Job Protection and Aggregate Employment Fluctuations: A Reappraisal

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Abstract

This paper tackles the issue of the link between the variability of employment and job protection. On this purpose, we build an equilibrium matching model of unemployment that handles both idiosyncratic and aggregate shocks. In opposition with standard labor demand models, the simultaneous decisions of job creation and job destruction do not underline any obvious relationship between aggregate employment variability and firing costs. Still, our quantitative analysis leads us to argue that an increase in job protection is likely to enhance employment variability at the aggregate level for reasonable parameters values.

JEL Codes: J23, J38, 241, J64

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1 Introduction

Most European countries encountered high and persistent unemployment rates during the last decade whereas, at the same time, the U.S. labor market performed relatively well. Much attention has been devoted to the analysis of this phenomenon and as a result the stringent labor market regulations have often been blamed as a source of the poor unemployment performance of many European countries. However, the related literature mainly focuses on the unemployment rate whereas little attention has been paid to the consequence of...
such regulation on the aggregate employment fluctuations. The aim of the paper is definitely to tackle this issue and therefore to analyze how job protection shapes employment fluctuations. Actually, most economists would argue that job protection reduces employment fluctuations. The framework we develop hereafter outlines that this assertion is far from being right when one takes into account the simultaneous flows of job creation and job destruction. Broadly speaking, this paper is related to two but complementary strands of literature: partial equilibrium labor demand models and equilibrium search and matching models.

First, partial equilibrium labor demand models in the vein of the seminal work by Bertola (1990), Bentolila and Bertola (1990) and Bentolila and Saint Paul (1994) describe the behavior of firms subject to idiosyncratic shocks, adjustment costs of labor and constrained by an exogenous wage. It is therefore argued that job protection tends to reduce both firing and hiring and consequently induces opposite effects on employment with an ambiguous prediction about its net effect. Regulations are also found to have a significant negative impact on employment variability and to considerably affect unemployment persistence. However these models suffer some obvious limitations. On one hand, the stochastic structure is restricted to idiosyncratic shocks and therefore the model is likely to be irrelevant to account for employment fluctuations at the aggregate level where both hiring and firing are simultaneous. On the other hand, the wage is exogenous.

Second, a rapidly emerging literature in the fashion of the search and matching framework developed by Mortensen and Pissarides is focusing on labor market flows. Equilibrium matching models of unemployment offer useful insights on both wage formation and labor market flows when job protection is more or less stringent. Mortensen and Pissarides (1993, 1994) extend the standard equilibrium matching model for aggregate shocks and characterize the cyclical properties of job creation and job destruction in such framework. They show that models are consistent with stylized facts related to the observed behavior of job creation and job destruction but they do not address the question of labor market policies in such a framework. Millard and Mortensen (1997) were among the first to explore their implications in a search and matching framework. They suggest that some key labor market institutions explain the main differences in unemployment spells and unemployment levels between the U.S. and the U.K. In particular, they show that job protection has an ambiguous effect on unemployment but unambiguously increases the unemployment spell. Obviously, their analysis suffer two limitations. Their model does not handle the two tiers bargaining levels required to accurately study job protection and the model is restricted to idiosyncratic shocks and is therefore unable to account for aggregate employment fluctuations. Both limits were in turn wiped out. The first one was canceled by the seminal works of Mortensen and Pissarides (1999a) and Cahuc and Zylberberg (1999) but the models are still restricted to idiosyncratic shocks or comparative static results on a deterministic measure of aggregate productivity. Finally, Garibaldi (1998) was the first to attempt to narrow the gap between labor market policies and the effects of such policies in
an aggregate framework. His model focuses on the consequence of a determined
labor market policy—namely the notice period—on the cyclical behavior of job
creation and job destruction, but suffers some limitations due to the particu-
lar bargaining structure and does not address the question of the effects of job
protection on the aggregate variability of employment. Our paper takes a first
step towards tackling this issue in a somehow standard equilibrium search and
matching model where the wages are settled according to a bilateral bargaining
and where the economy is subject to both microeconomic and macroeconomic
productivity shocks. Job protection is likely to put out a wide range of forms.
Accordingly, our paper relies on a standard and simple measure of job protection
—namely the firing costs—and investigates how this particular measure shapes
the aggregate employment fluctuations. On this purpose, we perform several nu-
merical exercises on two labor markets as much dissimilar as the French and the
U.S. ones. In our simple framework, we show than an increase in job protection
has an ambiguous effect on employment and is likely to enhance employment
variability. This latter conclusion is at odd with the results assumed by par-
tial equilibrium models that conjecture that firing costs tend to lower aggregate
employment fluctuations. In addition, our results suggest that countries with a
high degree of job protection are likely to experience a sharp soaring in the un-
employment rate when the economy slumps. The paper is organized as follow:
sections 2 details the search and matching framework that allow us to mimic
the effects of firing costs on the aggregate employment fluctuations, section 3
focuses on some numerical exercises proving that our results are consistent with
a wide range of parameters values, section 4, finally, provides some concluding
remarks as well as some would be extensions.

2 The Model

We consider a continuous time search and matching model with endogenous
job destruction and macroeconomic shocks in the fashion of Mortensen and
Pissarides (1993, 1994). At first, we focus on the setting of the model then
the macroeconomic background is described as well as the general resolution
method.

2.1 The labor market

We consider an economy with two goods: labor which is the sole input, and a
numeraire good produced thanks to labor. There is an endogenous mass of firms.
Each firm has only one job that is in one of two states, filled and producing or
vacant and searching. The labor force is composed of a continuum of infinite
lived individuals, which size is normalized to unity. Each worker supplies one
unit of labor and can be either employed and producing or unemployed and
searching. Individuals have identical preferences, represented by a linear utility
function.

The cost of a vacant job per unit of time is denoted by \( h \). This transaction
cost implies that vacant jobs and unemployed workers are matched together in pairs through an imperfect matching process. The rate at which vacant jobs and workers meet is given by a matching function. The matching function satisfies the standard properties: it is increasing, continuously differentiable, homogenous of degree one, and yields no hiring if the mass of unemployed workers or the mass of vacant jobs is nil. The model is meant to account for the macroeconomic environment and more accurately for the cyclical behavior of job creation and job destruction. In this perspective, it is important to index the model for each aggregate state. We let the aggregate conditions move stochastically between \( n \) states indexed by the subscript \( i \) where \( i = 1 \ldots n \) and according to an arbitrary Markov process with persistence. More accurately, the law of motion of aggregate shocks is defined by a \( n \times n \) stochastic matrix containing the transition probabilities that the aggregate component of productivity move from one state to another. The aggregate states are ranked in a decreasing order so that \( i = n \) corresponds to the worst macroeconomic state. Accordingly, the matching function is defined by \( M(v_i, u_i) \) where \( v_i \) and \( u_i \) represent the vacancy and the unemployment in the aggregate state \( i \) respectively. The linear homogeneity of the matching function allow us to write the transition rate for vacancies as \( M(v_i, u_i)/v_i = M(1, u_i/v_i) = m(\theta_i) \), where \( \theta_i = v_i/u_i \) stands for the labor market tightness in the aggregate state \( i \). Similarly, the flow out of unemployment is given by \( M(v_i, u_i)/u_i = \theta_i m(\theta_i) \). The properties of the matching function imply that \( m(\theta_i) \) and \( \theta_i m(\theta_i) \) are decreasing and increasing functions of the labor market tightness respectively.

