an expected utility $\underline{U}$ under hidden action. This function, which plays an important role in our problem, will be interpreted in details in section 4. As finally the optimal action is chosen so as to maximize $S(a,\theta) - \Psi(\underline{U}, a, \theta)$, provided some concavity assumptions, it then solves equation (7):

$$S_a(a,\theta) = \Psi_a(\underline{U}, a, \theta) \tag{7}$$

The features of the function Ψ reflect the trade-off existing between insurance and efficiency. Note that little is known about this action. We will explain hereafter what mechanism affects this choice.

We turn now to present our results concerning the case where the two sources of asymmetric information simultaneously arise.

3 Properties of the optimal contract

Surprisingly in this model, as seen in the equation defining RC, the marginal rent $v'(\theta)$ depends only on type and action. The agent is then indifferent between two incentive compatible transfers (provided that $v(\theta_1)$ is identical), even if one of them for example is more variable. In other words the revelation constraint is identical in the two models (HIHA and HI) where adverse selection appears, hidden action being present or not. Integrating RC with respect to the type gives then the agent's rent:

$$v(\theta) = v(\theta_1) + \int_{\theta_1}^{\theta} c_\theta(a(y), y)dy$$

So provided that the participation constraint is binding at the top under HIHA, introducing hidden action in an adverse selection problem does not affect the rent design. Such a property then implies the next proposition.

Proposition 3 *Let $a^*(\theta)$ and $v^*(\theta)$ the optimal action and rent solution of P_{HIHA}. For given a^* and v^*, the optimal transfer minimizes the expected transfer subject to IC and UC. The participation constraint is binding for the highest type, $v^*(\theta_1) = \underline{U}$ and the optimal action solves equation (8) given below.*

$$\{S_a(a,\theta) - \Psi_a(a,v,\theta)\}g(\theta) - c_{\theta a}\int_{\theta_0}^{\theta} \Psi_v(a(x), v(x), x)g(x)dx = 0 \qquad (8)$$

Proposition 3 deserves two kinds of comments. A first set of comments compare the three models (HI, HA, HIHA) assuming that the action is identical. In a second time we discuss how is chosen this optimal action.

• As suggested at the beginning of the section, for identical actions, the agent obtains the same rent in a hidden information model, moral hazard being present or not, (that is under HIHA or HI).

• As a consequence, the optimal reward, which is a solution of P_{HIHA}, solves also P_{HA}, for the same given $a(.)$ and $v(.)$. So hidden action payments are not modified by introducing hidden information. Optimal transfers have then all the properties of hidden action contracts, the only difference resting on the fact that the agent's reserve utility turns into rent. As noted in the previous section, payment depends on the likelihood ratio $\dfrac{f_a(x,a)}{f(x,a)}$, and increases with it (see Holmström 1979). [7][8]

• Proposition 3 provides a general method for solving this mixed problem, following two steps:

- first, focus on the hidden action contract, and determine for all (v, a) the optimal reward of which expectation is the smallest possible. Such a transfer satisfies equation (6) where the reserve utility $\underline{U}$ is replaced by v, so finally its expected value equates $\Psi(a, v, \theta)$;

- second, solve the adverse selection problem, knowing that, thanks to the preceding step, the principal's objective is now $\int_\Theta (S - \Psi) g(\theta) d\theta$ and the relevant constraints are PC and RC.

As $\Psi(a, v, \theta)$ increases with v (as proved in the appendix), the highest type's participation constraint binds, so $v(\theta_1) = \underline{v}$, and it is slack for the other types, thanks to the monotony assumption under which $c_\theta(a, \theta) > 0$. Comparing equation (5) with equation (8) shows that the implementation cost φ turns into Ψ whenever hidden action arises. So this new cost function enters in the calculus of optimal action.

• This separation result generalizes what happens in the case of risk neutrality, as studied by Laffont and Tirole (1986), Picard (1987) and Guesnerie, Rey and Picard (1992). Indeed, imagine for a moment that the principal faces a risk neutral agent, whose VNM utility function is $t - c$. Taking into account such a linearity, the incentive compatibility constraints

[7] More precisely, all the results concerning the optimal choice of information structures, for instance the sufficient statistics theorem, or the comparison of two signals due to Kim (1999), remain valid. In the same way, the optimal auditing policy is not affected by hidden knowledge (Sinclair-Desgagné 1999).

