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Buy or Wait, That is the Option
The Buyer's Option in Sequential Laboratory Auctions[*]

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Buy or wait, that is the option

The buyer's option in sequential laboratory auctions*

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Abstract

This paper reports the results from an experiment on two-unit sequential auctions with and without a buyer's option (which gives the first-auction winner the right to buy the second unit at the winning price). The 4 main auction institutions (first-price, Dutch, second-price, English) are studied. Observed bidding behavior is closer to risk-neutral Nash equilibrium bidding in the second auction than in the first auction. In Dutch and first-price auctions, the deviations from theory can be attributed to risk aversion among buyers; in the English and second-price auctions, they are a consequence of either myopic or punitive behavior. The revenue ranking of the 4 auction institutions is the same as in single-unit experiments. The buyer's option decreases (resp. increases) revenue in first-price (resp. second-price) auctions, but there is no significant effect in the "oral" auctions. The buyer's option causes declining price patterns in our experimental auctions.

Keywords: Experimental economics, sequential auctions, buyer's option.
***JEL* Classification:** C91; D44.

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1 Introduction

In sales of multiple units of a particular good, auction houses often choose to sell the items sequentially, i.e. the items are auctioned separately, one after the other. The advantage of a sequential auction is that it well fits the needs of both small and large buyers, whereas the alternative auction procedure that consists in selling all available units simultaneously, in one shot, typically excludes buyers who set low values on the items, thereby reducing competition at auction. The main disadvantage of a sequential method is that it can be very time-consuming, especially when the total number of units on sale is large. For this reason, auctioneers sometimes provide a so-called buyer's option, which gives the winner of the first auction the right to buy any number of units (1, 2, ..., or all units available). For each unit he/she must pay the winning price established at the first auction. If the winning bidder decides to purchase only part of the total quantity, the remaining items are reauctioned, in the same manner, through a second auction; and this scheme is repeated until all units are eventually sold.

The buyer's option thus clearly offers the best of both worlds: it allows the auctioneer to speed up sales, while keeping the auction mechanism sufficiently flexible to be of interest for different types of buyers. Not surprisingly therefore, the buyer's option is used in many auctions throughout the world. Cassady (1967) describes how the buyer's option is practiced in fur auctions in Leningrad and London, and fish auctions in English port markets. At the auction market in Aalsmeer, the Netherlands, huge quantities of flowers are sold through sequential descending auctions with a buyer's option (see van den Berg, van Ours, and Pradhan (1999)). Well-known auction houses such as Christie's and Sotheby's (see Ashenfelter (1989) and Ginsburgh (1998)) and Drouot (see Février, Roos, and Visser (2001)) systematically use the buyer's option in their sequential ascending auctions of fine wines.

Despite the practical importance of the buyer's option, little attention has been paid to the subject in the literature. The only theoretical article we are aware of is Black and De Meza (1992). They consider the Independent Private Value (IPV) paradigm, and derive optimal bidding strategies in two-unit sequential second-price auctions with and without the buyer's option. All buyers in their model have decreasing demand for the two units (the additional value of the second unit is less than the value of the first unit), or flat demand (both units are valued the same). Empirical studies are also rare. Ashenfelter (1989) and Ginsburgh (1998) report that the option is exercised by many buyers in ascending wine auctions at Christie's and Sotheby's.[1] Van den Berg, van Ours, and Pradhan (1999) study price patterns at sequential descending auctions of roses and argue that the presence of the option is the main determinant of the observed price decline. Finally, Février, Roos, and Visser (2001), using data on

[1] Ashenfelter (1989) claims that auctioneers feel uneasy and uncomfortable about revealing the declining price phenomenon (the fact that in sequential auctions of identical items successive prices tend to decline) to buyers, and use the option as a device for hiding it. The auctioneers with whom we have discussed at Drouot argue, however, that speed is the main reason for making the option available.

ascending auctions of wine held at Drouot, structurally estimate their optimal bidding model and use their estimations to analyze the impact of the option on revenue.

The main contribution of this paper is to study both theoretically and experimentally the role of the buyer's option in two-unit sequential auctions. We adopt the IPV paradigm and assume that the 2 units are sold to 2 risk-neutral buyers. Buyers desire both units, and their demand for the items is either decreasing, flat, or increasing (implying that the value of the second unit exceeds the value of the first unit). The 4 main auction institutions are considered: first-price, descending (Dutch), second-price (Vickrey), and ascending (English) auctions. Although there are apparently no field examples of first-price and second-price sequential auctions with or without a buyer's option,[2] it is nonetheless of interest to study these sealed-bid auctions. Like in standard one-unit auction theory, it is shown in this paper that first-price (resp. second-price) and Dutch (resp. English) sequential auctions with or without a buyer's option are theoretically isomorphic. Furthermore the 4 auction formats generate the same expected revenue. By analogy with experimental studies on single-unit auctions (see Kagel (1995)) for a survey), our experimental design thus allows us to test whether bidding behavior is identical and whether there is an equivalence in revenue.

Many other theoretical predictions are confronted with the experimental data. For each auction institution with and without buyer's option, and each form of the demand curve, we test if observed bidding behavior corresponds to risk-neutral Nash equilibrium bidding. We analyze to what extent the experimental subjects exercise their buyer's option. Referring to the title of the paper, we thus analyze to what extent winners of the first auction directly *buy* the second unit, or instead *wait* and attempt to obtain the additional unit (at a lower price!) in the second auction. Observed frequencies of buying/waiting are compared with optimal frequencies. Predictions on the degree of efficiency of auction outcomes are also tested. Finally we compare observed price patterns with their predicted counterparts. The design of our experiment is such that all types of price patterns are theoretically possible. Depending on the auction mechanism, the form of the demand function, and the presence or not of the buyer's option, theory predicts that successive prices are either decreasing, constant, or increasing.

Experimental work on sequential auctions is still very rare.[3] Burns (1985) considers sequential English auctions. The experiment is designed to mimic the Australian wool market, and the paper's main objective is to study the effect of market size on auction prices. The paper is essentially theory-free in that observed behavior is not confronted with any equilibrium bidding behavior.

[2]Cassady (1967, p. 197) describes the electronic auction market in Osaka, Japan, where lots of fruit and vegetables are sold via sequential first-price auctions, but he never explicitly mentions that the successive lots on sale are identical.

[3]Spurred by the recent FCC auctions, experimental papers on all sorts of *simultaneous* multi-demand auctions are, however, flourishing (see for example Kagel and Levin (2001) and the references therein, and the special issue of the *Journal of Economics & Management Strategy* (1997, Number 3)).

Keser and Olson (1996) consider sequential first-price auctions and suppose that buyers have single-unit demand functions. Their main objective is to compare observed price-sequences with the predicted patterns derived in Weber (1983), under different design parameters. Similarly as in Burns, the paper focuses on one particular auction mechanism, and no attempt is made to examine outcomes under alternative institutions. Robert and Montmarquette (1999) do consider several auction institutions, and also provide theoretical foundations for each of them. In their models, the number of items desired by each buyer is a random variable and demand functions are decreasing. They consider sequential Dutch, English and mixed auctions, and compare observed behavior with predicted behavior. None of these 3 experimental papers on sequential auctions analyzes the buyer's option.

The paper proceeds as follows. In the next section the theoretical background is presented. In deriving the risk-neutral Nash equilibrium bidding functions and the expected revenues in the different auction institutions, we partly draw on Black and De Meza (1992), Donald, Paarsch, and Robert (1997) and a recent paper by Février (2000). But most results in this section are actually new. Section 3 describes the experimental design, section 4 the experimental results, and section 5 concludes.

2 Theoretical background

Suppose that 2 units of a good are auctioned to 2 potential buyers. Each buyer is assumed to be risk-neutral and desires to purchase both units. Adopting the IPV paradigm, let v_i denote the value that buyer i places on the first unit. The value v_i and the value of i's opponent are independently drawn from a uniform distribution on the interval $[0, \overline{v}]$. It is assumed that the value that i places on the second unit is kv_i. The parameter k can take three values: $k \in \{\frac{1}{2}, 1, 2\}$. The value of k is common knowledge. Note that $k = \frac{1}{2}$ implies that the second unit is valued less than the first unit (decreasing demand), $k = 1$ that both units are valued the same (flat demand), and $k = 2$ that the second unit is valued more than the first (increasing demand).

The 2 units are sold sequentially. The first unit of the good is sold in the first auction. The manner in which it is auctioned depends on the auction institution. Let a indicate the auction institution, $a \in \{D, E, F, S\}$, where D stands for Dutch auction, E for English auction, F for First-price auction, and S for Second-price auction, and let p_1 denote the price the winner of the first auction has to pay for the first unit. When $a \in \{D, E\}$, the unit is auctioned using a clock. When $a = D$, the clock starts very high, and descends until one of the players stops the clock. This player wins the unit and p_1 equals the price at which the clock was stopped. When $a = E$, the clock starts at 0, and increases until one of the players stops the clock. Here the winner of the auction is the player who *did not stop* the clock. The price p_1 he/she has to pay for the first unit is again the amount at which the clock stopped. When $a \in \{F, S\}$, the unit is sold via sealed-bid auctions. Both players submit their sealed bid to the auctioneer who awards the unit to the highest bidder. When $a = F$ the winner pays his/her own bid, i.e. p_1 equals the highest submitted bid. When $a = S$ the winner pays the bid of his opponent, i.e. here p_1 equals the second highest submitted bid. For all institutions a, the price p_1 is revealed to both players once the first auction has ended.

The way in which the second unit is sold depends on whether the buyer's option is available or not. Let o be the indicator for the availability of the buyer's option, $o = N$ if it cannot be used, and $o = Y$ otherwise. For any auction institution a, if $o = N$ the second unit is auctioned under the prevailing rules of institution a. Let p_2 be the price paid for the second unit. If instead $o = Y$ the winner of the first auction has the option to buy 1 or 2 units, at the price of p_1 per unit. When he decides to purchase only 1 unit, a second auction is held under the conditions of institution a. When he/she exercises the buyer's option, no second auction is held. Note that in this case we automatically have $p_2 = p_1$.

The theoretical model presented here is essentially based on the framework built by Black and De Meza (1992). These authors, however, only considered the second price auction ($a = S$) and they do not analyze the case of increasing marginal valuation ($k = 2$). The hypothesis that each bidders' valuation for the second unit is connected, in a deterministic way, to the valuation of the first unit, is certainly restrictive, and might no necessarily reflect behavior at real auctions. The hypothesis on the number of players is also restrictive as real world auctions the number of participant is typically larger than two. These simplifying hypotheses are, however, needed to ease solving for the equilibrium strategies. Also, as mentioned in the introduction, this is the first experimental paper on the buyer's option, justifying a rather simple setup, that can be refined and generalized in future work.

For any given value of a, o, and k, let $G(a, o, k)$ denote the bayesian two-stage game described above. We are looking for perfect bayesian equilibria of the game $G(a, o, k)$ with pure and symmetric strategies in the first auction. Let $b_1(v)$ denote the equilibrium strategy of the bidders in the first auction. If $o = Y$, let $bo(p_1) \in \{0, 1\}$ indicate whether the winner exercises the buyer's option or not given the auction price p_1, with $bo(p_1) = 1$ meaning that he/she uses his/her option, and $bo(p_1) = 0$ that he/she does not. Finally, let $b_2^w(v, p_1)$ denote the second auction strategy of the winner of the first auction, and $b_2^l(v, p_1)$ the second auction strategy of the loser of the first auction. For practical reasons given in section 4.3, these strategies are only confronted with the data when the buyer's option is not available. In the following proposition, the strategies are therefore only given for $o = N$. But in the proof of the proposition (given in the appendix), explicit use is made of the strategies for $o = Y$.