For a given aggregate state \( i \), each job is endowed with an irreversible technology requiring one unit of labor to produce \( p_i + \sigma_i \varepsilon \) units of output where \( p_i \) is an aggregate productivity parameter common to all jobs, \( \sigma_i \) is an indicator of the dispersion in the idiosyncratic component, and \( \varepsilon \) is a job specific productivity parameter. This latter parameter is a random variable drawn from a general distribution function \( F(\varepsilon) \) with support in the range \( [−\infty, \varepsilon_u] \). Every new job starts at the upper bound of the distribution i.e. with the highest productivity \( \varepsilon_u \). On every continuing job, productivity changes according to a Poisson process with arrival rate \( \lambda \). Every time a match is hit by an idiosyncratic shock, a new value of \( \varepsilon \) is drawn from the distribution \( F(\varepsilon) \). If the new value of \( \varepsilon \) is below the current endogenous threshold denoted by \( \varepsilon_{di} \), the job is destroyed. Thus, the job destruction rate for the aggregate state \( i \) follows a Poisson process with parameter \( \lambda F(\varepsilon_{di}) \). Assuming there is no on the job search the law of motion of unemployment on the labor market for the aggregate state \( i \) is given by:

\[
\dot{u}_i = \lambda F(\varepsilon_{di})(1 - u_i) - \theta_i m(\theta_i)u_i \tag{1}
\]

If the aggregate shock takes on the same value repeatedly, the economy converges to a state in which unemployment is constant. Assuming a long

\footnote{This assumption is made without any loss of generality in order to avoid any creation threshold and to preserve the main intuitions of the model.}
sequence of realizations of aggregate shock $i$, one gets a Beveridge curve which equation is given by:

$$u_i = \frac{\lambda F(\xi_{di})}{\lambda F(\xi_{di}) + \theta_i m(\theta_i)}$$

Following Cole and Rogerson (1999), one denotes $u_i$ as the conditional steady states unemployment rate the economy will converge to if the aggregate shock remains unchanged for many periods. This curve shows that the unemployment rate depends on the rates of job destruction as well as on the labor market tightness.

2.2 Values of jobs and expected utilities

A vacant job cost $h$ per unit of time and is filled at rate $m(\theta_i)$. The asset value of a vacancy in the aggregate state $i$, denoted by $\Pi_{vi}$, reads as:

$$r \Pi_{vi} = -h + m(\theta_i) [\Pi_{0i}(\xi_u) - \Pi_{vi}] + \sum_{i \neq j}^n t_{ij} [\Pi_{vj} - \Pi_{vi}]$$

with $i, j = 1 \ldots n$ (3)

where $r$ is the exogenous interest rate, $t_{ij}$ is the transition probability from aggregate state $i$ to aggregate state $j$ and $\Pi_{0i}(\xi_u)$ is the expected value of a newly created job in the aggregate state $i$. We consider that the wages are bargained over while setting up a new contract and each time a shock hits the match. One needs here to make a sharp distinction between newly created jobs and the continuing ones due to the firing costs. At the very beginning of a new match i.e. while the negotiation, firms do not support any firing cost since no contract has been signed up. However, once the contract is signed firms support firing costs in case the value of the job falls below the state contingent reservation productivity $\xi_{di}$.

The asset value of a newly created job in the aggregate state $i$ satisfies:

$$r \Pi_{0i}(\xi_u) = p_i + \sigma_i \xi_u - w_{0i} + \lambda \left[ \int_{-\infty}^{\xi_u} \text{Max} [\Pi_{ei}(\xi), \Pi_{vi} - f_i] dF(\xi) - \Pi_{0i}(\xi_u) \right]$$

$$+ \sum_{i \neq j}^n t_{ij} [\Pi_{0j}(\xi_u) - \Pi_{0i}(\xi_u)]$$

where $w_{0i}$ is the wage bargained at the beginning of the match, $\Pi_{ei}(\xi)$ is the expected value of continuing job and $f_i$ is the firing costs supported by the firm. One needs here to point out that we explicitly distinguish for redundancy payment, denoted by $f_e$, and administrative costs, denoted by $f_a$, so that firing costs are defined by $f_i = f_e + f_a$ and are contingent to the aggregate state. One can also remark that jobs are not destroyed if the economy switches from
an aggregate state to another one. As a matter of fact, an aggregate transition does not affect the idiosyncratic component of the match and since the jobs are assumed to begin with the highest productivity, the labor contracts are assumed to hold. Things turn out to be slightly different when considering the asset value of job whose productivity is spread in the range $]-\infty, \varepsilon_u[$. The asset value of a continuing job in the aggregate state $i$ reads as:

$$r\Pi_{ci}(\varepsilon) = p_i + \sigma_i \varepsilon - w_i(\varepsilon) + \lambda \left[ \int_{-\infty}^{\varepsilon_u} \text{Max} \left[ \Pi_{ci}(\xi), \Pi_{vi} - f_i \right] dF(\xi) - \Pi_{ci}(\varepsilon) \right] + \sum_{i \neq j} t_{ij} \left[ \text{Max} \left[ \Pi_{cj}(\varepsilon), \Pi_{vj} - f_j \right] - \Pi_{ci}(\varepsilon) \right].$$

Contrary to the previous equation, aggregate switch may cause jobs to be destroyed. Even though the aggregate shock does not affect the idiosyncratic component of the productivity, it induces a shift of the endogenous threshold that may lead to end up a match since $\varepsilon \in ]-\infty, \varepsilon_u[$. For the sake of simplicity, let’s deal with an illustrative example. The reservation productivities are ranked as on Figure (1) from the best aggregate state to the worst aggregate state.

If the aggregate state switches from state $n-1$ to state $n$, the macroeconomic environment worsens and the reservation threshold increases from $\varepsilon_{dn-1}$ to $\varepsilon_{dn}$. Accordingly all jobs whose productivity is below the new threshold value are destroyed whereas the idiosyncratic component of the jobs remains the same. Thus the sources of job termination are twofold. Matches are destroyed either after a macroeconomic shock that affects the reservation threshold or after a microeconomic shock that affects the idiosyncratic component of the match. As a matter of fact, for a given aggregate state $i$, the only source of job destruction is the microeconomic one. Accordingly, jobs are destroyed after a low value of $\varepsilon$ has been drawn from the distribution $F(\varepsilon)$ and such that $\varepsilon$ is lower than the current state contingent reservation threshold $\varepsilon_{di}$. On the contrary, for a given idiosyncratic productivity component $\varepsilon$, the only remaining job termination source is the macroeconomic one. Therefore jobs are destroyed when the aggregate conditions worsen due to the shift in the reservation productivity as depicted on Figure (1).

The expected value, $V_{ui}$, of the stream of income of an unemployed worker in the aggregate state $i$ satisfies:
where $b_i$ stands for the unemployment benefits and $V_{0i}$ is the expected stream of income of a newly hired worker. An unemployed worker gets an instantaneous income $b_i$ and expects the macroeconomic state to switch from state $i$ to state $j$ with the probability $t_{ij}$. In the meantime, she also expects to move back to employment with probability $\theta_i m(\theta_i)$. As previously, one needs to make the distinction between the expected utility of a newly hired worker and the expected utility of a titular worker due to the transfers associated with the firing costs.

The expected present utility, $V_{0i}$, of the stream of income of a newly hired worker is given by:

$$
rV_{0i} = b_i + \theta_i m(\theta_i) [V_{0i}(\varepsilon_u) - V_{ui}] + \sum_{i \neq j} t_{ij} [V_{uj} - V_{0i}]$$

(6)

where $b_i$ stands for the unemployment benefits and $V_{0i}$ is the expected stream of income of a newly hired worker. An unemployed worker gets an instantaneous income $b_i$ and expects the macroeconomic state to switch from state $i$ to state $j$ with the probability $t_{ij}$. In the meantime, she also expects to move back to employment with probability $\theta_i m(\theta_i)$. As previously, one needs to make the distinction between the expected utility of a newly hired worker and the expected utility of a titular worker due to the transfers associated with the firing costs.