[8] Some part of this property has already been shown by Baron and Besanko (1988) and Faynzilberg and Kumar (1997), when they prove that the payment increases with the likelihood ratio.

now become:

$$\int_X f(x, a(\theta))t(x, \theta)dx - c(a(\theta), \theta) = v(\theta) \qquad (\widetilde{UC})$$

$$v'(\theta) = -c_\theta \qquad (\widetilde{RC})$$

$$\int_X f_a(x, a(\theta))t(x, \theta)dx - c_a(a(\theta), \theta) = 0 \qquad (\widetilde{IC})$$

$(\widetilde{UC})$ gives straightforwardly the expected transfer. The principal's program can then be rewritten as Maximize $\int_\Theta (S - c - v)g(\theta)d\theta$ with respect to v and a, and subject to $\widetilde{RC}$ and PC. So hidden action has disappeared, and every reward function solution of $\widetilde{IC}$ and $\widetilde{UC}$ is solution of our problem. But this is exactly what proposition 3 states because under risk neutrality, every payment such that $\widetilde{IC}$ and $\widetilde{UC}$ hold is an obvious solution of Holmström's moral hazard problem.

• This comparison with the case of risk neutrality suggests that, as the two models (HI and HIHA) have the same features under risk neutrality, risk behavior is surely the source of differences. As a consequence the properties of function Ψ will have to be related to risk aversion.

• Let us now discuss the choice of action and compare (5), (7) and (8). We have already remarked that (5) and (8) are identical, except that φ in equation (5) turns into Ψ in equation (8). The first term of equation (8) comes from the surplus of the relation, benefit S minus cost Ψ. The second term is due to hidden information mechanism, it represents the rent effect already pointed out in the second section. A first conclusion is then available: because of hidden action, the principal has to account with the agent's risk behavior. The function $\varphi(u)$ which represents the lowest expected transfer necessary for providing the agent with an expected utility u under perfect insurance, has been modified in $\Psi(a, v, \theta)$, the similar lowest expected transfer under hidden action. Moreover, comparing (7) and (8) shows that hidden information introduces the rent mechanism in the moral hazard model.

• Because the rent mechanism induces inefficiency, we then are able to compare the actions solving (7) and (8). Assuming that $S(a,\theta) - \Psi(a,v,\theta)$ is concave with respect to action, for $\theta > \theta_0$, the agent will exert more effort under moral hazard (HA) (knowing that the principal gives him the same expected utility v) than under both hidden information and hidden action (HIHA). This is due to the fact that Ψ rises with v. So paying a rent makes the action to fall. Note that this property was already true in the absence of moral hazard, that is when comparing perfect information to adverse selection.

• As we will see hereafter, comparing the adverse selection action (HI) with the action under hidden information and hidden action (HIHA) is much more complex. A first step in this direction is to understand in what the functions φ and Ψ differ, but the literature dealing with moral hazard provides few intuitions about this function. In section 4, we propose an interpretation of Ψ in terms of risk premium. Such an approach enlightens the working mechanisms, and reveals that some wealth effects appear in this principal-agent relationship.

4 An approach in term of risk premium

Changing some variables helps to present a simple interpretation of the implementation cost Ψ. Define the certainty equivalent e and the risk premium Π of the lottery associated with a random transfer $t(x), x \in X$ and an action a. By definition we have:

$$u(e) \equiv \int_X u(t(x))f(x,a)dx \tag{9}$$

$$\Pi \equiv \int_X t(x)f(x,a)dx - e \tag{10}$$

As by definition of expected utility, $v = u(e) - c$, for given action and utility $a(.)$ and $v(.)$, one can deduce the corresponding certainty equivalent $e(.)$. Recall that φ is the inverse function of u, (9) gives $e(\theta) = \varphi(v(\theta) + c(a(\theta),\theta))$. And (10) implies that $\int_X t(x)f(x,a)dx =$