Proposition 1. *A symmetric-first-auction perfect bayesian equilibrium of the game $G(a, o, k)$ is:*

1. *If $a \in \{E, S\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = kv$, $b_2^l(v, p_1) = v$, $b_2^w(v, p_1) = kv$.*

2. *If $a \in \{D, F\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1\right\}$, then no such equilibrium exists.*

3. *If $a \in \{D, F\}$, $o = N$, and $k = 2$, then $b_1(v) = \frac{1}{2}v$, $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.*

4. *If $a \in \{E, S\}$, $o = Y$, and $k = \frac{1}{2}$, then $b_1(v)$ is solution of $b_1(v) - \frac{v}{2} = 2\lambda(v - b_1(v))\, b_1'(v)$, with $\lambda = 0$ if $b_1(v) \geq \frac{1}{2}\overline{v}$ and $\lambda = 1$ otherwise; $bo(p_1) = 1$ if $p_1 \leq \frac{1}{2}v$ and $bo(p_1) = 0$ if $p_1 > \frac{1}{2}v$.*

5. *If $a \in \{E, S\}$, $o = Y$, and $k = 1$, then $b_1(v) = v$, $bo(p_1) \in [0, 1]$.*

6. *If $a \in \{E, S\}$, $o = Y$, and $k = 2$, then $b_1(v) = 2v$, $bo(p_1) = 0$.*

7. *If $a \in \{D, F\}$, $o = Y$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = \frac{1+k}{4}v$, $bo(p_1) = 1$.*

Let us first comment on the predictions for the English and second-price auctions. As mentioned in the introduction, the behavioral predictions are always the same for these 2 mechanisms. When $o = N$, theory requires bidders to bid kv in the first auction, that is they have to bid the value for the second unit. While this result is intuitive for flat demand, it is less so when demand is decreasing or increasing. With decreasing demand, bid shading is required because losing the first auction is not necessarily bad news, as it implies a weaker rival in the second auction. With increasing demand, over-bidding is required as the winner of the first auction is also going to be the winner of the second auction. In the second auction (still when $o = N$), it is a dominant strategy for each player to bid the value of the unit for which he/she is bidding. That is, the loser of the first auction should bid v, and the winner of the first auction kv.

When $o = Y$ and $k \in \{1, 2\}$, optimal first-auction bidding is the same as in the absence of the buyer's option. Put in other words, the buyer's option has no effect on first-auction bidding behavior. However, when $k = \frac{1}{2}$, first-auction bidding should be more aggressive than in the absence of the option. The optimal use of the buyer's option is fairly simple when $k = \frac{1}{2}$ or $k = 1$. In the former case it should be used if the first-auction price is lower than the second unit value, and in the latter case the first-auction winner is indifferent between exercising the option or not, which is the meaning of $bo(p_1) \in [0, 1]$. When $k = 2$ it is not optimal to use the option because the loser of the first auction is expected to bid less aggressively in the second auction, so the first-auction winner has a higher expected gain by waiting for the second auction.

Let us next comment on the predictions for the Dutch and first-price auctions. Again theory predicts that behavior is strictly identical under the 2 institutions. When $o = N$, there does not exist a symmetric pure strategy equilibrium for $k = \frac{1}{2}, 1$. An explanation for this result is the following. If such an equilibrium were to exist, the loser of the first auction would learn the valuation of the winner (since p_1 is revealed at the end of the first auction). The first-auction winner would then clearly be in an uncomfortable situation in the second auction. The equilibrium in the second auction would take the following form: the winner of the first auction would play a mixed strategy and the loser a pure strategy. However, this second-auction equilibrium is not compatible with a first-auction pure strategy, since we can show that there always exists a profitable deviation. This means that both players should hide their valuation by playing a mixed strategy in the first auction.

When $o = N$ and $k = 2$, a symmetric pure strategy equilibrium does exist for the Dutch and the first-price auctions. This equilibrium is not simple to compute and is not very intuitive as it implies a relatively low first-auction bid. At first sight one might indeed think that it should be rewarding for player 1 to deviate from equilibrium by bidding $\frac{x}{2}$ (with $x > v_1$) in the first auction in order to increase the probability to win the first unit, and thereby to enter the second auction with a stronger valuation $2v_1$. But this deviation is not profitable. Indeed, this deviation decreases the expected gain in the first auction (since bidding half of one's valuation is optimal in a single-unit auction), and, as can be shown, it does not affect the expected gain in the second auction. Note that the equilibrium given in the proposition is such that the winner of the first auction, say bidder 1, automatically wins the second auction: his/her valuation for the second unit is $2v_1$ while his/her opponent's valuation for the first unit is[4] $v_2 \leq v_1$, so by bidding v_1 he/she wins the second auction with probability one. Therefore, in equilibrium it is as if both bidders only compete for the first unit.

When $o = Y$, a symmetric pure strategy equilibrium does exist for all values of k. Note that in equilibrium, bidders behave exactly as in standard single-unit Dutch or first-price auctions. Indeed, in equilibrium each player bids $\frac{1+k}{4}v$ in the first auction and the winner *always* exercises his/her option. It is thus as if players submit a single bid equal to $\frac{1+k}{2}v$, for a "single good" with a value $(1 + k)v$.[5] Note finally that for $k = 2$, first-auction bidding should be more aggressive when the option is available than when it is not available.

3 Experimental design

The experiment was conducted on 28 and 29 March 2001 at the *Ecole Nationale de Statistique et de l'Administration Economique* (ENSAE).[6] Students were recruited through personal emails, and fliers that we dispatched in their mailboxes. Seventy four students (out of roughly 360 students that studied at the time at ENSAE) actually participated in the experiment. We organized a total of 10 experimental sessions in the computer rooms at ENSAE, and each student took part in only one session. Only one type of auction mechanism was used per session. Table 1 lists for each session the type of auction mechanism that was studied and the number of participants. From Table 1 it can be seen

[4]Because the first-auction strategy is symmetric.

[5]Recall that, given our model assumptions, the optimal single-unit bid (in first-price and Dutch auctions) for a good valued at v is $\frac{1}{2}v$.

[6]The ENSAE is one of the leading French institutions of higher learning in the fields of statistics, economics, finance, and actuarial sciences. After completing the three-year curriculum of this school, graduates have a training comparable to the level attained by first-year Ph.D. students at a good North American university.

Table 1: Sessions

Session	Type of auction	Number of subjects
1	First-price	8
2	Second-price	8
3	Dutch	6
4	English	6
5	Dutch	10
6	English	8
7	First-price	8
8	Second-price	8
9	Dutch	6
10	English	6

that 22 students participated in the Dutch auctions, 20 in the English auctions, 16 in the first-price auctions, and 16 in the second-price auctions.

Each session was made up of two parts. The first part was devoted to sequential auctions without a buyer's option, and the second part to sequential auctions with a buyer's option.

We start by describing the first part of a session. We began by reading aloud the instructions about the auction's rules without a buyer's option. Written versions of the instructions were distributed to the participants and could be consulted at any time during the experimental session.[7] The first part had 12 periods. Since we focus in this paper on auctions with 2 buyers, participants were told that they were in competition with a single person. At the beginning of each period the computer randomly matched each student to another student present in the room (all sessions had an even number of participants), so participants were aware of the fact that their opponent differed from period to period. Participants were also told that in each period 2 units of a fictitious good were sold at auction to each couple.

At the start of each period, valuations were independently drawn from a uniform distribution on $[0;\overline{v}]$=[0;FFr50.00]. On the computer screen of participant i appeared his/her valuation for the first unit of the good v_i, the prevailing value of k, and his/her valuation for the second unit kv_i. The value of k changed every 4 periods ($k = \frac{1}{2}$ in periods 1-4, $k = 1$ in periods 5-8, and $k = 2$ in periods 9-12). Participants could observe this information for 30 seconds, after which the first auction started (but the information remained on the screen even during the auction). The manner in which participants could bid depended on the type of auction mechanism that was used during the session. The auction-specific bidding devices will be described later on.

Once the first auction was over, some information concerning the first auction was added to the screen of each subject i. It indicated whether i was the winner or not, his/her own bid (if any), the winning price p_1, i.e. the price he/she or

[7]The instructions can be obtained from the authors upon request.

his opponent had to pay for the first unit, and his/her gain associated with the auction ($v_i - p_1$ if i was the winner, 0 otherwise). Since the identity of the winner of the first auction is crucial knowledge in our experiment, we emphasized this by coloring the box marked "Winning bid" blue if i had won the first auction, and red otherwise. Note that the exact nature of information released between the two auctions differed slightly with the type of auction mechanism. For instance, for the winner of an English auction the box marked "Your bid" remained empty, while for the winner of a Dutch auction this box indicated the price at which he had stopped the clock.

Before the start of the second auction, participants again had a thirty-seconds reflection period during which they could, if they wished, consult all information on their screen (again, all information remained visualized during the second auction). The second auction functioned in the same way as the first auction. We stressed the fact that the gain associated with the second auction depended on the outcome of the first auction. Thus, winner i of the second auction had a gain of $kv_i - p_2$ if he had also won the first auction, and a gain equal to $v_i - p_2$ if he had lost the first auction. Once the second auction was terminated for all couples in the room, we proceeded with the next period.

The 12 periods of the first part of each experimental session were preceded by 6 "dry" periods (2 for each value of k). This gave participants the opportunity to familiarize themselves with the bidding method, determine their strategy for the different values of k, and ask questions to the experimenter.

Next we describe the second part of the session, the one that was designed to study the buyer's option. We began by reading aloud the instructions about this part of the experiment. Like the first part it consisted of 12 periods. Each period started exactly like in the first part of the experiment: the valuations and the value of k (the values of k alternated as in the first part) showed up on the screen, the first auction started after 30 seconds, and once the first auction was over for player i and his/her rival, their screens updated them on the relevant auction results. Unlike the first part of the session, subjects were told that the winner of the first auction could, if he/she desired, use the buyer's option. If winner i chose to execute his/her option, the period was over for him and his/her opponent, and his/her total gain in the period was $(v_i - p_1) + (kv_i - p_1) = (1 + k)v_i - 2p_1$. If he/she choses not to do so, his/her gain associated with the first auction was $v_i - p_1$, and a second auction was held after the thirty-seconds pause. The second auction was in all respects identical to the second auction conducted in the first part of the experiment.

The 12 "wet" periods of the second part of each experimental session were again preceded by dry periods, but now just 3 of them (1 for each value of k) since, at least from a practical point of view, the second part differed little from the first.

As mentioned above, the way in which participants had to submit their bids depended on the auction format. In the first-price and second-price auctions participants could submit their bid by entering a number in a box marked "Submit your bid here". The number could be any positive real integer, i.e. we did not forbid subjects to bid in excess of their valuations.