The expected present utility, $V_{0i}$, of the stream of income of a newly hired worker is given by:

$$
rV_{0i}(\varepsilon_u) = w_{0i} + \lambda \left[ \int_{-\infty}^{\varepsilon_u} \max [V_{ei}(\xi), V_{ui} + f_{ei}] dF(\xi) - V_{0i}(\varepsilon_u) \right]
+ \sum_{i \neq j} t_{ij} [V_{0j}(\varepsilon_u) - V_{0i}(\varepsilon_u)]$$

(7)

where $V_{ei}$ is the expected present utility of a titular worker. The newly hired worker gets an instantaneous income $w_{0i}$ and expects the microeconomic and the macroeconomic environment to change with probability $\lambda$ and $t_{ij}$ respectively. One can remark, that the aggregate disturbance do not cause any job destruction for the newly hired worker for the same reasons we dealt with above.

Similarly, the expected present utility, $V_{ei}$, of the stream of income of a titular worker reads as:

$$
rV_{ei}(\varepsilon) = w_{i}(\varepsilon) + \lambda \left[ \int_{-\infty}^{\varepsilon_u} \max [V_{ei}(\xi), V_{ui} + f_{ei}] dF(\xi) - V_{ei}(\varepsilon) \right]
+ \sum_{i \neq j} t_{ij} [\max [V_{ej}(\varepsilon), V_{uj} + f_{ej}] - V_{ei}(\varepsilon)].$$

(8)

Again, the aggregate disturbance by shifting up or down the state contingent threshold may lead to terminate some jobs. Fired workers receive a redundancy payment that amounts to $f_{ej}$.

### 2.3 Job destruction and job creation conditions

For any $i$, one can now solve the model in the steady state for all the unknowns, more noticeably the labor market tightness $\theta_i$, the destruction threshold $\varepsilon_i$, and the unemployment rate $u_i$. Henceforth, we will define a set of two non-linear equations, namely the job creation and the job destruction, that determines the
The equilibrium values of $\theta_i$ and $\varepsilon_{di}$, the others endogenous variables being easily deduced from those values. To derive the job destruction condition, it is useful to define the surplus for both a new and a continuing job. Broadly speaking, the surplus of a job is the sum of the expected present value of the workers’ and the employers’ future income on the job less the expected present value of their income in case of separation. Thus the above equations allow us to write the surplus associated with each match, contingent to the aggregate state $i$. As previously one needs to make a distinction between the surplus associated to a new job and the surplus associated to a continuing job. At the very beginning of the match, an employer and a worker can split up at no cost since no contract has been signed up and thus the surplus of the match can be written as:

$$S_{0i}(\varepsilon_u) = \Pi_{0i}(\varepsilon_u) - \Pi_{vi} + V_{0i}(\varepsilon_u) - V_{ui}. \quad (9)$$

On the contrary, once a contract is signed things turn out to be different. In case of separation the firm has to pay the administrative costs to the government as well as the redundancy payment to the worker. On every continuing job with current productivity $\varepsilon$, an employer gets either $\Pi_{ei}(\varepsilon)$ or $\Pi_{vi} - f_{ai} - f_ei$ in case of separation. Similarly a titular worker obtains either an expected income equal to $V_{ei}(\varepsilon)$ or $V_{ui} + f_ei$ in case she becomes unemployed. Thus the threat points for the employer and the worker are $\Pi_{vi} - f_{ai} - f_{ei}$ and $V_{ui} + f_{ei}$ respectively and the surplus satisfies:

$$S_i(\varepsilon) = \Pi_{ei}(\varepsilon) - \Pi_{vi} + f_{ai} + V_{ei}(\varepsilon) - V_{ui}. \quad (10)$$

Wage is the outcome of a bilateral bargaining over surplus and is negotiated according to a Nash sharing rule which provide a share $\beta \in [0,1]$ of the surplus generated by the match to the worker. This latter parameter $\beta$ can be interpreted as the bargaining power of workers. As usual, one needs again to make distinction for the wage bargaining processes for a new job and a continuing one. The negotiated wages are the solution of the following Nash bargaining rules:

$$w_{0i} = \text{Arg max}[V_{0i}(\varepsilon_u) - V_{ui}]^\beta[\Pi_{0i}(\varepsilon_u) - \Pi_{vi}]^{1-\beta} \quad (11)$$

$$w_i(\varepsilon) = \text{Arg max}[V_{ei}(\varepsilon) - V_{ui} - f_{ei}]^\beta[\Pi_{ei}(\varepsilon) - \Pi_{vi} + f_i]^{1-\beta} \quad (12)$$

The resulting sharing rules for both workers and employers contingent to aggregate state $i$ and to the bargaining process (negotiation and renegotiation) read as:

$${\Pi_{0i}(\varepsilon_u) - \Pi_{vi} = (1 - \beta)S_{0i}(\varepsilon_u), \quad V_{0i}(\varepsilon_u) - V_{ui} = \beta S_{0i}(\varepsilon_u)} \quad (13)$$
\[\Pi_{ei}(\varepsilon) - (\Pi_{ei} + f_i) = (1 - \beta)S_i(\varepsilon), \quad V_{ei}(\varepsilon) - (V_{ui} + f_ei) = \beta S_i(\varepsilon). \tag{14}\]

The related wage equations can be easily derived (see appendix 5.2 for details) but are not required to define equilibrium. Meanwhile, one needs to expand the surplus equation (10) to define the job destruction condition of the model. Making use of equations (3), (5), (6), (8), (10) and (14), it can be shown (see appendix 5.1 for details) that the surplus of a continuing job with idiosyncratic component \(\varepsilon\) and contingent to the aggregate state \(i\) satisfies the following asset pricing equations:

\[
(r + \lambda + \sum_{i \neq j} t_{ij} S_i(\varepsilon)) = p_i + \sigma_i \varepsilon - b_i - \frac{\theta_i \beta h}{1 - \beta} + rf_{ai} + \lambda E(S_i) \\
+ \sum_{i \neq j} t_{ij} (f_{ai} - f_{aj}) \\
+ \sum_{i \neq j} t_{ij} [Max [S_j(\varepsilon), 0]] \tag{15}\]

where \(E(S_i)\) stands for the mean over the idiosyncratic component \(\varepsilon\) of the expected value of the surplus in the aggregate state \(i\). Similarly, making use of equations (3), (4), (6), (7), (9) and (13), it can be shown (see appendix 5.1 for details) that the surplus of a new job contingent to the aggregate state \(i\) satisfies the following asset pricing equation:

\[
(r + \lambda + \sum_{i \neq j} t_{ij} S_{0i}(\varepsilon_u)) = p_i + \sigma_i \varepsilon_u - b_i - \frac{\theta_i \beta h}{1 - \beta} - rf_{ai} + \lambda E(S_i) \\
+ \sum_{i \neq j} t_{ij} S_{0j}(\varepsilon_u). \tag{16}\]

Jobs are destroyed as soon as their surplus become nil. The formal condition reads as \(S_i(\varepsilon_{di}) = 0\). Using this latter condition together with equation (15) one gets the reservation productivity contingent to aggregate state \(i\):

\[
p_i + \sigma_i \varepsilon_{di} = b_i + \frac{\theta_i \beta h}{1 - \beta} - rf_{ai} - \lambda E(S_i) \\
- \sum_{i \neq j} t_{ij} (f_{ai} - f_{aj}) - \sum_{i \neq j} t_{ij} [Max [S_j(\varepsilon_{di}), 0]] \tag{17}\]