$\varphi(v + c(a, \theta)) + \Pi$. Then minimizing expected transfer subject to UC and IC is equivalent to minimizing the risk premium Π. Let $\Pi^*(v, a, \theta)$ this minimized risk premium, we get:

$$\Psi(a, v, \theta) = \varphi(v + c(a, \theta)) + \Pi^*(v, a, \theta) \tag{11}$$

The implementation cost (i.e. expected transfer for a type θ) is made with two ingredients. On the one hand, the principal has to pay for the certainty equivalent necessary to provide an expected utility v, on the other hand, she must reimburse the risk the agent bears. This risk premium appears only when hidden action is present, and existing hidden information only, the risk premium would vanish.

Replacing $\Psi(a, v, \theta)$ by its value in (8) gives equation (12):

$$\{S_a(a, \theta) - \varphi'(v + c)c_a - \Pi_a^*\}g(\theta) - c_{\theta a} \int_{\theta_0}^{\theta} (\varphi' + \Pi_v^*)g(x)dx = 0 \tag{12}$$

And likewise, equation (7) determining the optimal decision under hidden action becomes:

$$\{S_a(a, \theta) - \varphi'(v + c)c_a - \Pi_a^*\} = 0 \tag{13}$$

As previously remarked, the principal has to pay the certainty equivalent and the risk premium of an agent, so as rising action for a type θ affects the risk premium. Note that, as far as I know, this interpretation of optimal expected transfer is new. It suggests that the optimal action could be sometimes higher than the perfect information one, according to the sign of Π_a^*.

An intuition of this result is the following. Recall that the density function of the signal x depends on the action, that means two different actions lead to two different random variables, then two different information systems. By choosing the optimal action, the principal decides three elements of the agency relationship: the cost level $c(a, \theta)$, the marginal cost $c_a(a, \theta)$, and the density of the random signal. So if increasing action improves the information system, it might be profitable. We illustrate this intuition by an example hereafter.

Comparing (6) and (12) shows that no risk premium is payed under adverse selection alone. This is due to the fact that the principal perfectly insures the agent in this latter case. The rent effect of equation (5) measured by the term $\int_{\theta_0}^{\theta}(\varphi')g(x)dx$ appears in equation (12), confirming the intuition according to adverse selection plays the same role with or without hidden action.

When the two asymmetric information sources are mixed, evidently these both mechanisms (rent and risk premium) interact together. But they create something new, designed by the quantity $\int_{\theta_0}^{\theta}\Pi_v^* g(x)dx$. When the principal wants to increase the effort made by an agent, she has to enhance the rent of all the agents whose type is smaller. This modifies how they perceive risk, generating a wealth effect. In other words, when the risk premium decreases (respectively increases) with rent, rising effort for a type θ diminishes (increases) the risk premium of all smaller types. A key point is then the sign of this third effect.[9]

The comparison between the adverse selection action and that under hidden information and hidden action depends on the signs of these information and wealth effects. As following subsections confirm, no general result exists concerning the values of these signs, which may be sometimes positive or sometimes negative. Both information effect (Π_a^*) and wealth effect (Π_v^*) have an indeterminate sense. Remark that to obtain such findings, we only need some counter-examples, presented herafter.

4.1 Example 1

Assume that the agent's VNM utility function is $u(t) = t^{1/2}$. This numerical example of the moral hazard literature was first developed by Holmström (1979). Taking into account equations (6), (UC) and (IC), a simple algebra leads to rewrite Ψ as:

$$\Psi(a, v, \theta) = (v + c)^2 + \frac{c_a^2}{I(a)} \text{ where } I(a) = \int f(x, a)(\frac{f_a(x, a)}{f(x, a)})^2 dx$$

[9]This third effect is pointed out by Baron and Besanko (1988), even if an interpretation in terms of risk premium is not made.