In the Dutch and English auctions bidding took place via numerical clocks. After the 30-seconds reflection period, the clock appeared on the screens of the participants. In the English auctions the clock started at 0.00FFr, augmented continuously at a rate of 50.00FFr per minute, and stopped automatically at FFr120.00. The clock started and operated simultaneously on the screens of participant i and his/her rival. They could stop the clock at any time by pressing the "Enter" key or "Space bar", or click on a window marked "Stop the clock". If neither i nor his rival had stopped the clock before it reached FFr120.00, the computer randomly selected i or his/her rival as the winner (actually this never happened during our experiments). In the Dutch auctions the clock started at FFr60.00 (if $k = \frac{1}{2}$ or $k = 1$) or FFr120.00 (if $k = 2$), descended continuously at the speed of 50.00FFr per minute, and stopped automatically at FFr0.00. The Dutch clock started and operated simultaneously for subject i and his/her opponent and they could stop it, at any time, as the English clock. If neither i nor his/her rival had stopped the clock before it reached FFr0.00, there was no auction winner (again, this never occurred during our experiments). Note that as in the sealed-bid auctions, subjects could bid above their valuations (up to a reasonable limit) in the clock auctions.

At the start of an experimental session, i.e. at the beginning of the first period, all participants were given a capital balance of FFr50.00. At the end of each period, the gains made during the period were added to the balance, and losses were subtracted from it. We informed the experimental subjects that if the end-of-period balance of a participant was negative (as a result of his/her bidding behavior in the period), the balance would immediately be readjusted to 0. We stressed that balances would only be readjusted at *the end* of a period, in view of the end-of-period balance, and not at some point *during* a period. The reason for censoring the start-of-period balances at 0 is to incite subjects to play well all along the experiment.[8] As it turned out, for none of the experimental subjects the capital balance went negative, so it was never necessary to implement the readjustment procedure.

At the end of the session participants were paid in cash their final capital balance divided by two. This 50% cut does not affect bidding behavior and could be interpreted by the participants as a tax due to the auctioneer. On average we paid FFr229 to the students, the minimum payment was FFr60, and the maximum payment FFr360. Experimental sessions lasted between 1.5 and 2 hours.

Before turning to the experimental results, we want to comment on the number of treatment levels in our experiment. In each session subjects went through 2×3 different treatments (with or without a buyer's option and three forms of the demand function). A drawback of our design is this high number of treatment levels as it might have made the subjects susceptible to hysteresis effects. However, we do not think that this occurred. Each change in the value of k was clearly indicated both on the screen and orally by the experimenter.

[8]Had we not done this, a subject with a balance of say minus FFr300 at the beginning of period 24, would clearly not have been incited to behave optimally in this last period.

Moreover, the introduction of the buyer's option was made very clear since we began the second part of the experiment by oral instructions about the rules of this mechanism. Therefore, subjects have not been confused nor by the shifts in the value of k nor by the introduction of the buyer's option. Instead, the frequent changes in the treatment helped to keep the subjects alert and attentive. On the other hand, the advantage of having several treatments within a session is that the estimation of treatment effects is facilitated since it is not necessary to control for inter-individual differences.

4 Experimental results

4.1 Bidding behavior in the first auction

Figures 1-12 show all first-auction bids for the different values of k for the 4 auction formats without buyer's option. The figures thus graph all first-auction bids submitted during the first part of the experiment, that is during periods 1-12. They depict the losing bids for the English auctions, the winning bids for the Dutch auctions, and both winning and losing bids for the sealed bid auctions. Whenever there is a theoretical prediction (see Proposition 1), the optimal equilibrium bid function $b_1(.)$ is drawn in green. For instance, in Figure 2 (second-price auction, $k = \frac{1}{2}$) the green line is the function $b_1(v) = \frac{1}{2}v$, but in Figure 1 (first-price auction, $k = \frac{1}{2}$) no green line is drawn since no prediction is available. Since all optimal bidding strategies are linear functions of the valuations, equal to zero when $v = 0$, each figure also shows, in red, the fitted line $\hat{\beta}v$ where $\hat{\beta}$ is the OLS estimate of the coefficient in the regression $b_{1it} = \beta v_{it} + \varepsilon_{it}$ where b_{1it} and v_{it} are i's bid and valuation in period t, and ε_{it} an error term that is assumed independent over i and t. A comparison of the green and red lines is therefore a quick eyeball test of the theoretical predictions.

Looking at Figures 1-12, one can distinguish, roughly speaking, three types of graphs. First there are graphs where the fitted and predicted lines more or less coincide, suggesting that observed bidding behavior is coherent with theory. This is the case for Figures 6 and 8, i.e. the English and second-price auctions with flat demand. In the second category of graphs, the red and green lines are distinct and the large majority of bids is closely concentrated around the red line, suggesting that most subjects deviate, *in the same manner*, from optimal behavior. This is the case for Figures 9, 10, 11 and 12. In the third category of graphs the red and green lines are again distinct, but part of the bids is now closely concentrated around the green line. Apparently there is a group of subjects that bid according to theory. Another part of the bids is clearly not located near the optimal bidding line. The bids are closely scattered however, so there is again the impression that the subjects who deviate behave quite similarly. There is not a "continuum" of types of observed behavior, but instead the number of different behavioral strategies that can be observed seems limited. Figures 2 and 4 fit in this category.

Table 2 reports the OLS results of the bid regressions, that is the red lines in

Table 2: First auction without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	64	0.46 (0.03)	0.78	$\emptyset$	$\emptyset$
Second-price	64	0.83 (0.04)	0.87	0.5	No
Dutch	44	0.49 (0.02)	0.93	$\emptyset$	$\emptyset$
English	40	0.64 (0.04)	0.85	0.5	No
				$k = 1$	
First-price	64	0.55 (0.03)	0.8	$\emptyset$	$\emptyset$
Second-price	64	1.03 (0.02)	0.98	1	Yes
Dutch	44	0.54 (0.01)	0.97	$\emptyset$	$\emptyset$
English	40	1.00 (0.02)	0.98	1	Yes
				$k = 2$	
First-price	64	1.01 (0.04)	0.91	0.5	No
Second-price	64	1.36 (0.05)	0.93	2	No
Dutch	44	0.90 (0.04)	0.94	0.5	No
English	40	1.30 (0.06)	0.92	2	No

Figures 1-12. The table also reports the predicted slopes of the optimal bidding strategies and test results of the hypothesis that observed behavior is in line with predicted behavior. The null hypothesis can be tested simply by testing whether the coefficient β equals some specific value. For example, the OLS estimate $\hat{\beta}$ equals 0.83 for the second-price auctions with decreasing demand, the estimated standard error is 0.04, the number of observations in the regression is 64 (16 subjects $\times 4$ periods), the R^2 is 0.87 (defined for a regression model without a constant), the predicted slope is $\frac{1}{2}$, and the null hypothesis that $\beta = \frac{1}{2}$ is rejected at the 5% level.

As Table 2 shows, the null is accepted just 2 out of 8 times. Theory is accepted precisely in the two cases where the optimal strategies are relatively transparent; subjects have indeed understood that in second-price and English auctions with flat demand it is optimal to bid the value v in the first auction. However, in second-price and English auctions with decreasing (resp. increasing) demand, subjects had a tendency to over-bid (resp. under-bid); although subjects have understood that optimal behavior calls for bid shading (resp. bidding above value), the extent to which they did this was too modest. In first-price and Dutch auctions with increasing demand, observed bidding is on average close to v, whereas optimal behavior requires agents to bid $\frac{1}{2}v$. Note that as predicted in Proposition 1, observed behavior in respectively the first-price and Dutch auctions, and second-price and English auctions, is very similar.

Figures 13-24 show all first-auction bids for the different values of k for the 4 auction formats with buyer's option, i.e. they are based on all first-auction bids submitted during periods 13-24 of the experiment. Table 3 presents the corresponding estimation and test results. For second-price and English auctions with decreasing demand the solution (approximated using simulations) of the differential equation given in Proposition 1 is $b_1(v) = 0.99v - 0.009v^2$. This explains why the green line in Figures 14 and 16 is curved. The red line in each of these pictures is the fitted line $\hat{\beta}_1 v + \hat{\beta}_2 v^2$ where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the OLS estimates of the coefficients in the regression $b_{1it} = \beta_1 v_{it} + \beta_2 v_{it}^2 + \varepsilon_{it}$. The test of the theory in these 2 cases amounts to testing, using a standard Fisher-test, the joint hypothesis that $\beta_1 = 0.99$ and $\beta_2 = 0.009$.

The results for the first auctions with buyer's option are quite similar to those obtained for the first auctions without the option. Indeed, as Table 3 shows, the theoretical predictions are again rejected in most cases: the null is accepted only 2 out of 12 times. Theory is again accepted for the English auction with flat demand, and now also for the English auction with decreasing demand (not at the 5% level, but at the 1% level, which is indicated by "Yes*" in the table). Unlike the first-auction results without the option, observed bidding behavior is not in line with theory for the second-price auction with flat demand. Observed bidding in this case is on average slightly above the value v. As in the first auctions without option, the degree of bid shading is too modest in the second-price auctions with decreasing demand; the degree of over-bidding is too modest in the English and second-price auctions with increasing demand. Table 3 shows that in all Dutch and first-price auctions, observed bidding is on average above equilibrium bidding. Note finally that there is again much behavioral similarity between Dutch and first-price auctions on the one hand, and English and second-price auctions on the other.

The impact of the buyer's option on first-auction bidding behavior can be studied by comparing the results of Table 2 and Table 3. Let us first consider the Dutch and first-price auctions. Here the theoretical effect of the buyer's option can be confronted with the data only for $k = 2$. With increasing demand, theory predicts that bidders should be more aggressive when the option is available (the predicted slope is 0.75 with the option, and 0.50 without). However, running an appropriate regression model and testing for a buyer's option effect,[9] it turns out that bidding behavior in the Dutch auction is *not* affected by the presence of a buyer's option, whereas bidding in the first-price auction is actually significantly *less* aggressive when the option is available.

Concerning the English and second-price auctions, we only test for an impact of the buyer's option when $k = 1$ and $k = 2$. In these cases, theory predicts that the buyer's option does not modify first-auction bidding behavior. Performing the same kind of tests as described just before, we find that for both auction institutions the theory is verified when demand is flat. When demand is increasing, the data are again in line with theory for the English auction but not for the second-price auction. In the latter case, first-auction bidding is actually more aggressive when the buyer's option is available.

[9] Pooling all first-auction bids submitted in the Dutch (resp. first-price) auctions, we consider the model $b_{1it} = \beta v_{it} + \gamma v_{it} 1\{o_{it} = Y\} + \varepsilon_{it}$, with $1\{.\}$ the indicator function and o_{it} the option indicator for individual i in period t, and check for the buyer's option effect in Dutch (resp. first-price) auctions by testing whether $\gamma = 0$.