First, one can remark that total productivity must exceeds the opportunity cost of employment \(b_i + \theta_i \beta h/(1 - \beta)\) to pursue the match. Second, one needs
to take into account both institutional and voluntary labor hoarding at the microeconomic – match specific – and the macroeconomic level. For a given state $i$, institutional labor hoarding refers to the capitalized value of administrative firing costs $rf_{ai}$. Accordingly, both the worker and the firm agree on a productivity loss in order to avoid the termination costs. In turn, the third term $\lambda E(S_i)$ refers to the microeconomic voluntary labor hoarding due to expected variations in the idiosyncratic productivity component $\varepsilon$. These two elements are common to standard matching models that handle job protection. However, a shift in the aggregate state creates two additional sources of labor hoarding. As the level of job protection is contingent to the macroeconomic state, an aggregated productivity shock causes the level of firing costs to be shifted. This expected changes in the institutional environment is taken in account in the job termination rule. More accurately, an expected increase in the firing costs induces firms to destroy more jobs in the current state in order to avoid higher termination costs later on. Finally, the aggregate productivity shocks also creates, for a given idiosyncratic component, a voluntary labor hoarding phenomenon due to the shift in the surplus. An improvement in the aggregate condition shifts down the reservation threshold and makes the worker-firm pair willing to keep the job at the current productivity.

To solve the model in $\theta_i$ and $\varepsilon_i$, it is necessary to define the job creation condition. The free entry condition $\Pi_{vi} = 0$ for $i = 1,...,n$ together with the sharing rules (13) define:

$$\frac{h}{m(\theta_i)} = (1 - \beta)S_{m}(\varepsilon_u).$$

(18)

Finally, using equation (9), the job creation condition can be easily derived and thus satisfies:

$$(r + \lambda + \sum_{i\neq j} t_{ij}) \frac{h}{m(\theta_i)} = (1 - \beta)(p_i + \sigma_i \varepsilon_u - b_i - \frac{\theta_i \beta h}{1 - \beta})$$

$$- (1 - \beta)\lambda f_{ai} + (1 - \beta)\lambda E(S_i)$$

$$+ \sum_{i\neq j} t_{ij} \frac{h}{m(\theta_j)}. \quad (19)$$

The left hand side of the job creation condition is the expected capitalized value of the firm’s hiring cost. The right hand side of the equation stands for the expected profit of a vacant job and can be divided in four terms. The first one refers to the net instantaneous profit of the firm. The second term is the expected loss to the firm due to a renegotiation of the labor contract. The third term represents the expected gains associated with an increase in the idiosyncratic component $\varepsilon$. Finally, the last term reflects the gains for the firm to hire a worker in the current aggregate state.
The job destruction (17) and the job creation conditions (19), when the wage is bargained according to a Nash rule, allow to solve the model at the steady state for the equilibrium values \((\theta_i, \varepsilon_{di})\) for \(i = 1...n\). Hence the model exhibits \(2n\) non linear equations in \((\theta_i, \varepsilon_{di})\) which need to be jointly solved in order to determine the \(n\) steady states equilibria of the model.

We investigate now the issue of employment fluctuation at the steady state. Let \(\pi_i\) be the ergodic probability for aggregate state \(i\), that is to say its unconditional probability. Thus, the average unemployment rate across the \(n\) states is \(\bar{u} = \sum_{i=1}^{n} \pi_i u_i\) and its differential is \(d\bar{u} = \sum_{i=1}^{n} \pi_i du_i\). The total differential of the equilibrium rate of unemployment contingent to aggregate state \(i\) solves, according to equation (2):

\[
du_i = u_i (1 - u_i) \left[ \frac{\lambda F^{'}(\varepsilon_{di})}{\lambda F(\varepsilon_{di})} d\varepsilon_{di} - \frac{[\theta_i m(\theta_i)]'}{\theta_i m(\theta_i)} d\theta_i \right] 
\]

where \(d\varepsilon_{di}\) and \(d\theta_i\) are endogenous and linked to the level of firing costs. Differentiating equations (17) and (19) with respect to \(\varepsilon_{di}\), \(\theta_i\) and \(f_{ai}\) for a given aggregate state \(i\) and replacing in the above formula, one can show that the sign of \(d\bar{u}\) is ambiguous. For the clarity of the statement, these calculus are not reported here due to their complexity. One just needs here to remind that an increase in the degree of job protection tends to lower \(\varepsilon_{di}\) and consequently the job destruction but it also decreases the labor market tightness and then the job creation. Accordingly, the effect on the unemployment rate is ambiguous for any aggregate state and a fortiori on the average unemployment rate. The variance of the unemployment rate across steady states reads as \(V(u) = \sum_{i=1}^{n} \pi_i u_i^2 - \bar{u}^2\) and its differential as \(dV(u) = 2 \left[ \sum_{i=1}^{n} \pi_i u_i du_i - \bar{u} d\bar{u} \right]\). Obviously, the effect of job protection on the variability of employment is also unknown. One have here to note that this ambiguity does not hold in partial equilibrium model. As a matter of fact, job protection only reduces job creation in booms and job destruction in slumps. Thus, the impact of firing costs on the variability of employment is clearly determined. Things turn out to be different in our equilibrium matching model since job creation and job destruction take place simultaneously. To evaluate the effects of firing costs on both the average employment rate and its variability, one needs to proceed to some quantitative exercises allowing us to get rid of the ambiguities we underlined and this for a large set of parameters.

### 2.4 Unemployment Dynamic

We now turn out to the analysis of the dynamic law of motion for employment and the worker flows implied by the macroeconomic model we have developed

\[\pi_i = \sum_{j=1}^{n} \pi_{ij} \text{ and } \sum_{i=1}^{n} \pi_i = 1.\]
above. Both $\theta$ and $\varepsilon_d$ are forward-looking variables that jump on the impact to their new steady state equilibrium values as the aggregate state changes (Pissarides, 2000). On the contrary, unemployment is a sticky variable that is driven by the co-movement in the two forward looking variables. In order to specify the dynamic of unemployment, we divide time into discrete periods indexed by the subscript $t$ where $t = 0, 1, \ldots$ represents a quarterly sequence. Let $N_t, C_t$ and $D_t$ denote the employment at the beginning of period $t$, the job creation and the job destruction flows at period $t$ respectively. Thus, the aggregate law of motion of employment is given by the following equation:

$$N_{t+1} = N_t + C_t - D_t.$$  \hspace{1cm} (21)

To describe more accurately the model’s dynamic, one needs to keep track of the law of motion for employment for each idiosyncratic component of productivity $\varepsilon$. For the sake of simplicity, we assume that the aggregate shock only occurs at the beginning of the time period i.e. each quarter for the case at purpose. Hence, once the macroeconomic environment is defined, the only remaining source of job destruction is the idiosyncratic one. Let $n_t(\varepsilon)$ represent the number of workers employed at the current productivity $\varepsilon$ at the beginning of period $t$. Accordingly, the number of workers whose productivity is $\varepsilon$ at the beginning of period $t$ + 1 satisfies:

$$n_{t+1}(\varepsilon) = \begin{cases} 
(1 - \lambda)n_t(\varepsilon) + \lambda F'(\varepsilon) \left[ N_t - \int_{\varepsilon_1}^{\varepsilon_d} n_t(\zeta) d\zeta \right] & \text{if } \varepsilon_u \geq \varepsilon \geq \varepsilon_{dit} \\
0 & \text{if } \varepsilon < \varepsilon_{dit} 
\end{cases}$$  \hspace{1cm} (22)

where $\varepsilon_{dit}$ is the reservation productivity contingent to the current aggregate state $i$ and for the time period $t$. The first term of equation (22) represents the jobs which idiosyncratic productivity is $\varepsilon$ and that are not hit by a job-specific shock. The second term refers to all the surviving occupied jobs which idiosyncratic productivity becomes $\varepsilon$ due to the change in the idiosyncratic component. Finally, the dynamic law of motion for employment is given by the first line of equation (22) provided the idiosyncratic component is in the range $[\varepsilon_{dit}, \varepsilon_u]$ and by the second term for all others remaining values.