and then the risk premium equates:

$$\Pi^*(a, v, \theta) = \frac{c_a^2}{I(a)}$$

The optimal schemes depends on the reserve utility v, on production cost, and on Fisher's information index $I(a)$. The more informative (in Fisher's sense) is the signal, the smaller will be the variance of the agent's utility as a function of x, and smaller the risk premium. Increasing the effort has two consequences: as the agent's marginal production cost rises, the optimal reward must become more incentive, so the agent will get a riskier scheme. But perhaps a higher effort improves (or does not improve) available information[10]: this information effect is captured by Fisher's information index. Taking the derivative of $\Pi^*(a, v, \theta)$ with respect to action gives:

$$\Pi_a^* = \frac{c_a}{I(a)^2}\{2c_{aa}I(a) - c_a I'(a)\}.$$

Then, $\Pi^*(a, v, \theta)$ may decrease with action if $I(a)$ increases quickly with it. But the converse may also arise. As a consequence, when $\Pi_a^* < 0$, the optimal action under moral hazard will be higher than the perfect information one (which is defined by $S_a = \varphi'(v + c)c_a$). In such a case the agent will exert more effort under hidden action and information (HIHA) than under adverse selection (HI), at least for some intervals of types.

4.2 Example 2

Recall that the optimal action under both hidden action and hidden information depends also on the variation of $\Pi^*(v, a, \theta)$ with respect to v. But the sign of this derivative may be positive or negative, as lemma 4 shows.

[10] Kim (1995) compares two signals for the same action. The key point here is the fact that two actions a_1 and a_2 create two different random variables X_1 and X_2, which have the same support but different densities $f(x, a_1)$ and $f(x, a_2)$.

Lemma 4 *If* $3(u'')^2 - u'''u' > 0$ *(respectively* $=, <$*)*, $\Pi^*(v, a, \theta)$ *increases (is constant, decreases) with* v*.*

Note that the condition $3(u'')^2 - u'''u' > 0$ can also be expressed as $\rho < 3R$, where $\rho = \dfrac{-u'''}{u''}$ is the absolute prudence index and R that of absolute risk aversion. Such a condition often appears in hazard moral models (see Sinclair-Desgagné (1999) or Thiele and Wambach (1999) for instance). Note moreover that the derivative of R with respect to wealth is $R' = R(R - \rho)$, so if utility is $NIARA$, the condition $R \leq \rho$ always holds.[11] One interpretation of this condition could be the following: if prudence is not too high relative to risk aversion, increasing reserve utility leads to propose a riskier scheme.

This result can be compared to Thiele and Wambach (1999). In a hidden action model without using first order approach, they prove the principal's implementation cost increases with the agent's private wealth whenever $\rho \leq 3R$. The principal then dislikes facing a rich agent[12], or in other words $\Pi^*(v, a, \theta)$ decreases with v.

Note that even in this case, the trade-off between rent and efficiency never vanishes, because the sum of the rent effect and the information effect is always positive. But lemma 3 confirms that the wealth effect can play with or against the rent phenomenon.

When the absolute prudence index is high enough (relative to that of risk aversion, $\rho > 3R$), the wealth effect works against the rent mechanism: rising action increases the smallest types' rent as a consequence, but reduces their risk premium. Surprisingly, it is then

[11] For a constant relative risk aversion utility function, $u(t) = \dfrac{w^\alpha}{\alpha}$, the parameter of risk aversion is $1 - \alpha > 0, (R = \dfrac{1-\alpha}{w})$ and Π increases with v if $\alpha < \dfrac{1}{2}$ and decreases if not. For $u(t) = \log t$ or $u(t) = -exp(-rt)$, Π always increases with v.

[12] This analogy can be read as follows. Consider $e(\theta)$ as a private exogenous revenue and let $t(x)$ the agent's wealth for an outcome x, the expected payment is then equal to $\int_X t(x) f(x, a) dx - e(\theta)$ which is our risk premium. In a standard hidden action model, minimizing the risk premium would then be the principal's objective when she faces an agent who has a private revenue e. Note that our lemma 3 generalizes Thiele and Wambach result to the case where $\rho > 3R$, on condition that the first order approach applies.

possible that introducing hidden action in an adverse selection model alleviates the problem of underproduction.