Table 3: First auction with buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	64	0.51 (0.02)	0.92	0.375	No
Second-price	64	1.25 (.09) -0.008 (.002)	0.95	0.99 -0.009	No
Dutch	44	0.48 (0.02)	0.95	0.375	No
English	40	1.02 (.06) -0.008 (.001)	0.94	0.99 -0.009	Yes*
				$k = 1$	
First-price	64	0.61 (0.01)	0.97	0.5	No
Second-price	64	1.09 (0.03)	0.95	1	No
Dutch	44	0.59 (0.02)	0.97	0.5	No
English	40	0.99 (0.02)	0.99	1	Yes
				$k = 2$	
First-price	64	0.91 (0.03)	0.95	0.75	No
Second-price	64	1.48 (0.03)	0.97	2	No
Dutch	44	0.88 (0.03)	0.96	0.75	No
English	40	1.33 (0.05)	0.94	2	No

4.2 The buyer's option

In this subsection we study to what extent the buyer's option has been exercised by the subjects in our experiment. The results can be found in Table 4. The second column reports the relative number of times the buyer's option has been used for each auction institution. For ease of presentation, the third column recalls the optimal frequencies given in Proposition 1. Let us first discuss the results for the Dutch and first-price auctions. For each value of k, Nash equilibrium behavior requires that the first-auction winner should always use the buyer's option. Table 4 shows that observed behavior is quite well in line with this prediction for flat and increasing demand. However, when the demand function is decreasing, the observed frequency of exercising the option is much too low (only 32% in the Dutch auctions and 41% in the first-price auctions). Given that in both these auction institutions many first-auction bids are clearly out of equilibrium (see Figures 13 and 15), the question arises whether the deviating bidders are responsible for the low frequency observed in our data. To answer this question, we have run the logit model $Prob(bo_{it} = 1) = 1/(1 + \exp[\beta_1 + \beta_2 1\{b_{1it}$ "close" to $b_1(v_{it})\}])$ with $bo_{it} = 1$ if i uses the buyer's option in period t and 0 otherwise, and the observed bid b_{1it} is "close" to the optimal bid $b_1(v_{it})$ if the relative difference between the two is smaller than 30%. The ML estimate $\hat{\beta}_2$ (standard error) is -1.70 (0.75) for the Dutch auction and -2.61 (0.94) for the first-price auction. In both cases the coefficient is negative and significant, suggesting that subjects whose first-auction bids are at or close to equilibrium are more likely to exercise the buyer's option, and are thus more likely to remain coherent with theory. See also the discussion

Table 4: The buyer's option

Auction	Relative frequency[1]	Prediction
		$k = 0.5$
First-price	41%	$bo(p_1) = 1$
Second-price	93%	$bo(p_1) = 1$ if $p_1 \leq 0.5v$
	0%	$bo(p_1) = 0$ if $p_1 > 0.5v$
Dutch	32%	$bo(p_1) = 1$
English	96%	$bo(p_1) = 1$ if $p_1 \leq 0.5v$
	0%	$bo(p_1) = 0$ if $p_1 > 0.5v$
		$k = 1$
First-price	91%	$bo(p_1) = 1$
Second-price	78%	$bo(p_1) \in [0, 1]$
Dutch	84%	$bo(p_1) = 1$
English	68%	$bo(p_1) \in [0, 1]$
		$k = 2$
First-price	81%	$bo(p_1) = 1$
Second-price	69%	$bo(p_1) = 0$
Dutch	98%	$bo(p_1) = 1$
English	70%	$bo(p_1) = 0$

[1] Relative number of times the buyer's option is used.

in section 4.5.1.

Next consider the English and second-price auctions. Under decreasing demand the observed frequencies are very close to the frequencies predicted by the theory. Subjects have well understood that the option should be exercised when $p_1 \leq \frac{1}{2}v$, and inversely that it should not be used when $p_1 > \frac{1}{2}v$. That the last prediction is verified may seem somewhat obvious (exercising the option while $p_1 > \frac{1}{2}v$ implies a loss on the second unit acquired) but is nonetheless reassuring since it means that subjects have apparently well understood the rules of the game. Under flat demand the theory predicts that the winner of the first auction is indifferent between using or not using the buyer's option, so in this case there is no testable implication of the theory. Under increasing demand the theory predicts that the winner should never use the option, but should instead wait and try to win the second unit of the good in the second auction. This prediction is mostly rejected by the data since 70% (resp. 69%) of the winners in the English (resp. second-price) auctions *did* use the buyer's option.[10]

[10] Figures 22 and 24 show that for the English and second-price auctions with increasing demand practically all bids appear out of equilibrium. Performing the logit analysis would therefore not make sense in these 2 cases.

Table 5: Second auction (for loser of first auction) without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	32	0.57 (0.03)	0.93	$\emptyset$	$\emptyset$
Second-price	32	0.95 (0.03)	0.96	1	Yes
Dutch	18	0.55 (0.03)	0.95	$\emptyset$	$\emptyset$
English	26	0.97 (0.03)	0.97	1	Yes
				$k = 1$	
First-price	32	0.76 (0.03)	0.96	$\emptyset$	$\emptyset$
Second-price	32	1.07 (0.03)	0.97	1	Yes*
Dutch	15	0.74 (0.03)	0.74	$\emptyset$	$\emptyset$
English	38	0.99 (0.03)	0.99	1	Yes
				$k = 2$	
First-price	32	1.04 (0.06)	0.91	1	Yes
Second-price	32	1.21 (0.08)	0.88	1	No
Dutch	5	1.12 (0.14)	0.94	1	Yes
English	40	1.29 (0.06)	0.92	1	No

4.3 Bidding behavior in the second auction

We only discuss the second-auction results for the 4 auction formats without buyer's option, that is we focus on second-auction bids submitted during periods 1-12 of the experiment. The reason is that the buyer's option has been frequently used by our experimental subjects, so relatively few second auctions were actually held during periods 13-24, leaving us, in most cases, with too few data to reliably estimate the second-auction bidding strategies.[11] Since winners and losers of the first auction should generally behave differently in equilibrium, the results for the 2 groups are presented separately.

Let us first describe second-auction bidding behavior for the losers of the first auction. They are presented in Table 5.[12] Unlike the first-auction results, the majority of predictions are now verified by the data: the null hypothesis is accepted 6 out of 8 times. In the Dutch and first-price auctions with increasing demand, we again find the by now familiar pattern, namely that observed bidding in these 2 auction formats is on average in excess of equilibrium bidding. However, the deviations from optimality are not significant here.

In English and second-price auctions, and for each value of k, it is a dominant strategy for the first-auction loser to bid v in the second auction. Thus, like in standard single-unit English and second-price auctions, it is optimal for bidders to reveal their valuation. Because of this theoretical equivalence, it is of interest to compare our results with those obtained in the experimental literature on single-unit English and single-unit second-price auctions (see Coppinger, Smith,

[11] This is especially true for the auction periods with flat or increasing demand.

[12] To economize on space, the figures depicting the second-auction bids are not given here.

and Titus (1980), Cox, Roberson, and Smith (1982), Kagel, Harstad, and Levin (1987), and Kagel and Levin (1993)).[13] It should be stressed however that bidding in our experiment and bidding in the single-unit experiments took place in slightly different contexts. First because our experimental subjects were more informed about their opponents (when subjects in our experiment submitted their second-auction bid, they knew the first-auction price p_1) than subjects in the single-unit experiments. And also because, under decreasing and increasing demand, bidders in the second auction are no longer symmetric as in the single-unit experiments. Therefore, any differences between the results obtained in the single-unit auction literature and our's can be attributed to these contextual differences.

For $k = \frac{1}{2}$ and $k = 1$ we find the same results as in the experimental single-unit literature: in English and second-price auctions bidders play the dominant strategy as predicted by the theory. For $k = 2$ we find that bidding in English auctions is significantly above value, contradicting both the theory and the results from the single-unit experiments. Bidding in the second-price auctions is also significantly above value, which is contradictory to theory but coherent with the single-unit experiments. Kagel, Harstad, and Levin (1987) explain over-bidding in their single-unit second-price auctions by arguing that bidding above value does not necessarily entail losses and increases the probability of winning, so that subjects can have the illusion that such a strategy increases expected profits. It is unlikely that this explanation also holds in our experiment since it is incompatible with the fact that for second-price auctions with $k = \frac{1}{2}$ or $k = 1$ we *do not* observe over-bidding. A more plausible explanation for overbidding in English and second-price auctions is given in section 4.5.

Next we describe second-auction bidding behavior for the winners of the first auction. They can be found in Table 6. Observed behavior is again in line with theory in the majority of cases: the null is accepted 5 out of 7 times. In English and second-price auctions, and for all values of k, the dominant strategy for the first-auction winner is to reveal his/her value for the second unit, i.e. $2v$. In all English and second-price auctions, observed behavior is in line with theory and the single-unit literature (except the English auction with $k = 1$, but here there are just 2 observations, and the result is probably not very reliable; for $k = 2$ there are no observations at all for the English auction, so theory can not be tested). In the first-price auction with increasing demand, the data are in line with theory at the 1% level, but in the Dutch auction observed bidding is below equilibrium bidding.

[13] This literature shows that subjects bid according to equilibrium behavior in single-unit English auctions. Regarding the single-unit second-price auctions, Coppinger, Smith, and Titus (1980) and Cox, Roberson and Smith (1982) find that average bidding is below (but not always significantly) value; Kagel, Harstad, and Levin (1987) and Kagel and Levin (1993) find, however, that the subjects in their study bid significantly above value. They point out that a likely explanation for these conflicting findings is that, unlike their experiments (and our's!), the designs of Coppinger, Smith, and Titus (1980) and Cox, Roberson and Smith (1982) *did not allow* subjects to bid in excess of their valuation.

Table 6: Second auction (for winner of first auction) without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	32	0.41 (0.01)	0.98	$\emptyset$	$\emptyset$
Second-price	32	0.53 (0.02)	0.94	0.5	Yes
Dutch	26	0.34 (0.02)	0.95	$\emptyset$	$\emptyset$
English	14	0.54 (0.03)	0.96	0.5	Yes
				$k = 1$	
First-price	32	0.61 (0.02)	0.97	$\emptyset$	$\emptyset$
Second-price	32	1.00 (0.01)	0.99	1	Yes
Dutch	29	0.57 (0.02)	0.96	$\emptyset$	$\emptyset$
English	2	0.98 (0.00)	0.99	1	No
				$k = 2$	
First-price	32	1.05 (0.02)	0.99	1	Yes*
Second-price	32	2.34 (0.23)	0.77	2	Yes
Dutch	39	0.93 (0.03)	0.97	1	No
English	0	- (-)	-	2	$\emptyset$

4.4 Efficiency, price patterns, and revenue comparisons

We start this subsection by comparing observed revenues with their theoretical counterparts. Results are given separately for auctions without a buyer's option (Table 7) and auctions with a buyer's option (Table 8). The third column in these tables gives the revenues as predicted by the theory. These predictions follow from Proposition 1. In the absence of a buyer's option, and for each value of k, the 4 auction institutions are equivalent in terms of the expected revenue they generate. When the buyer's option is available, there is again revenue-equivalence when $k = 1$ and $k = 2$. However, when $k = \frac{1}{2}$, the first-price and Dutch auctions generate more revenue than the English and second-price auctions. Comparing Table 7 and Table 8, it can be seen that in theory the buyer's option has no effect on expected revenue. The only exceptions are the English and second-price auctions with decreasing demand. In these cases the buyer's option increases expected revenue.

To test the revenue predictions, we define for each couple and for each period the seller's revenue $REV = p_1 + p_2$. For each value of k and each auction mechanism the empirical average of REV is calculated, and using a T-test we test the hypothesis that REV has a mean equal to the predicted revenue. For instance, in Table 7, the average revenue in English auctions without a buyer's option when $k = 2$ (the average is thus calculated over 10 couples $\times 4$ periods $=$ 40 observations) equals 40.95 (standard error equal to 5.45), and the hypothesis that the mean of REV equals the predicted value 50.00 is accepted at the 5% level.