Assuming, firms have only one job that is either vacant or filled and jobs that are quit are destroyed, the job creation rate is equal to the rate vacant jobs are getting matched. Thus, the job creation flow in period $t$ reads as:

$$C_t = \theta_t m(\theta_t)(1 - N_t)$$  \hspace{1cm} (23)

where $\theta_t m(\theta_t)$ is the job finding rate.

Jobs are destroyed for one of two reasons. First, a bad aggregate shock may occur and causes the reservation productivity threshold to be shifted up. Accordingly, all jobs which idiosyncratic productivity lies between the old and the
new threshold are terminated. Second, jobs may be hit by an adverse microeconomic shock causing the job-specific productivity to fall below the current reservation threshold. The destruction flow is then given by:

\[
D_t = \int_{\varepsilon_t}^{\varepsilon_{dit}} n_t(\zeta)d\zeta + \lambda F(\varepsilon_{dit}) \left[ N_t - \int_{\varepsilon_t}^{\varepsilon_{dit}} n_t(\zeta)d\zeta \right].
\]

(24)

Finally, assuming population is normalized to unity, the law of motion of unemployment is:

\[
U_t = 1 - N_t.
\]

(25)

3 Numerical exercises

The results of the model are consistent with a large set of parameters. In order to evaluate its effects, we calibrate the model for both the French and U.S. labor market. We show that an increase in firing costs tends to increase the variability of employment and the net effect on employment is slightly positive. For each numerical exercise, one begins to study the quantitative effect of an increase in job protection in the steady state. Then, we focus on the impulse response functions to an increase in firing costs, hereafter IRF, and to the cyclical behavior of the key variables.

We follow Mortensen and Pissarides (1999a) in the calibration exercise. A matching function of the Cobb-Douglas form is assumed, such that \( m(v_i, u_i) = k u_i^{0.5} v_i^{0.5} \) where \( k \) is a mismatch parameter. This specification is consistent with the empirical results developed by Blanchard and Diamond (1989). The distribution of the idiosyncratic shocks is assumed to be uniform on the support \([0,1]\). The aggregate productivity shocks are modeled as a three states Markov chain. This assumption is consistent with Christiano (1990) who shows that such process as the same Wold representation than a first order autoregressive process of the form \( y_t = \rho y_{t-1} + (1-\rho)\mu + \nu_t \). Assuming that, it is impossible to jump from an extreme aggregate state to another, the transition matrix is represented by:

\[
\begin{pmatrix}
\rho & 1-\rho & 0 \\
\frac{1-\rho}{\rho} & \frac{1+\rho}{\rho} & \frac{1-\rho}{\rho} \\
0 & 1-\rho & \rho
\end{pmatrix}
\]

where \( \rho \) is the autocorrelation coefficient from the first order autoregressive process estimated on the stationary productivity process. The baseline parameters for each numerical exercise are reported in Table (1) and Table (3). Finally, we assume a symmetric bilateral Nash bargaining, so that \( \beta = 0.5 \), and that the Hosios (1990) condition holds. The interest rate is set to 1% per quarter.
Table 1: Baseline parameters for the French labor market

<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>h</th>
<th>λ</th>
<th>σ</th>
<th>b</th>
<th>fₗ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.075</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

3.1 France

We begin the quantitative analysis by calibrating the model for the French labor market. We take parameter values which are supposed to represent the main features of the French economy and are consistent with the empirical studies. Following Karamé and Mihoubi (1998), we set the autocorrelation coefficient for the autoregressive process to 0.946 and the variance of the innovation to 0.007. Accordingly, the vector of aggregate productivity components satisfies $p₁ = 0.0375$, $p₂ = 0$ and $p₃ = -0.0375$ where the subscripts 1, 2 and 3 stand for the high, median and low aggregate state respectively. The scale parameter $k$ and the cost of vacant jobs $h$ are set to approximate the mean unemployment rate across steady states to 11%. The idiosyncratic dispersion indicator $σ_i$ and the arrival rate of the job specific shocks are fixed in order to mimic the employment variability across steady states as well as the main features of the employment flows documented by Duhautois (1999). The reference firing costs amount to 50% of the yearly wage (Goux and Maurin (2000))\(^3\). Finally, the unemployment benefits are worth 50% of the average wage at the steady state. The baseline parameters are reported in Table (1).

3.1.1 Steady States

One now focuses on the steady states implications of the model. Figures (2), (3) and (4) represent the effects of an increase in job protection on the unemployment rate across states, the average employment and its variance. At first glance, it is striking that an increase in firing costs tends to lower the unemployment rate for the best aggregate states. One needs here to remind that the effects of firing costs are twofold and ambiguous on the unemployment rate. First, they tend to reduce job creation. Second, they tend to increase the labor hoarding phenomenon by lowering the reservation productivity. Hereafter, we will refer to a job creation effect and to a job destruction effect respectively. For the exercise at purpose this second effect rules over the first one in the high and median states and consequently unemployment rates tend to decrease. Figure (3) corroborates this analysis. For the worst aggregate state, the job creation effect is greater and then the unemployment rate increases with firing costs. More noticeably, one can also remark on Figure (2) that the slope of unemployment rate is steeper when the aggregate conditions improve. This result addresses the question of the effects of firing costs on the variability of employment. Figure (4) plots the employment variability when job protection becomes more stringent.

\(^3\)In our numerical exercises, the reference level of administrative firing costs are set to 0.3 on a quarterly basis so that to amount for 50% of the average yearly wage.
Figure 2: Unemployment rates for the high state, the median state and low state (French labor market).

Figure 3: Average employment (French labor market).
Adm inistrative firing costs

Figure 4: Standard deviation of employment (French labor market).

From this latter figure, it is obvious that an increase in the firing costs tends to enhance the aggregate employment fluctuations. This result is at odd with the standard conclusion of partial equilibrium models that assess that job protection is likely to smooth the cyclical behavior of employment over the business cycle. In order to analyze the mechanism behind this result, it is convenient to refer to the surplus that sheds light on this effect. This analysis will mainly rely on equations (17) and (19) namely the job destruction and the job creation conditions.

First, one focuses on the job destruction condition (17). Job protection has two main asymmetrical effects on job destruction or more accurately an idiosyncratic labor hoarding effect and an aggregate labor hoarding effect. As we have seen, an increase in the firing costs tends to directly enhance the surplus and consequently to decrease the productivity threshold. This effect induces an asymmetrical reaction of the idiosyncratic surplus, which differs whether one considers a “good” aggregate state or a “bad” aggregate state. As a matter of fact, this reaction is proportional to the number of jobs protected at the margin. For a given aggregate state, the mass of jobs taken into account in the expected surplus will be greater the higher the aggregate condition. Accordingly, the job destruction effect is an increasing function of the aggregate productivity level.