This result can be connected to Sinclair-Desgagné (1999), who studies a hidden action model in a multitask and auditing context. More precisely, the principal observes a random signal about one task and has to pay to obtain information about the second action. Sinclair-Desgagné argues that an auditing procedure introduces a kind of complementary between the two tasks, solving in such a way the problem of low powered incentives schemes underlined by Holmström and Milgrom (1991). This mechanism arises when $\rho > 3R$. For the same reason, this remark applies to our model. As a matter of fact, type and effort are substitutes in our framework[13], as the two tasks in Sinclair-Desgagné. That means that paying for one element (revelation of type for instance) reduces the incentive cost of the other one (choosing the right action). So the trade-off between rent and efficiency is counterbalanced by the trade-off between insurance and efficiency, because a higher rent makes insurance less expensive (in incentive terms). Although, note that when such a mechanism succeeds in increasing effort, it rises rent in the same time relative to adverse selection !

When conversely prudence is low enough (relative to risk aversion, $\rho < 3R$), the wealth effect works with the rent mechanism. As a higher rent makes insurance more expensive, the two trade-offs between rent and efficiency on one hand and insurance and efficiency on the other hand go in the same direction. Surprisingly when this mechanism yields in reducing effort, as a consequence the rent is also reduced (relative to adverse selection)! In this latter case the agent would get a lower rent when he controls his action rather than when the principal monitors it.

The two effects described in the two previous last subsections arising together, a conclusion comparing the actions of the two hidden information models, with and without hidden action, is impossible in a general context. In some examples, the adverse selection action will

[13] This is due to the single crossing which assumes that $c_{\theta a} > 0$.

be found higher than the decision under both hidden action and hidden information, but the converse will appear in other contexts. What happens in this model is totally due to the interaction of wealth and information effects.

5 Conclusion

First let us provide a summary of our results.

Under the separability assumptions made by most papers dealing with hidden action, we apply the first order approach to our hidden action and hidden information context. That means we ignore the second order conditions, assuming that they are satisfied by the optimal contract. We observe that the agent's rent depends only on action and type. As a consequence, for a given action, the adverse selection rent is not affected by introducing hidden action, and the optimal reward is exactly the one of a hidden action model, in which the agent's reserve utility is replaced by his adverse selection rent. This pleasant separation result has a lot of consequences, because all the properties of hidden action contracts still hold when both sources of asymmetric information are present. Only optimal actions then differ.

This optimal action is deeply affected by wealth effects. We adopt an approach in terms of risk premium to explain the characteristics of the expected optimal transfer in our model. It is the sum of two terms, the rent on one hand, and a risk premium on the other hand which reflects the fact that the agent bears some risk. Depending on the agent's utility function, it becomes possible that increasing rent diminishes this risk premium, or conversely increases it. This interaction makes a general conclusion unfeasible.

Many questions are left open in this area. In particular, one can wonder about the role devoted to separability. In an adverse selection situation, paying the agent a rent is a way to reimburse him the production cost he supports. Is it reasonable to assume that paying a rent generates a wealth effect when getting a higher cost does not? So it would be important in future research to understand how our conclusions are modified when the agent's utility function is not separable with respect to action and transfer. In this case, we know that it is possible that the participation constraint is not binding at the optimum in the moral hazard model, so that in order to diminish risk premium, principal could prefer that the agent receives a rent. Two rents would then appear, one due to adverse selection and the other to moral hazard. Understanding how these two rents interact would surely provide some new elements to this problem.

6 Appendix

6.1 Proof of lemma 1

Define

$$V(m,a,\theta) \equiv \int_X U(t(x,m))f(x,a)dx - c(a,\theta)$$

By definition 1, a profile of utility $v(\theta) = V(\theta, a(\theta), \theta)$ and action $a(\theta)$ is implementable under adverse selection if $\forall\theta \in \Theta$ we have $V(\theta, a(\theta), \theta) \geq V(m, a(m), \theta), \forall m \in \Theta$. This inequality implies that the function of m, $\Lambda(m,\theta) = V(m, a(m), \theta)$ is maximum when $m = \theta$, and the necessary conditions of optimality gives us $\Lambda_m(\theta,\theta) = 0$ and $\Lambda_{mm}(\theta,\theta) \leq 0$ for every $\theta \in \Theta$. Taking the derivative of $v(\theta) = \Lambda(\theta,\theta)$ with respect to θ and using the first order conditions gives us

$$v'(\theta) = \Lambda_\theta(\theta,\theta) = V_\theta(\theta, a(\theta), \theta) = -c_\theta(a(\theta), \theta) \tag{14}$$

The second order conditions can be written $\Lambda_{m\theta}(\theta,\theta) \geq 0$, or $-c_\theta(a(\theta),\theta)a'(\theta) \geq 0$.