As Table 7 and Table 8 show, the results for all the English auctions are

Table 7: Seller's revenue without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	31.41 (2.00)	$\emptyset$	$\emptyset$
Second-price	27.52 (2.45)	20.83	No
Dutch	29.81 (1.75)	$\emptyset$	$\emptyset$
English	20.12 (2.46)	20.83	Yes
		$k = 1$	
First-price	46.46 (2.80)	$\emptyset$	$\emptyset$
Second-price	33.22 (3.75)	33.33	Yes
Dutch	37.82 (1.74)	$\emptyset$	$\emptyset$
English	31.10 (3.33)	33.33	Yes
		$k = 2$	
First-price	70.91 (4.79)	50.00	No
Second-price	38.91 (4.13)	50.00	No
Dutch	64.43 (3.81)	50.00	No
English	40.95 (5.45)	50.00	Yes

Table 8: Seller's revenue with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	33.98 (2.09)	25.00	No
Second-price	29.15 (3.14)	23.65	Yes
Dutch	27.07 (1.93)	25.00	Yes
English	24.56 (2.70)	23.65	Yes
		$k = 1$	
First-price	39.85 (2.69)	33.33	Yes*
Second-price	37.48 (4.93)	33.33	Yes
Dutch	37.86 (2.51)	33.33	Yes
English	30.27 (3.69)	33.33	Yes
		$k = 2$	
First-price	59.49 (4.48)	50.00	Yes*
Second-price	45.49 (5.94)	50.00	Yes
Dutch	63.87 (2.63)	50.00	No
English	39.84 (4.19)	50.00	Yes*

in line with the theoretical predictions. Concerning the Dutch, first-price and second-price auctions, the null is generally accepted when the buyer's option is available, but rejected when it is not. In most cases where the null hypothesis is rejected, mean revenue is significantly above the predicted revenue. These deviations from theory are sometimes considerable. For instance, when a buyer's option is not proposed by the auctioneer, a first-price auction ($k = 2$) generates almost FFr21 more per period than predicted by the theory. Table 7 and Table 8 also indicate that generally the first-price auction generates the highest revenue, followed by the Dutch auction, then the second-price auction, and lastly the English auction. Note that our revenue-ranking of auction formats is exactly identical to the ordering found by Cox, Roberson and Smith (1982) in their experimental study on one-unit auctions.[14] Two-sample T-tests on the equality of mean revenues (not reported in the Tables) suggest that the difference between respectively the Dutch and first-price auctions and the first-price and second-price auctions are significant at the 5% level, but the difference between the second-price and English auctions is generally not significant. Two-sample T-tests also suggest that the buyer's option significantly decreases (resp. increases) revenue in first-price (resp. second-price) auctions; the buyer's option does not significantly affect expected revenue in Dutch or English auctions.

Table 9 and Table 10 report for each value of k and each auction mechanism the mean and standard deviation of the difference in prices $p_2 - p_1$. The Tables also indicate the theoretical predictions on the expected value of $p_2 - p_1$ and whether these predictions are rejected by the data or not. The predicted price variations follow immediately from Proposition 1.

As shown in Table 10, the winning price p_1 is expected to be equal to the winning price p_2 when the buyer's option is available. The only exceptions are the second-price and English auctions for $k = \frac{1}{2}$ and $k = 2$ where on average the sequence of prices is expected to be declining. Note that expected price variations in second-price and English auctions for $k = 2$ are almost 19 times larger compared to the predicted variations for $k = \frac{1}{2}$. Note also that in the case of first-price and Dutch auctions with buyer's option the prediction of constant prices is not surprising since, according to Proposition 1, winners of the first auction should always execute their buyer's option. Table 9 shows that in the absence of a buyer's option the theoretical predictions vary considerably with the auction format and the value of k. The sequence of prices in second-price and English auctions is expected to be increasing when $k = \frac{1}{2}$, and constant when $k = 1$ (no predictions for first-price and Dutch auctions for these values of k). It is quite striking that for $k = 2$ the predicted patterns in the first-price and Dutch auctions are completely opposite to those of the second-price and English auctions: in the former two auction formats the theory predicts a price increase of FFr16.67, while in the latter two auction types a price decline of FFr16.67 is expected.

Table 9 and Table 10 show that for $k = 1$ the results are in line with the

[14] Cox et al. experimentally study first-price, second-price, and Dutch auctions, but not the English auctions.

Table 9: Price variation ($p_2 - p_1$) without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	-1.09 (1.17)	$\emptyset$	$\emptyset$
Second-price	-1.18 (1.28)	4.17	No
Dutch	-3.53 (0.81)	$\emptyset$	$\emptyset$
English	0.57 (0.75)	4.17	No
		$k = 1$	
First-price	0.99 (1.49)	$\emptyset$	$\emptyset$
Second-price	0.75 (0.83)	0.00	Yes
Dutch	1.40 (0.43)	$\emptyset$	$\emptyset$
English	-0.39 (0.49)	0.00	Yes
		$k = 2$	
First-price	2.02 (2.24)	16.67	No
Second-price	0.41 (1.28)	-16.67	No
Dutch	1.95 (1.03)	16.67	No
English	0.11 (1.50)	-16.67	No

Table 10: Price variation ($p_2 - p_1$) with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	-3.74 (1.00)	0.00	No
Second-price	-5.29 (1.11)	-0.88	No
Dutch	-3.31 (0.64)	0.00	No
English	-1.00 (0.45)	-0.88	Yes
		$k = 1$	
First-price	-0.18 (0.14)	0.00	Yes
Second-price	-0.07 (0.13)	0.00	Yes
Dutch	-0.94 (0.40)	0.00	Yes*
English	-0.72 (0.77)	0.00	Yes
		$k = 2$	
First-price	-0.71 (0.34)	0.00	Yes*
Second-price	-0.30 (0.89)	-16.67	No
Dutch	-0.07 (0.07)	0.00	Yes
English	-3.77 (1.73)	-16.67	No

theoretical predictions for all auction institutions, with and without buyer's option: when demand is flat, observed price differences are indeed not significantly different from 0. For $k = \frac{1}{2}$ and $k = 2$ the results are somewhat less satisfactory. When the buyer's option is not available, theory predicts (strong) decreasing or increasing price patterns, but the hypothesis that prices remain constant can never be rejected; when the buyer's option is available, the observed price patterns are in line with theory for English auctions with decreasing demand, and Dutch and first-price auctions with increasing demand.

Table 9 and Table 10 show that observed price sequences are mostly constant when the buyer's option is not available, but significantly decreasing when it is available. These findings are compatible with the field-data studies mentioned in the introduction: they are in support of Van den Berg, Van Ours, and Pradhan (1999), who think that the buyer's option is responsable for the price declines in their Dutch auctions of flowers; and they are in line with the studies on sequential English auctions of wine at Christie's, Drouot and Sotheby's, (see Ashenfelter (1989), Ginsburgh (1998), and Février et al. (2001)), where successive prices are generally found to be declining.

In the last part of this subsection we study auction efficiency. The results can be found in Table 11 and Table 12. For each k and auction mechanism, the first column reports the mean and standard deviation of the relative efficiency $RE = \frac{1}{2}(RE_1 + RE_2)$, where RE_j is the value that the j-th unit winner places on unit j, divided by the maximum of this value and his/her rival's value. For example, if bidder 1 wins the first unit, and bidder 2 the second unit, $RE = \frac{1}{2}(\frac{v_1}{\max(v_1,v_2)} + \frac{v_2}{\max(kv_1,v_2)})$. We also report the predicted values of RE (these predictions follow from Proposition 1), and whether the predictions are accepted or not in the data.

As Table 11 and Table 12 show, all auction institutions are, in theory, efficient mechanisms. The only exceptions are the auctions with buyer's option and decreasing demand. The auction institutions are slightly inefficient in these cases since the buyer's option allows the first-auction winner to buy the second unit while having a lower valuation than his opponent. Actual efficiency is generally remarkably close to predicted efficiency, and theory is accepted in most cases. In spite of the high degree of out-of-equilibrium behavior observed in the data, the 4 auction institutions are highly efficient in our experiments.

4.5 Understanding deviations from optimal bidding behavior

The purpose of this subsection is to understand and interpret the deviations from optimal behavior described in sections 4.1-4.3. Depending on the type of auction mechanism, we find different explanations for the deviations. In the Dutch and first-price auctions, observed bidding behavior turns out to be compatible with risk-averse Nash equilibrium theory. As in single-unit auction experiments (see Kagel (1995) for a survey), the deviations from theory can thus be attributed to risk aversion among experimental subjects. Risk aversion is modeled as in

Table 11: Relative efficiency without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	0.93 (0.02)	$\emptyset$	$\emptyset$
Second-price	0.98 (0.01)	1.00	Yes
Dutch	0.96 (0.01)	$\emptyset$	$\emptyset$
English	0.98 (0.01)	1.00	Yes*
		$k = 1$	
First-price	0.95 (0.02)	$\emptyset$	$\emptyset$
Second-price	0.99 (0.01)	1.00	Yes
Dutch	0.94 (0.01)	$\emptyset$	$\emptyset$
English	0.99 (0.01)	1.00	Yes
		$k = 2$	
First-price	0.97 (0.02)	1.00	Yes
Second-price	1.00 (0.00)	1.00	Yes
Dutch	0.96 (0.01)	1.00	Yes*
English	1.00 (0.00)	1.00	Yes

Table 12: Relative efficiency with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	0.94 (0.03)	0.92	Yes
Second-price	0.98 (0.01)	0.98	Yes
Dutch	0.92 (0.03)	0.92	Yes
English	0.96 (0.02)	0.98	Yes
		$k = 1$	
First-price	0.98 (0.01)	1.00	Yes*
Second-price	0.99 (0.01)	1.00	Yes
Dutch	0.97 (0.01)	1.00	No
English	1.00 (0.00)	1.00	Yes
		$k = 2$	
First-price	0.99 (0.01)	1.00	Yes
Second-price	0.99 (0.01)	1.00	Yes
Dutch	1.00 (0.00)	1.00	Yes
English	1.00 (0.00)	1.00	Yes

the single-unit experiments (see for example Cox, Roberson and Smith (1982) and Kagel, Harstad, and Levin (1987)).[15] All agents are thus assumed to have the same concave utility function $u(\cdot)$ over money income. Furthermore, the utility function is assumed to be of the form $u(x) = x^{\alpha}$ with $\alpha \in [0, 1)$. This is a constant relative risk-aversion (CRRA) model. In analyzing the deviations, it is assumed that the participants in our experiment have a coefficient of relative risk aversion α equal to 0.6.[16]

In the English and second-price auctions, most of the Nash equilibrium strategies (the only exceptions are the auctions with buyer's option and decreasing demand) stated in Proposition 1 are robust to the form of risk aversion that we consider; put in other words, risk-neutral Nash equilibrium bidding behavior in English and second-price auctions remains optimal under the CRRA assumption. The implication of this invariance property is that something other than risk aversion is responsible for the observed deviations in English and second-price auctions. As will be seen below, the deviations from theory are a consequence of either myopic behavior or punitive behavior. By myopic behavior is meant that agents' bidding behavior in the first auction is identical to bidding behavior in a single-unit auction. Although agents fully understand that 2 units are on sale instead of 1 unit, their first-auction behavior does not reflect this crucial difference. By punitive behavior is meant that first-auction losers attempt to harm their opponents by bidding above their value v in the second auction (the dominant strategy), thereby reducing the second-auction profits of their opponents.