A second effect of job protection on the labor hoarding will obviously go along with the expected change in the aggregate condition. For the sake of simplicity and as a departure, let us deal with an illustrative example. Let us assume that the economy is stuck in the worst aggregate state. For a given idiosyncratic productivity, the macroeconomic environment can therefore only improve and thus maintains all existing jobs (see Figure (1)). Consequently, the value of
the job will increase in accordance with the improvement in the surplus for any state. For a given level of firing costs, this improvement will be greater the lower the initial aggregate condition. However, one needs to note that this gain can be nil according to the initial state of the economy since a change in the aggregate state may lead to terminate the job. An increase in firing costs will strengthen this effect since they tend to increase the overall surplus in any state. Accordingly, the aggregate labor hoarding will be higher the lower the aggregate productivity level. This creates a negative link between the job destruction effect and the aggregate condition. For the numerical exercises at purpose, the idiosyncratic labor hoarding response always offsets the aggregate one. Consequently, the job destruction effect is greater the higher the aggregate condition.

Second, let us swivel to the job creation condition (19). Job protection handles two straight and traditional effects on job creation. First, an increase in firing costs reduces the mean profit of a new job and therefore induces firms to create fewer jobs. Second, one needs to rely to an idiosyncratic labor hoarding effect similar to the one described above. An increase in the firing costs and/or an improvement in the aggregate condition will elevate the idiosyncratic expected surplus and therefore lower the negative effect on job creation. For a given labor market tightness, the second effect will be greater the higher the aggregate state. Accordingly, the negative effect of firing costs on job creation is more important in low aggregate state.

Hence, an increase in the degree of job protection will reduce both job creation and job destruction as in any standard equilibrium matching models. But in opposition to these models the width of each effect will be contingent to the aggregate state. Broadly speaking, the negative effect on job creation is stronger in bad states and the effect on job destruction seems to be stronger in good states. This creates the asymmetrical impact of firing costs on unemployment. Thus, the cyclical behavior of job creation and job destruction sheds light on the increase of employment variability consecutive to an increase in job protection.

3.1.2 Impulse Response Functions (IRF)

Let’s now turn out to the analysis of the IRF when the economy is hit by a positive or a negative aggregate shock. Assuming that the initial state of the economy is the median one, we successively analyze the effects of the aggregate shocks for various levels of firing costs.

First, we focus on the effects of a positive aggregate shock. Figure (5) and (6) plot the IRF of job creation and job destruction for a level of firing costs that amounts to 0.3 and 0.6 respectively. The initial response is an increase in creation and a fall in destruction. The aggregate shock causes the productivity threshold to be shifted down. Accordingly, destructions falls due to the fact a shock induces a new range of productive jobs. In this case, some of the jobs hit by an adverse idiosyncratic shock and that would have been previously destroyed, remains productive. On the contrary, job creations increase, because
the improvement of the aggregate environment leads to open up some new vacancies. As the job creation process is costly and takes time, the employment level adjust slowly to its new level as well as the rate of job creation. In turn, the comparison of Figure (5) and (6) gives significant insights on the effect of job protection on the cyclical behavior of the job destruction rate. One can remark that the response of job destruction is positively correlated with the level of job protection due to the increase in the range of the reservation productivities. One can also note that the adjustment to the new steady state is longer the higher the degree of job protection.

Second, we deal with the effect of a negative aggregate shock. Figure (7) and (8) plot the IRF of job creation and job destruction for the same levels of firing costs we used above. The initial response to a negative aggregate shock is an increase in the rate of job destruction and a decrease in the rate of job creation. The marginal jobs which productivity is now less than the new reservation productivity are immediately destroyed. After the shock, the only remaining destruction source is the idiosyncratic one. The job destruction rate joins its new level. The reduction in the profit opportunities tends to instantaneously decrease the job creation rate. These initial jumps increase unemployment and consequently decrease the labor market tightness. Accordingly, there is a positive jump in job creation that overshoots its new steady state level. One have here to remind that job creation process is costly and time consuming, therefore the job creation rate will slowly adjust to its new steady state level. In turn, the comparison of Figure (7) and (8) sheds lights on the effects of an increase in the degree of job protection. The variation in the rate of job destruction

Figure 5: Impulse response function for a positive aggregate shock when the firing costs amount to 0.3 (French labor market).
Figure 6: Impulse response function for a positive aggregate shock when the firing costs amount to 0.6 (French labor market).

is positively correlated with the level of firing costs. A tighter job protection increases the range between the threshold productivities and consequently an adverse aggregate shock tends to terminate more jobs the higher the firing costs. A tighter job protection tends to increase the initial negative effect on the job creation. It also decreases the creation overshooting due to smaller labor market flows. One can remark that the adjustment process for the job creation rate is longer for a high level of firing costs.

3.1.3 Simulation Statistics

Table (2) summarizes the time series statistics for the French labor market following the method developed by Mortensen and Pissarides (1994). We build for three different levels of job protection the series of job creation, job creation and unemployment rates then we calculate the main statistics namely the means, the variances and the correlation coefficients. From this table it is obvious that the relevant means of this series tend to decrease with the level of job protection⁴. On the contrary, the variability of the series and, more noticeably the variance of unemployment, tends to increase with the level of firing costs. Therefore, the steady state analysis remains valid in such dynamic framework.

⁴Note that for high values of the unemployment benefits b, the mean unemployment rate tends to increase. However, the main findings about aggregate employment variability still holds (see appendix 5.3 for details).
Figure 7: Impulse response function for a negative aggregate shock when the firing costs amount to 0.3 (French labor market).

Figure 8: Impulse response function for a negative aggregate shock when the firing costs amount to 0.6 (French labor market).
<table>
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<th>( f_a = 0.3 )</th>
<th>( f_a = 0.6 )</th>
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<tbody>
<tr>
<td>Mean (JC)</td>
<td>5.7940</td>
<td>5.0419</td>
<td>4.3039</td>
</tr>
<tr>
<td>Mean (JD)</td>
<td>5.7939</td>
<td>5.0448</td>
<td>4.3015</td>
</tr>
<tr>
<td>Std. dev. (JC)</td>
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<td>0.2742</td>
<td>0.2793</td>
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<tr>
<td>Std. dev. (JD)</td>
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<td>0.3838</td>
<td>0.4013</td>
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<tr>
<td>Corr. (JC,JD)</td>
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<td>-0.3151</td>
<td>-0.2767</td>
</tr>
<tr>
<td>Mean (u)</td>
<td>10.47</td>
<td>10.03</td>
<td>9.56</td>
</tr>
<tr>
<td>Std. dev. (u)</td>
<td>1.0712</td>
<td>1.2110</td>
<td>1.2877</td>
</tr>
<tr>
<td>Corr. (u,v)</td>
<td>-0.3688</td>
<td>-0.3888</td>
<td>-0.3266</td>
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</table>

Table 2: Simulation statistics for 100 simulated 100 quarters samples (French labor market)

<table>
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<tr>
<th>( k )</th>
<th>( h )</th>
<th>( \lambda )</th>
<th>( \sigma )</th>
<th>( b )</th>
<th>( f_a )</th>
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<tbody>
<tr>
<td>1.9</td>
<td>0.6</td>
<td>0.075</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Baseline parameters for the U.S. labor market

### 3.2 The United States

We turn next to the analysis of the U.S. labor market. We take parameter values which are supposed to represent the main features of the U.S. economy and are consistent with the empirical studies. The autocorrelation coefficient is set to 0.933 and the variance of the innovation to 0.011 according to the findings of Mortensen and Pissarides (1994). Hence, the vector of aggregate productivity components satisfies \( p_1 = 0.053, \ p_2 = 0 \) and \( p_3 = -0.053 \) where the subscripts 1, 2 and 3 stand for the high, median and low aggregate state respectively. The scale parameter \( k \) and the cost of vacant jobs \( h \) are set to approximate the mean unemployment rate across steady states to 8%. The idiosyncratic dispersion indicator \( \sigma_i \) and the arrival rate of the job specific shocks are fixed in order to mimic the employment variability across steady states as well as the main features of the employment flows documented by Davis and Haltiwanger (1999). The labor market does not handle job protection and then firing costs are nil. Finally the unemployment benefits are neglected. The baseline parameters are reported in Table (3).