By definition 3, when hidden action and adverse selection are present together, a profile of utility $v(\theta) = V(\theta, a(\theta), \theta)$ and action $a(\theta)$ is implementable if $\forall\theta \in \Theta$ we have $V(\theta, a(\theta), \theta) \geq V(m, a, \theta), \forall m \in \Theta, \forall a \in A$. This inequality implies that the function of m and a $V(m, a, \theta)$ is maximum with respect to m and a when $m = \theta$ and $a = a(\theta)$, so the first order necessary conditions of optimality give us

$$V_m(\theta, a(\theta), \theta) = 0 \text{ and } V_a(\theta, a(\theta), \theta) = 0. \tag{15}$$

and by the second order conditions we have,

$$V_{aa}(\theta, a(\theta), \theta) \leq 0 \tag{16}$$

$$V_{mm}(\theta, a(\theta), \theta)V_{aa}(\theta, a(\theta), \theta) - (V_{am}(\theta, a(\theta), \theta))^2 \geq 0. \tag{17}$$

As equations (15) and (14) hold whatever θ, taking their derivatives with respect to θ gives:

$$V_{mm}(\theta, a(\theta), \theta) + V_{ma}(\theta, a(\theta), \theta)a'(\theta) + V_{m\theta}(\theta, a(\theta), \theta) = 0$$

$$V_{am}(\theta, a(\theta), \theta) + V_{aa}(\theta, a(\theta), \theta)a'(\theta) + V_{a\theta}(\theta, a(\theta), \theta) = 0$$

Using these formulas and the fact that $V_{\theta m}(m, a, \theta) = 0$ to simplify (17) leads to, after a little algebra:

$$\begin{aligned} V_{mm}V_{aa} - (V_{am})^2 &= V_{am}V_{a\theta} \\ &= c_{a\theta}[V_{aa}a' - c_{a\theta}] \geq 0. \end{aligned} \tag{18}$$

and finally the second order conditions are (16) and (18). ■

6.2 Proof of lemma 2

Changing the variables $u(x, \theta) = U(t(x, \theta))$ leads to an utility constraint:

$$\int_X u(x, \theta)f(x, a)dx - c(a, \theta) = v$$

and the principal's objective becomes:

$$H = \int_\Theta g(\theta)[S(a(\theta), \theta) - \int_X \varphi(u(x, \theta))f(x, a(\theta))dx]d\theta$$

Assume that the principal does not provide perfect insurance, then, as $\varphi'' > 0$, we have

$$\int\limits_X \varphi(u(x,\theta))f(x,a(\theta))dx > \varphi(\int\limits_X u(x,\theta)f(x,a(\theta))dx) = \varphi(v+c).$$

So, offering to the agent the transfer $t(x,\theta) = \varphi(v+c), \forall x \in X$ increases the principal's objective, hence a contradiction. The optimal contract under adverse selection is then a perfect assurance one, that is $t(x,\theta) = t(\theta), x \in X$.

Moreover, as φ is increasing, the participation constraint is binding at the top $v(\theta_1) = \underline{U}$. The necessary optimality condition (see for example Hestenes 1966) states that there exists a multiplier $\rho(\theta)$, with $\rho(\theta_0) = 0$, and a Hamiltonian function $H = \{S(a,\theta) - \varphi(v + c(a,\theta))\}g(\theta) - \rho c_\theta$ such that the optimal solution satisfies $\rho' = -\dfrac{\partial H}{\partial v} = \varphi'(v + c(a,\theta))g(\theta)$ and $a(\theta)$ maximizes H, that is:

$$\{S_a(a,\theta) - \varphi'(v + c(a,\theta))c_a\}g(\theta) - \rho c_{\theta a} = 0$$

Integrating ρ' leads to $\rho(\theta) = \displaystyle\int_{\theta_0}^{\theta} \varphi'(v(y) - c(a(y),y))g(y)dy$ and replacing $\rho(\theta)$ by its value in the above condition defining the optimal effort gives equation (5). ■

6.3 Proof of proposition 3

Let $v^*(\theta), a^*(\theta), t^*(x,\theta)$ the solution of the general form of P_{HIHA} defined in section 2.