4.5.1 Dutch and first-price auctions

In sections 4.1-4.3 we have seen that observed bidding behavior in Dutch and first-price auctions is generally above risk-neutral equilibrium behavior. Furthermore, experimental subjects have used the buyer's option much too rarely when demand is decreasing. In this subsection it is shown that these deviations can be explained once agents are allowed to be risk-averse. We can thus rationalize all observed behavior in terms of risk aversion. We give the risk-averse Nash equilibrium bidding functions in all relevant cases but omit the proofs of their derivations (obtainable from the authors).

In the absence of a buyer's option and when demand is decreasing, there again does not exist a symmetric pure Nash equilibrium. When the option is available an equilibrium does exist, and the first-auction optimal bidding function under CRRA is $b_1(v, \alpha) = \frac{3}{4(1+\alpha)}v$ (note that when $\alpha = 1$ we find

[15]See Harrison (1989) and the subsequent debate in the *American Economic Review*, Vol. 82 No 5, pp. 1374-1443, for an alternative explanation for the overbidding phenomenon. In particular, see Cox, Smith, and Walker (1992) (and the references herein) for an extensive discussion of risk-aversion models.

[16]This value is obtained in a somewhat ad hoc way by minimizing, over α, the sum of squared deviations between observed bids and optimal bids under risk aversion. Our estimate is a bit higher than the one found in Kagel, Harstad, and Levin (1987). They report an average estimate of α equal to 0.49, suggesting that students in our experiment were slightly less risk-averse than their North American counterparts.

the risk-neutral bidding function $b_1(v) = \frac{3}{8}v$ given in point 7 of Proposition 1). Taking $\alpha = 0.6$, we have $b_1(v, \alpha) = 0.47v$, and performing the same tests as in section 4.1 (i.e. we test whether $\beta = 0.47$, etc.), we accept, at the 5% level, for both the Dutch and first-price auctions, the hypothesis that observed bidding is in accordance with risk-averse bidding behavior.

The question that is still unanswered is whether risk-aversion provides a rationale for the fact that the option is exercised too rarely under decreasing demand? Given that under risk aversion it is still optimal to always use the buyer's option, the answer is no. Note however that the optimal bidding function under risk aversion $b_1(v, \alpha) = 0.47v$ is very close to the threshold curve $0.50v$ above which it is not profitable to use the buyer's option. The fact that risk-neutral equilibrium behavior is accepted of course only means that *average* bidding is according to the function $b_1(v, \alpha)$, and does obviously not exclude that part of the observations are located above the nearby located threshold curve. In our data all persons with bids above the threshold curve did indeed not use the option (and inversely, those with bids under the threshold curve did exercise the option), explaining why the observed frequency of using the option is lower than predicted by optimal behavior under CRRA.

Let us next consider consider the auctions under flat demand. Again, an equilibrium only exists when the buyer's option is available, and the first-auction optimal bidding function under CRRA is now $b_1(v, \alpha) = \frac{1}{(1+\alpha)}v$. Given $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.625v$, and again risk-averse equilibrium theory is accepted for both the Dutch and first-price auction. Under CRRA it remains optimal to always use the option, so regarding the use of the buyer's option, the data remain in line with theory. Note that the problem that was mentioned above does not play a role here since the threshold curve here is v, i.e. well above the optimal function.

Finally consider the auctions under increasing demand. As in the case with risk-neutrality, equilibria exist for the auctions with and without the buyer's option. For the auctions without buyer's option, we have $b_1(v, \alpha) = \frac{2-\alpha}{(1+\alpha)}v$, and taking $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.875v$. The hypothesis that bidding behavior is according to equilibrium behavior under CRRA is only accepted for the Dutch auction. The second-auction strategies are not affected by risk-aversion, so the data remain coherent with the theory except for first-auction winners in the Dutch auction (see Table 5 and Table 6). When the buyer's option is available, we obtain $b_1(v, \alpha) = \frac{3}{2(1+\alpha)}v$, and taking $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.938v$, implying that equilibrium behavior under CRRA is once again accepted for both auctions institutions. The optimal use of the buyer's option is not affected by risk-aversion, so observed frequencies remain coherent with predicted frequencies.

4.5.2 English and second-price auctions

In sections 4.1-4.3 it was shown that subjects' behavior is quite well in line with the predictions of Proposition 1 when the demand function is flat. Important deviations are however observed when the demand function is decreasing or increasing: in the first auctions the degree of bid shading (resp. over-bidding) observed in the data is clearly too small under decreasing (resp. increasing) demand; in the second auctions with increasing demand, first-auction losers are found to be bidding significantly above the dominant strategy which consists in revealing the value v; the final deviation that needs our attention concerns the higher-than-optimal use of the buyer's option when demand is increasing. As in the previous subsection, we discuss each of these deviations separately.

First consider the deviations under decreasing demand. In analyzing the first-auctions without buyer's option, it is helpful to look again at Figures 2 and 4. In both figures there is evidence of there being 2 groups of bids: one group of bids closely scattered around the optimal bidding line $b_1(v) = \frac{1}{2}v$, and another group of bids concentrated around the line v. Apparently part of the subjects play the optimal strategy, while others bid in a myopic way.[17] This can be checked more formally by running the following switching regression model

$$b_{1it} = \beta_1 v_{it} + \varepsilon_{1it} \text{ with probability } \pi$$
$$b_{1it} = \beta_2 v_{it} + \varepsilon_{2it} \text{ with probability } 1\text{-}\pi.$$

The estimates (standard error) of β_1 and β_2 are 1.04 (0.04) and 0.52 (0.06) for the second-price auction, and 0.98 (0.01) and 0.49 (0.04) for the English auction. The estimate (standard error) of the probability π is 0.62 (0.07) for the second-price auction, and 0.43 (0.09) for the English auction. Our switching regression estimates thus confirm that there are 2 groups of agents, one made up of rational bidders and the other of myopic bidders, and that the proportion of myopic agents is quite important in the data. The fact that the prevalence of myopic agents is so high explains the rejection of the theory in section 4.1.

The only strategies that are sensitive to the introduction of CRRA are the English and second-price auctions with buyer's option (still $k = \frac{1}{2}$). Under CRRA, the optimal bid function is the solution of $-(v-b_1(v,\alpha))^\alpha + (\frac{v}{2})^\alpha = 2(v-b_1(v,\alpha))^\alpha b_1'(v,\alpha)$. As in section 4.1, the solution can be precisely approximated by a second-order polynomial in v. Taking $\alpha = 0.6$, and performing the same tests as in section 4.1, the results are in support of risk-averse Nash equilibrium behavior, at the 5% level for the second-price auction, and at the 1% level for the English auction. CRRA does not affect the optimal use of the buyer's option, so observed frequencies remain in line with predictions.

Next consider the deviations under increasing demand. To understand the deviations in the auctions without buyer's option, we first look at the second-auction results. As Tables 5 and 6 show, winners of the first auction bid their valuation for the second unit, as predicted by theory, but losers of the first auction bid significantly above the dominant strategy v. As pointed out in

[17] Subjects are quite consistent in their behavior over the 4 periods for which $k = \frac{1}{2}$ and $o = N$: most optimal bidders are optimal in all 4 periods, and similarly, most deviators persistently deviate in all 4 periods.

section 4.3, the latter result contrasts with the experimental literature on single-unit auctions. However, unlike single-unit experiments, first-auction losers have information about the value of their opponent (the contextual difference mentioned in section 4.3), which allows them, without taking any personal risk, to punish their competitor by bidding above the dominant strategy. This punitive behavior in the second auction can also explain the deviation observed in the first auction. It can be formally shown that, in anticipation of punitive behavior in the second auction, players should bid somewhere between v and $2v$ (depending on the degree of punitive behavior among subjects) in the first auction to compensate for the smaller gain in the second auction. As Figures 10 and 12 show, this line of reasoning is well supported by the data, since practically all first-auction bids are indeed between v and $2v$ (with some exceptions for the second-price auction without buyer's auction).

The deviations that are observed in the auctions with buyer's option can be explained in the same manner. In the first auctions, player's anticipate future punitive behavior by bidding between v and $2v$ (Figures 22 and 24 show that this is indeed the case). Furthermore, under punitive behavior, it can be shown that it is optimal for first-auction winners to always exercise the buyer's option, justifying why the subjects in our experiment do not wait but buy instead.

5 Conclusion

This paper experimentally studies two-unit sequential auctions with and without the buyer's option. The 2 identical units are sold to 2 potential buyers. Each buyer desires both units, and their demand function is either decreasing, flat, or increasing. The four best known auction mechanisms are considered: Dutch, English, first-price and second-price auctions. Experimental papers on sequential auctions are still very rare and none analyzes the buyer's option despite its practical importance.

Observed bidding behavior in English and second-price auctions is closer to risk-neutral Nash equilibrium bidding in the second auction than in the first auction. This is not surprising since the first-auction strategies are more subtle and less transparent than the second-auction strategies. In the first auction, buyers face a complex situation because they need to anticipate that the first-auction winning price is revealed, that a second unit is going to be sold, and that the winner has the right to exercise the buyer's option (if the option is available). The inexperienced subjects in our experiment have nonetheless understood the basic strategic effects called for by optimal bidding behavior. Subjects have indeed understood that under flat demand it is optimal to bid their valuation in the first auction; under decreasing (resp. increasing) demand, subjects have understood that optimal behavior requires bid shading (resp. over-bidding), but the extent to which they did this was too modest. In the second auction, buyers face a relatively simple situation. As in single-unit English and second-price auctions, it is a dominant strategy for bidders to reveal their valuation. Apart from the first-auction losers under increasing demand, who bid in excess of their valuation, we find, as in the experimental literature on single-unit auctions, that our subjects play according to the dominant strategy. Bidders in our experiment have exercised the buyer's option quite adequately under decreasing and flat demand, but not when demand is increasing. In the latter case the bidders have made too much use of the option.

For the Dutch and first-price auctions without buyer's option, there only exists an equilibrium when the demand function is increasing. Compared to the English and second-price auctions, there are therefore less theoretical predictions that can be tested. As in the English and second-price auctions, risk-neutral Nash equilibrium behavior organizes the data better in the second auction than in the first auction. Practically all deviations that we observe share a common feature which is that bidding behavior is above optimal bidding behavior. This is a phenomenon that is also observed in experiments on single-unit first-price and Dutch auctions. Bidders in our experiment have exercised the buyer's option very often under flat and increasing demand, as theory predicts them to do, but too little when demand is decreasing.

Depending on the type of auction mechanism, we find different explanations for the deviations. In the Dutch and first-price auctions, observed bidding behavior turns out to be compatible with risk-averse Nash equilibrium theory. As in the single-unit auction experiments, the deviations from theory can thus be attributed to risk aversion among experimental subjects. In the English and second-price auctions, most of the Nash equilibrium strategies are robust to the form of risk aversion that we consider. For these auction institutions, the deviations from theory can be explained by either myopic or punitive behavior.

The paper also looks at the revenue and the price patterns in the different auction mechanisms with and without buyer's option. It is quite remarkable that the revenue ranking of the 4 auction institutions is the same as in single-unit experiments. We also find that the buyer's option decreases (resp. increases) revenue in first-price (resp. second-price) auctions, but that there is no significant effect in the clock auctions. Successive prices are found to be declining in the auctions with buyer's option, but are constant when the option cannot be used. This result, in conjunction with the fact that subjects in our experiment are found to be risk-averse, suggests that the buyer's option, and not risk-aversion, is responsable for the declining price anomaly.