#### 3.2.1 Steady States

We investigate now the steady states implications of the model. Figures (9), (10) and (11) show the impact of an increase in the firing costs on the unemployment rate across states, the average employment and its variance respectively. The results remain quite similar to the ones depicted on the French labor market and therefore seems quite robust on a wide range of labor markets. As a matter of fact, while comparing two labor markets as much dissimilar as the French and the U.S. markets, one remarks that our main quantitative results hold.
increase in the firing costs tends to increase the employment variability for low values of firing costs.

Therefore, the analysis developed for the case of France and described in the previous section remains valid. The dynamic behavior of job creation and job destruction after a negative or positive aggregate shock are also similar to the ones depicted on the French labor market. The related impulse response functions are reported in appendix (5.4). The only noticeable points are the higher responses of both job creation and job destruction to shocks.

### 3.2.2 Simulation Statistics

Table (4) summarizes the time series statistics for the U.S. labor market. As previously, we have built for three different levels of job protection the series of job creation, job creation and unemployment rates then calculated the main statistics. First, one can remark that the means of job creation and job destruction also decrease with job protection as well as the average unemployment rate. Second, the standard deviation for job destruction increases with the level of firing costs. Although, the standard deviation of job creation seems to slightly decrease, its value remain in the same range according to the standard error statistic (unreported here). Overall the increase of the aggregate employment variability still holds due to the wide variance of job destruction.
Figure 10: Average employment (U.S. labor market).

Figure 11: Standard deviation of employment (U.S. labor market).
Table 4: Simulation statistics for 100 simulated 100 quarters samples (U.S. labor market)

<table>
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<tr>
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<th>$f_a = 0.15$</th>
<th>$f_a = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (JC)</td>
<td>6.2067</td>
<td>5.4322</td>
<td>4.6242</td>
</tr>
<tr>
<td>Mean (JD)</td>
<td>6.2030</td>
<td>5.4313</td>
<td>4.6233</td>
</tr>
<tr>
<td>Std. dev. (JC)</td>
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<td>0.8434</td>
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<td>Std. dev. (JD)</td>
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<tr>
<td>Corr. (JC,JD)</td>
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<td>-0.0591</td>
</tr>
<tr>
<td>Mean (u)</td>
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<td>7.51</td>
<td>6.83</td>
</tr>
<tr>
<td>Std. dev. (u)</td>
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<td>1.997</td>
</tr>
<tr>
<td>Corr. (u,v)</td>
<td>-0.0273</td>
<td>-0.0451</td>
<td>-0.0680</td>
</tr>
</tbody>
</table>

4 Conclusion

In this paper, we have provided a simple model of matching with both idiosyncratic and aggregate productivity shocks. This model leads us to argue that simultaneous job creation and job destruction give significant insights on the link between employment variability and job protection. Our quantitative analysis suggests that an increase in firing costs is likely to enhance the variability of employment and to reduce average unemployment for a large set of parameters. In addition, our results suggest that countries with a high level of firing costs are likely to experience a sharp increase in the unemployment rate when the economy slumps. Obviously, our model has some limitations that future work should go beyond. First, one can investigate the ability of state dependent firing costs to reach a given variability objective in terms of aggregate production, unemployment or welfare. Second, one needs to note that job protection arises in many different ways on the Continental European labor markets. More accurately, the minimum wage and the temporary jobs play a key role on such markets. Accordingly, they are likely to provide new perspectives in our framework. Finally, the unemployment compensation system is also likely to interact with the features developed in this paper and in particular with job protection. More noticeably, experience rating (see Feldstein (1976) for details) which is a mean to require employers to contribute to the payment of unemployment benefits through their firing decisions is part of our research agenda.
References


5 Appendix

5.1 Surplus equations

5.1.1 Surplus of a continuing Job

The surplus associated with a continuing job is defined by equation (10). Using equations (5), (8) together with the zero-profit condition \( \Pi_{vi} = 0 \), one gets:

\[
(r + \lambda + \sum_{i \neq j} t_{ij}) S_i(\varepsilon) = p_i + \sigma_i \varepsilon - (r + \sum_{i \neq j} t_{ij}) (V_{ui} - f_{ai}) + \mu(V_{uj} - f_{aj}) \\
+ \lambda (E(\Pi_{ci}) + E(V_{ci}) - V_{ui} + f_{ai}) \\
+ \sum_{i \neq j} t_{ij} \max[\Pi_{cj}(\varepsilon) + V_{cj}(\varepsilon) - V_{uj} + f_{aj}, 0] \tag{26}
\]

where \( E(\Pi_{ci}) \) and \( E(V_{ci}) \) are the means over the idiosyncratic component \( \varepsilon \) of the expected value of a filled job and of the expected utility of an employed worker respectively. It is worth noting that the mean of the expected value of the surplus of a continuing job could be written as \( E(S_i) = E(\Pi_{ci}) + E(V_{ci}) - V_{ui} + f_{ai} \) since nor the instantaneous utility of an unemployed worker nor the firing costs are dependent of the idiosyncratic component of the productivity \( \varepsilon \).

Using this property as well as relation (10), the surplus reads as:

\[
(r + \lambda + \sum_{i \neq j} t_{ij}) S_i(\varepsilon) = p_i + \sigma_i \varepsilon - (r + \sum_{i \neq j} t_{ij}) (V_{ui} - f_{ai}) \\
+ \sum_{i \neq j} t_{ij} (V_{uj} - f_{aj}) \tag{27}
\]

\[
+ \lambda E(S_{ci}) + \sum_{i \neq j} t_{ij} \max[S_j(\varepsilon), 0]. \tag{28}
\]

The expected utility of an unemployed worker contingent to aggregate state \( i \) is given by equation (6). Making use of this relation together with the sharing rules (13) and (14), the expected value of a vacant job (3) and the free entry condition, allow to write the surplus of a continuing job as:

\[
\]
\( (r + \lambda + \sum_{i \neq j}^{n} t_{ij}) S_i(\varepsilon) = p_i + \sigma_i \varepsilon - b_i - \frac{\theta_i \beta h}{1 - \beta} + r f_{ai} + \lambda E(S_i) \)
\[
+ \sum_{i \neq j}^{n} t_{ij} (f_{ai} - f_{aj})
\]
\[
+ \sum_{i \neq j}^{n} t_{ij} [\text{Max} [S_j(\varepsilon), 0]].
\]  
(29)

5.1.2 Surplus of a new job

The surplus of a new job is defined by (9). Using equations (4), (7) and the free entry condition, one can rewrite the surplus as:
\[
(r + \lambda + \sum_{i \neq j}^{n} t_{ij}) S_{0i}(\varepsilon_u) = p_i + \sigma_i \varepsilon_u - \lambda f_{ai} - (r + \sum_{i \neq j}^{n} t_{ij}) V_{ui} + \sum_{i \neq j}^{n} t_{ij} V_{uj}
\]
\[
+ \lambda \left(E(\Pi_{ai}) + E(V_{ai}) - V_{ui} + f_{ai}\right)
\]
\[
+ \sum_{i \neq j}^{n} t_{ij} (\Pi_{oj}(\varepsilon_u) + V_{oj}(\varepsilon_u) - V_{uj}).
\]  
(30)

The expressions of the surplus for both a new job and a continuing one are respectively given by equations (9) and (10). Making use of these two relations and using the previously derived expression for the mean of the expected value of the surplus, one gets:
\[
(r + \lambda + \sum_{i \neq j}^{n} t_{ij}) S_{0i}(\varepsilon_u) = p_i + \sigma_i \varepsilon_u - \lambda f_{ai} - r V_{ui} + \lambda E(S_i)
\]
\[
+ \sum_{i \neq j}^{n} t_{ij} (V_{uj} - V_{ui})
\]
\[
+ \sum_{i \neq j}^{n} t_{ij} S_{0j}.
\]  
(31)