Denote $\widehat{P}(a,v,\theta)$ the hidden action program, when the principal faces an agent whose type is θ and wants to give him an expected utility equal to v. The principal has to determinate the reward function which minimizes the expected transfer $\int_X f(x,a)t(x)dx$ subject to UC and IC.

Let $\widehat{t}(x,\theta,a,v)$, $x \in X$ the solution of $\widehat{P}(a,v,\theta)$ and $\Psi(a,v,\theta)$ the expected optimal transfer $\int_X f(x,a)\widehat{t}(x,\theta,a,v)dx$. By definition we know that

$$\int\limits_\Theta g(\theta)\Psi(a^*(\theta),v^*(\theta),\theta)d\theta \le \int\limits_\Theta g(\theta) \int\limits_X f(x,a^*(\theta))t(x,\theta)dxd\theta$$

for every function t which satisfies IC et PC evaluated for v^* and a^*.

Let us assume that the optimal solution is such that

$$\int_\Omega g(\theta)\Psi(a^*(\theta), v^*(\theta), \theta)d\theta < \int_\Omega g(\theta) \int_X f(x, a^*(\theta))t^*(x, \theta)dxd\theta$$

for an interval $\Omega \subset \Theta$ with positive measure. It is then possible to modify the optimal transfer $t^*(x, \theta)$, $x \in X$ in this interval and to replace it by $\widehat{t}(x, \theta, a^*, v^*)$, $x \in X$. This new reward function satisfies all the constraints of the program P_{IM}. Such a modification is then possible and increases the principal's expected utility, so a contradiction.

If there exists an interval $\Omega \subset \Theta$ in which $t^*(x, \theta)$, $x \in X$ is different from $\widehat{t}(x, \theta, a^*, v^*)$, $x \in X$ then

$$\int_\Omega g(\theta)\Psi(a^*(\theta), v^*(\theta), \theta)d\theta = \int_\Omega g(\theta) \int_X f(x, a^*(\theta))t^*(x, \theta)dxd\theta$$

and so $t^*(x, \theta), x \in X$ is a solution of $\widehat{P}(a^*(\theta), v^*(\theta), \theta)$. In addition, every transfer $t(x, \theta) = \widehat{t}(x, \theta, a^*, v^*)$ for $\theta \in \Omega' \subset \Omega$ and $t(x, \theta) = t^*(x, \theta)$ for $\theta \in \Theta - \Omega'$ is a solution of P_{IA}.

- Let us first prove now that Ψ increases (strictly) with v. As in the body text, define:

$$s \equiv u(t) - c - v \text{ and } u(e) \equiv v + c(a, \theta), \text{ or equivently } t = \varphi(s + c + v) \text{ and } e = \varphi(c + v).$$

IC and UC can now be expressed as functions of s :

$$\int_X s(x, \theta)f_a(x, a)dx = c_a(a, \theta) \ (IC') \text{ and } \int_X s(x, \theta)f(x, a)dx = 0 \ (UC')$$

Moreover, as by definition of certainty equivalent e and risk premium Π we have $e \equiv \int tf - \Pi$, we can deduce that

$$\int tf = e + \Pi = \varphi(v + c) + \Pi = \int_X \varphi(v + c + s)f(x, a)dx$$

Changing the variable t in s provides a new writing of the moral hazard program $\widehat{P}(a, v, \theta)$. The principal has to choose $s(x, \theta), x \in X$ which minimizes $\int_X \varphi(v + c + s)f(x, a)dx$ subject to IC' and $UC^{'}$. Denoting by $s^*(x, v, a), x \in X$ the solution of this program, we have :

$$\Psi(v, a, \theta) = \int_X \varphi(v + c + s^*) f(x, a) dx$$

Note that because the constraints IC' and UC' do not depend on v, we have if $v < \widehat{v}$

$$\begin{aligned} & \Psi(v, a, \theta) - \Psi(\widehat{v}, a, \theta) \\ \leq & \int_X \varphi(v + c + s^*(x, \widehat{v}, a, \theta)) f(x, a) dx - \int_X \varphi(\widehat{v} + c + s^*(x, \widehat{v}, a, \theta)) f(x, a) dx \\ < & \ 0 \end{aligned}$$

since $s^*(., \widehat{v}, a, \theta)$ is possible when the expected utility of the agent is v and φ strictly increases. So Ψ increases with v.