In future work we plan to study the effect of an increase in the number of buyers at auction. We also plan to investigate the role of the buyers' option in the common value paradigm.

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Appendix

A Proof of proposition 1

A.1 $a \in \{E, S\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$

The second auction strategies are obtained by the standard dominated strategies argument. Therefore, both the loser and the winner of the first auction bid their valuation:

$$b_2^l\left(v, p_1\right) = v \text{ and } b_2^w\left(v, p_1\right) = kv.$$

To derive the first auction equilibrium strategies, we have to distinguish the English auction from the second-price auction as the available information is not the same in these two auction institutions.

A.1.1 $a = E$ and $k = \frac{1}{2}, 1$

See Donald, Paarsch, and Robert (1997).

A.1.2 $a = E$ and $k = 2$

Let $b_1\left(v\right)$ be the first auction equilibrium strategy and v_1 the value of player 1. Suppose the clock has reached p (close to $b_1\left(v_1\right)$) and player 1 has to decide to continue or to stop bidding. Let $G\left(\varepsilon, p\right)$ denote the expected total gain (for the first and second auctions) for player 1 if he decides to continue with bidding until $p + \varepsilon$:

$$G\left(\varepsilon, p\right) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} \left(v_1 - b_1\left(w\right) + 2v_1 - w\right) \frac{dw}{1 - b_1^{-1}\left(p\right)}.$$

The above expression follows because player 1 can only win the first auction if $p \leq b_1\left(v_2\right) \leq p + \varepsilon$, with v_2 being the valuation of player 2. If he wins the first auction, he also wins the second auction because, since p is close to $b_1\left(v_1\right)$, we have that $2v_1$ is larger than v_2. On the contrary, if he loses the first auction he also loses the second one. Note that the density in the integral is the conditional density of v_2 given $v_2 \geq b_1^{-1}\left(p\right)$. Derivation with respect to ε gives:

$$\frac{\partial G}{\partial \varepsilon}\left(\varepsilon, p\right) = \left(b_1^{-1}\left(p + \varepsilon\right)\right)'\left(p + \varepsilon\right) \frac{v_1 - \left(p + \varepsilon\right) + 2v_1 - b_1^{-1}\left(p + \varepsilon\right)}{1 - b_1^{-1}\left(p\right)}.$$

The equilibrium condition is:

$$\frac{\partial G}{\partial \varepsilon}\left(\varepsilon = 0, p = b_1\left(v_1\right)\right) = 0,$$

which leads to

$$b_1\left(v_1\right) = 2v_1.$$

To prove that it is indeed a Nash equilibrium, assume that player 2 follows the strategy $b_1(v_2) = 2v_2$ and assume that player 1 deviates from this strategy and stops at $p < v_1$. In that case he loses the first auction and can only win the second if $v_2 \in \left[\frac{p}{2}, \frac{v_1}{2}\right]$ which would lead to a gain of $v_1 - 2v_2$. But in that case, by bidding until $p = v_1$ player 1 would win both the first and the second auction (because $v_2 < v_1$), which leads to a larger profit of $v_1 - 2v_2 + 2v_1 - v_2$. Assume, now, that player 1 stops at p with $v_1 < p < 2v_1$. In that case losing the first auction also means losing the second because $v_1 < p < 2v_2$ (here in the second auction player 2 bids $2v_2$), therefore it is optimal to bid until $2v_1$. Finally, a deviation $p > 2v_1$ is weakly dominated: it does not improve the gain when $v_2 < v_1$ while it implies a loss when $v_1 < v_2 < \frac{p}{2}$ as the revenue of player 1 is then $3v_1 - 3v_2 < 0$.

A.1.3 $a = S$ and $k = \frac{1}{2}, 1$

See Black and De Meza (1992).

A.1.4 $a = S$ and $k = 2$

In order to characterize the equilibrium strategy $b_1(.)$, assume that player 1 deviates from $b_1(v_1)$ by bidding $b_1(x)$, with x close to v_1. If he loses the first auction while bidding $b_1(x)$, he is sure to lose the second auction as well. On the contrary, if he wins the first auction he is also sure to win the second auction. That is, the expected total gain of player 1 playing $b_1(x)$ is:

$$G(x) = \int_0^x [v_1 - b_1(w) + 2v_1 - w]\, dw.$$

In equilibrium such a deviation cannot be profitable which means that:

$$G'(x = v_1) = 0,$$

which leads to

$$b_1(v_1) = 2v_1.$$

To prove that it is a Nash equilibrium, assume that player 2 bids $2v_2$. It is then obvious that a bid equal to $2x$, $x < v_1$, gives player 1 a lower expected gain than a bid equal to $2v_1$ as it does not increase the gain when player 1 wins both auctions but it reduces the probability of winning. Next, a bid equal to $2x$, $v_1 < x$, also reduces the expected gain of player 1 because when player 1 wins the first auction with $2x$ but not with $2v_1$ he has a negative total gain.

A.2 $a \in \{D, F\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1\right\}$

The non-existence of a Nash equilibrium with symmetrical pure strategies in the first auction is proved in Février (2000).

A.3 $a \in \{D, F\}$, $o = N$, and $k = 2$

We first study the second auction assuming that the players bid according to $b_1(.)$ in the first auction. Suppose that player 1 with valuation v_1 won the first auction and let v_2 denote the valuation of player 2. Therefore the value of $b_1(v_1)$ is revealed before the second auction starts and both players know that $v_2 < v_1 < 2v_1$. In equilibrium the second player knows that he cannot win the second auction and his (weakly dominant) strategy is to bid v_2 in the second auction.

By bidding $x \leq v_1$ the expected gain of player 1 in the second auction is $\text{Prob}(x > v_2 | v_2 < v_1)(2v_1 - x) = \min\left\{\frac{x}{v_1}\ ;\ 1\right\}(2v_1 - x)$, which is maximized for $x = v_1$. Of course, it is not profitable to bid more than v_1. Consequently, both players bid their first-unit valuation in the second auction, i.e. $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.

We now study the first auction. Suppose player 2 bids $b_1(v_2)$ in the first auction and player 1 bids $b_1(x) > b_1(v_1)$. If he wins the first auction he learns that $v_2 < x$, and he maximizes over y (in the second auction) $\frac{y(2v_1 - y)}{x}$. On the other hand, if he loses the first auction then $v_2 > x > v_1$ and he loses the second auction as well. Therefore, his expected total gain is $x\left(v_1 - b_1(x) + \max_y \min\left\{\frac{y}{x}\ ;\ 1\right\}(2v_1 - y)\right)$. This expected gain must be maximized in equilibrium for $x = v_1$. The first order condition leads to $b_1(v_1) \geq \frac{v_1}{2}$.

Suppose, now, that player 1 bids $b_1(x) < b_1(v_1)$. If he wins the first auction he learns that $v_2 < x < v_1$ and he maximizes his second auction gain $\max_y \min\left\{\frac{y}{x}\ ;\ 1\right\}(2v_1 - y)$ by bidding x. On the other hand, if he loses the first auction he learns the value of v_2. If $v_2 > v_1$, he also loses the second auction. If $v_2 < v_1$, he wins the second auction by bidding just above v_2. Therefore the expected gain is:

$$G(x) = x\left[v_1 - b_1(x) + 2v_1 - x\right] + \int_x^{v_1} (v_1 - w)\, dw.$$

The first order condition leads to $b_1(v_1) \leq \frac{v_1}{2}$.

Therefore, the equilibrium first auction strategy is $b_1(v_1) = \frac{v_1}{2}$.

A.4 $a \in \{E, S\}$, $o = Y$, and $k = \frac{1}{2}, 1, 2$

The second auction strategies are obtained by the standard dominated strategies argument. Therefore, each player bids his valuation

$$b_2^l(v, p_1) = v \text{ and } b_2^w(v, p_1) = kv.$$

To derive the first auction equilibrium strategies, the English auction has to be distinguished from the second-price auction as the available information are not the same in the two auction mechanisms.

A.4.1 $a = E$ and $k = \frac{1}{2}$

We start with the buyer's option. If $\frac{v_1}{2} \geq p_1 = b_1(v_2)$, it is profitable to use the option because if he does not execute the option his gain in the second auction is $\max\left\{0, \frac{v_1}{2} - v_2\right\}$, which is lower than $\frac{v_1}{2} - b_1(v_2)$. On the contrary, if $\frac{v_1}{2} < p_1 = b_1(v_2)$, it is clear that the winner must not use the option.

We study now the first auction. As it will become clear later we can restrict ourselves to the search of a first auction equilibrium $b_1(v) \geq \frac{v}{2}$. Suppose the clock has reached p and player 1 has to decide to continue or to stop bidding. It is important to remark that as player 2 is still active at p, his valuation is greater than $b_1^{-1}(p)$.

To derive the equilibrium necessary conditions, we assume that p is close to $b_1(v_1)$. Let $G(\varepsilon, p)$ denote the expected total gain if player 1 decides to continue with bidding until $p + \varepsilon$.

If player 2 withdraws between p and $p + \varepsilon$, player 1 wins the first auction. As we have assumed that $b_1(v_1) \geq \frac{v_1}{2}$, and that p is close to $b_1(v_1)$, it is not profitable to use the buyer's option. Furthermore, player 1 loses the second auction (indeed, his valuation is divided by two, while player 2 valuation remains around v_1). The expected gain in this case is: $\int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)}$

If player 2 remains active at $p + \varepsilon$, player 1 loses the first auction. As seen before, player 2 uses his option if and only if $p + \varepsilon \leq \frac{v_2}{2}$. In case player 2 does not use the option, we have $\frac{v_2}{2} < p + \varepsilon \simeq b_1(v_1) < v_1$ which means that player 1 wins the second auction. The expected gain in this case is: $\int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon), \overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)}$.

Finally

$$G(\varepsilon, p) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)} + \int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon), \overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)},$$

The equilibrium condition is $\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = b_1(v_1)) = 0$. Under the assumption that $b_1(v_1) \leq \frac{\overline{v}}{2}$ this leads to:

$$\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) + 2(v_1 - b_1(v_1)) = 0.$$

On the contrary, if $b_1(v_1) \geq \frac{\overline{v}}{2}$ we obtain:

$$\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) = 0.$$

This second differential equation combined with the assumption $b_1(v) \geq \frac{v}{2}$ implies that $b_1(\overline{v}) = \frac{\overline{v}}{2}$. The first differential equation and this terminal condition define a unique bidding function which verifies $b_1(v) \geq \frac{v}{2}$.

To end the proof, it is necessary to show that this function constitutes indeed a Nash equilibrium of the game by checking that there is no profitable deviation which is straightforward.

A.4.2 $a = S$ and $k = \frac{1}{2}, 1$

See Black and De Meza (1992).

A.4.3 $a = S$ and $k = 2$ or $a = E$ and $k = 1$, 2

The proof is identical to the proof without buyer's option because it is optimal not to use the buyer's option. Indeed, assume that both players bid $2v$ and that $v_1 > v_2$. Player 1 wins the first auction and the price $p_1 = 2v_2$. If player 1 uses the option he pays the second unit $2v_2$, while if he waits he only have to pays v_2.