Finally, using the sharing rules (13) and (14), the expected utility of an unemployed worker (6), the expected value of a vacant job (3) together with the free entry condition, the expression of the surplus for new job contingent to aggregate state \( i \) satisfies:
\[(r + \lambda + \sum_{i \neq j} t_{ij})S_{0i}(\varepsilon_i) = p_i + \sigma_i \varepsilon_i - b_i - \frac{\theta_i \beta h}{(1 - \beta)} - \lambda f_{ai} + \lambda E(S_i) + \sum_{i \neq j} t_{ij}S_{0j}.
\]

### 5.2 Wage equations

#### 5.2.1 Renegotiated wage

The renegotiated wage contingent to aggregate state \(i\) is the solution of the following Nash bargaining rule:

\[w_i(\varepsilon) = \text{Arg}\max\{V_{ei}(\varepsilon) - V_{ui} - f_{ei}\} = \beta (\Pi_{ei}(\varepsilon) + f_i)\]

The outcome of this bargaining process satisfies:

\[(1 - \beta) (V_{ei}(\varepsilon) - V_{ui} - f_{ei}) = \beta (\Pi_{ei}(\varepsilon) + f_i)\]

and implies the following properties:

\[\Pi_{ei}(\varepsilon) + f_i \leq 0 \Leftrightarrow V_{ei}(\varepsilon) - V_{ui} - f_{ei} \leq 0,\]

\[(1 - \beta) \max (V_{ei}(\varepsilon) - V_{ui} - f_{ei}, 0) = \beta \max (\Pi_{ei}(\varepsilon) + f_i, 0).\]

The value functions (5) and (8) yield:

\[\left(r + \lambda + \sum_{i \neq j} t_{ij}\right) (\Pi_{ei}(\varepsilon) + f_i) = p_i + \sigma_i \varepsilon_i - w_i(\varepsilon) + \lambda \left[\int_{-\infty}^{\varepsilon_u} \text{Max} [\Pi_{ei}(\xi) + f_i, \Pi_{ei}] \, dF(\xi)\right] + \sum_{i \neq j} t_{ij} [\text{Max} [\Pi_{ej}(\varepsilon) + f_j, \Pi_{ej}]] + rf_i + \sum_{i \neq j} t_{ij} (f_i - f_j)\]

\[\left(r + \lambda + \sum_{i \neq j} t_{ij}\right) (V_{ei}(\varepsilon) - V_{ui} - f_{ei}) = w_i(\varepsilon) + \lambda \left[\int_{-\infty}^{\varepsilon_u} \text{Max} [V_{ei}(\xi) - V_{ui} - f_{ei}, 0] \, dF(\xi)\right] + \sum_{i \neq j} t_{ij} [\text{Max} [V_{ej}(\varepsilon) - V_{uj} - f_{ej}, 0]] - rf_{ei} - \sum_{i \neq j} t_{ij} (f_{ei} - f_{ej}) - \sum_{i \neq j} t_{ij} (V_{ui} - V_{uj}).\]
Taking into account the properties as well as the sharing rules, the condition \( \Pi_{vi} = 0 \) and the fact that \( f_i = f_{ei} + f_{ai} \), one gets a wage equation:

\[
w_i(\varepsilon) = (1 - \beta)(rV_{ui} - \sum_{i \neq j} t_{ij} (V_{uj} - V_{ui})) + \beta(p_i + \sigma_i \varepsilon_i + \sum_{i \neq j} t_{ij} (f_{ai} - f_{aj}) + r f_{ai}) + r f_{ei} + \sum_{i \neq j} t_{ij} (f_{ei} - f_{ej}) .
\]

The expected value of an unemployed worker (6) can be written as:

\[
rV_{ui} - \sum_{i \neq j} t_{ij} [V_{uj} - V_{ui}] = b_i + \theta_i m(\theta_i) [V_{0\imath}(\varepsilon_u) - V_{ui}]
\]

The sharing rule (13) and the zero profit condition on vacant jobs imply respectively:

\[
V_{0\imath}(\varepsilon_u) - V_{ui} = \frac{\beta}{1 - \beta} \Pi_{0\imath}(\varepsilon_u)
\]

\[
\Pi_{0\imath}(\varepsilon_u) = \frac{h}{\bar{m}(\theta_i)}
\]

Then, the renegotiated wage contingent to aggregate state \( i \) reads as:

\[
w_i(\varepsilon) = (1 - \beta)(b_i + \frac{\beta}{1 - \beta} \theta_i h) + \beta(p_i + \sigma_i \varepsilon_i + r f_{ai} + \sum_{i \neq j} t_{ij} (f_{ai} - f_{aj})) + r f_{ei} + \sum_{i \neq j} t_{ij} (f_{ei} - f_{ej}) .
\]

5.2.2 Negotiated wage

The negotiated wage \( i.e. \) the starting wage, contingent to aggregate state \( i \), is the solution of the following Nash bargaining rule:

\[
w_i(\varepsilon) = \text{Arg max} [V_{0\imath}(\varepsilon_u) - V_{ui}]^\alpha [\Pi_{0\imath}(\varepsilon_u) - \Pi_{vi}]^{1-\beta}
\]

The outcome of this bargaining process satisfies:
The value functions (4) and (7) yield:

\[
(1 - \beta) (V_{0i}(\varepsilon_u) - V_{ui}) = \beta \Pi_{0i}(\varepsilon_u)
\]

The value functions (4) and (7) yield:

\[
\begin{align*}
(1 - \beta) (V_{0i}(\varepsilon_u) - V_{ui}) &= \beta \Pi_{0i}(\varepsilon_u) \\
(1 - \beta) (V_{0j}(\varepsilon_u) - V_{uj}) &= \beta \Pi_{0j}(\varepsilon_u)
\end{align*}
\]

Making use as previously of the expected utility of an unemployed worker, the zero profit condition on vacant job and of the sharing rules (14) and (13) the negotiated wage contingent to aggregate state \(i\) satisfies:

\[
w_{0i}(\varepsilon_u) = (1 - \beta)(b_i + \frac{\beta}{1 - \beta} \theta) + \beta(p_i + \sigma \varepsilon_u - \lambda f_{ai}) - \lambda f_{ei}.
\]

### 5.3 Job protection and high unemployment benefits (France)

The figures below represent the steady states results of our numerical exercises for the french labor market when the unemployment benefits are high ranked. For the exercise at purpose, unemployment benefits are worth 65% of the average wage in the high, median and low aggregate state. One can remark that in this case the average unemployment tends to increase as well as the aggregate employment fluctuations. Therefore the main finding about employment variability still holds whereas the one relative to the average unemployment is reversed. Accordingly, the results of the model seem to be robust in a wide range of cases.
Figure 12: Unemployment rates and job protection, high levels of unemployment benefits (French labor market).

Figure 13: Average employment, high levels of unemployment benefits (French labor market).
Standard deviation of unemployment, high levels of unemployment benefits (French labor market).

5.4 Impulse Response Functions (IRF) for U.S. labor market

Impulse response function for a positive aggregate shock when there is no firing costs (U.S. labor market).
Impulse response function for a positive aggregate shock when firing costs amount to 0.5 (U.S. labor market).

Impulse response function for a negative aggregate shock when there is no firing costs (U.S. labor market).
Impulse response function for a negative aggregate shock when firing costs amount to 0.5 (U.S. labor market).