• Knowing the function Ψ, P_{HIHA} can be rewritten as: the principal maximizes $\int_\Theta \{S(a, \theta) - \Psi(v, a, \theta)\} g(\theta) d\theta$ with respect to $v(.)$ and $a(.)$, subject to RC' and PC $(v(\theta) \geq \underline{v})$. This program is a very standard hidden information one.

Thanks to the monotony assumption $c_\theta > 0$, $v(\theta)$ decreases with θ and then the participation constraint is satisfied if and only if $v(\theta_1) \geq \underline{v}$. Moreover, integrating the revelation constraint leads to $v(\theta) = v(\theta_1) + \int_\theta^{\theta_1} c_\theta(a(y), y) dy$. So a diminution of $v(\theta_1)$ without changing the shape of v, when it is possible, increases the expected principal's welfare, and so the participation constraint is binding at the optimum so $v(\theta_1) = \underline{v}$. Taking into account this result leads then to solve as in lemma 2 a dynamic optimization program with a state variable v and a control one a, the initial condition being slack. There exists a multiplier $\rho(\theta)$, with $\rho(\theta_0) = 0$, and a Hamiltonian function $H = \{S(a, \theta) - \Psi(v, a, \theta)\} g(\theta) - \rho c_\theta$ such that the optimal solution satisfies $\rho' = -\dfrac{\partial H}{\partial v} = \Psi_v(v, a, \theta) g(\theta)$ and $a(\theta)$ maximizes H, that is:

$$\{S_a(a, \theta) - \Psi_a(v, a, \theta)\} g(\theta) - \rho c_{\theta a} = 0$$

Integrating ρ' leads to $\rho(\theta) = \int_{\theta_0}^{\theta} \Psi_v(v, a, y) g(y) dy$ and replacing $\rho(\theta)$ by its value in the above condition defining the optimal effort gives us equation (8). ■

6.4 Proof of Lemma 4

We know that:

$$\Pi^*(v,a,\theta) = \int_X \varphi(v+c+s^*)f(x,a)dx - \varphi(v+c)$$

As the constraints IC' and UC' above do not depend on v, applying the envelop theorem to the Lagrangian gives:

$$\frac{\partial \Pi^*(v,a,\theta)}{\partial v} = \int_X \varphi'(v+c+s^*)f(x,a)dx - \varphi'(v+c)$$

If $\varphi''' > 0$, φ' is convex and $\dfrac{\partial \Pi^*(v,a)}{\partial v} > 0$, since $\int_X \varphi'(v+c+s^*)f(x,a)dx > \varphi'(v+c+\int_X s^* f(x,a)dx) = \varphi'(v+c)$. Likewise, we obtain that if $\varphi''' < (=)0$ then $\dfrac{\partial \Pi^*(v,a)}{\partial v} < (=)0$.

Consider now the convexity of v'. As φ is the inverse function of u, by definition $\varphi(u(x)) = x, \forall x \in X$. Taking the derivative of this expression three times with respect to x leads to $\varphi'(u(x))u'(x) = 1$, then $\varphi''(u')^2 + \varphi'u'' = 0$ and finally $\varphi'''(u')^3 + 3\varphi''u'u'' + \varphi'u''' = 0$. Thus φ''' has the same sign than $-(3\varphi''u'u'' + \varphi'u''')$. As in addition $3\varphi''u'u''u' + \varphi'u'''u' = \varphi'(-3(u'')^2 + u'''u')$ we can conclude that φ''' has the same sign than $-3(u'')^2 + u'''u'$. ■

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