A.5 $a \in \{D, F\}$, $o = Y$, and $k \in \frac{1}{2}, 1, 2$

The second auction strategies are obtained from Février (2000) (proposition 4.5). The second auction strategy for the loser of the first auction is

$$b_2^l(v, p_1) = \begin{cases} v & \text{if } v \leq \frac{2k}{1+k} p_1 \\ \frac{kp_1}{1+k}\left(1 - \frac{4kp_1}{(1+k)v}\right) & \text{if } v \geq \frac{2k}{1+k} p_1 \end{cases}$$

The winner of the first auction plays the following strategy: If $p_1 = b(v)$ (that is if he played in the first auction according to the equilibrium strategy but he did not use the option), he plays a mixed strategy, such that he bids x, $x \in \left[\frac{kv}{2}, \frac{k - \frac{k^2}{4}}{v}\right]$, with x having the distribution function

$$F(x) = \frac{1 - k + \frac{k^2}{4}}{1 - \frac{k}{2}} \frac{kv}{2x - kv} \exp\left[\frac{4x - 2\left(2k - \frac{k^2}{2}\right)v}{(2x - kv)(2 - k)}\right].$$

If $p_1 > b(v)$ (that is he played in the first auction a bid above the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{k}{2} v$.

If $p_1 < b(v)$ (that is he played in the first auction a bid below the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{4k - k^2}{1+k} p_1$.

Consider now the first auction. Assume that player 2 bids $b_1(v_2)$. If player 1 bids $b_1(x)$ and uses his option, then his expected gain is:

$$G(x) = x\left[(1 + k)v - 2b_1(x)\right].$$

The first-order condition is

$$G'(v) = 0 \Leftrightarrow \frac{1 + k}{4} v^2 = (v b_1(v))',$$

which leads to

$$b_1(v) = \frac{1 + k}{4} v.$$

If both players bid according $b_1(v)$ the expected gain of a player with a valuation v is $\frac{1+k}{2} v^2$. See Février (2000) for the proof that given the strategies described above, it is not profitable to deviate in the first auction and to abstain from using the buyer's option.

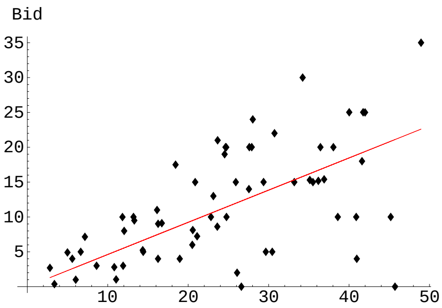

Figure 1: First-price, $k = \frac{1}{2}$, $o = N$

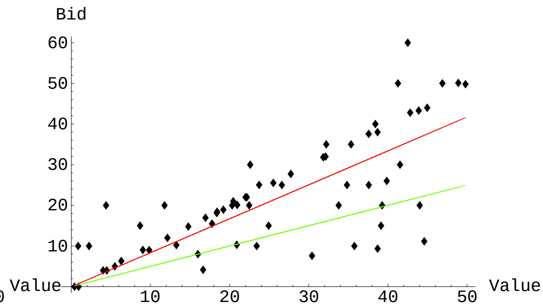

Figure 2: Second-price, $k = \frac{1}{2}$, $o = N$

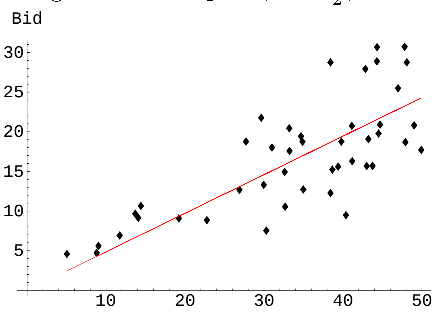

Figure 3: Dutch, $k = \frac{1}{2}$, $o = N$

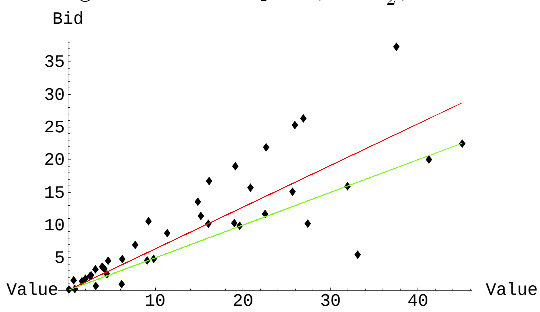

Figure 4: English, $k = \frac{1}{2}$, $o = N$

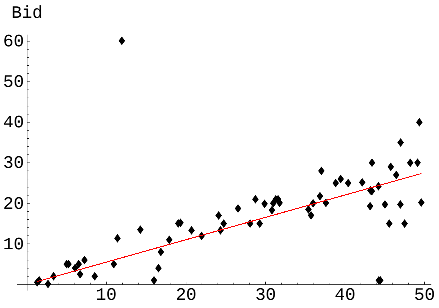

Figure 5: First-price, $k = 1$, $o = N$

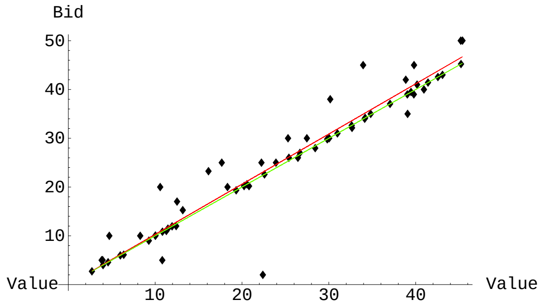

Figure 6: Second-price, $k = 1$, $o = N$

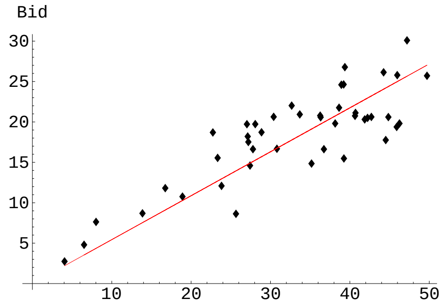

Figure 7: Dutch, $k = 1$, $o = N$

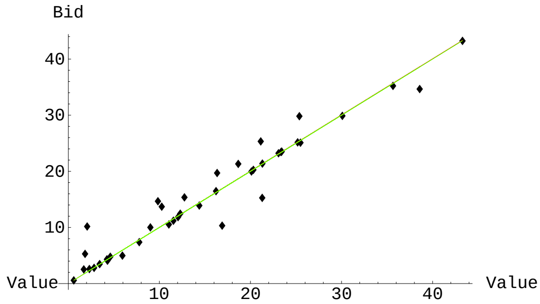

Figure 8: English, $k = 1$, $o = N$

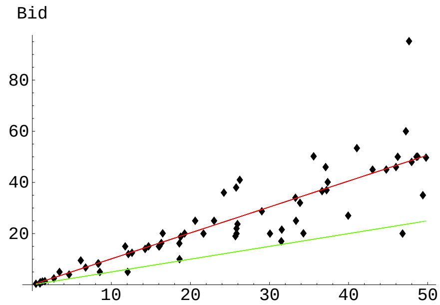

Figure 9: First-price, $k = 2$, $o = N$

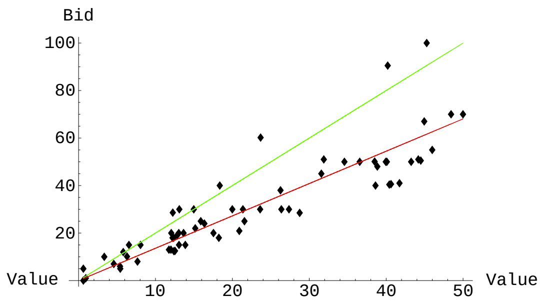

Figure 10: Second-price, $k = 2$, $o = N$

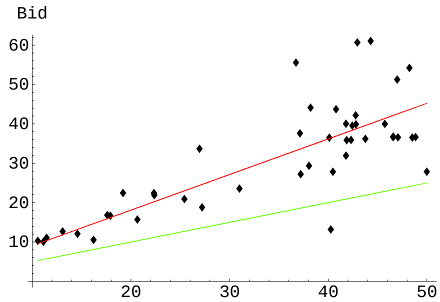

Figure 11: Dutch, $k = 2$, $o = N$

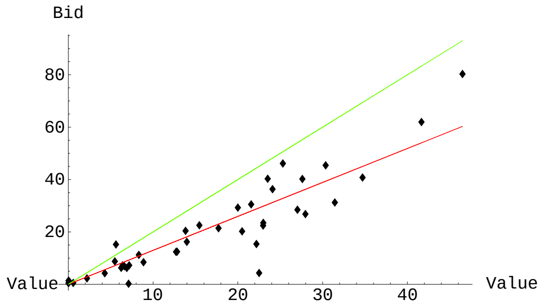

Figure 12: English, $k = 2$, $o = N$

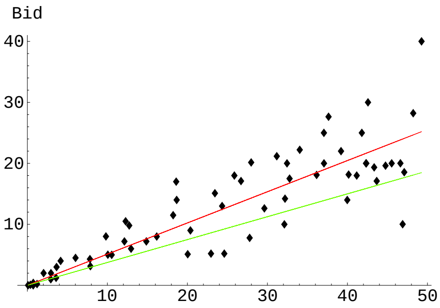

Figure 13: First-price, $k = \frac{1}{2}$, $o = Y$

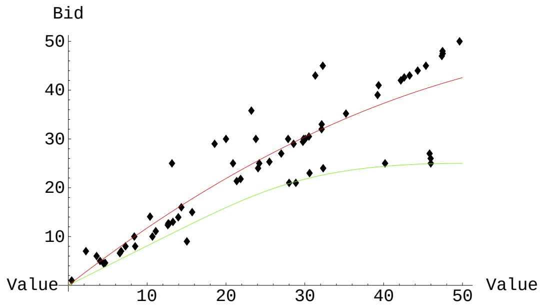

Figure 14: Second-price, $k = \frac{1}{2}$, $o = Y$

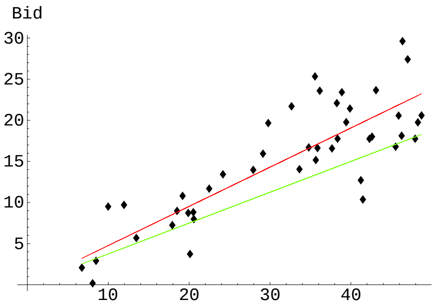

Figure 15: Dutch, $k = \frac{1}{2}$, $o = Y$

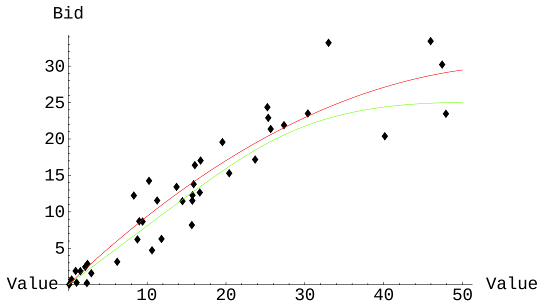

Figure 16: English, $k = \frac{1}{2}$, $o = Y$

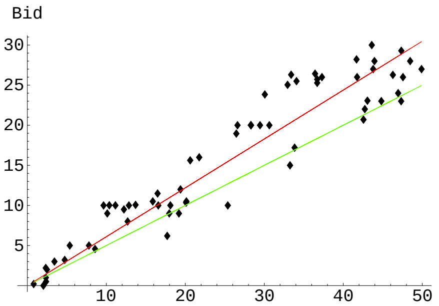

Figure 17: First-price, $k = 1$, $o = Y$

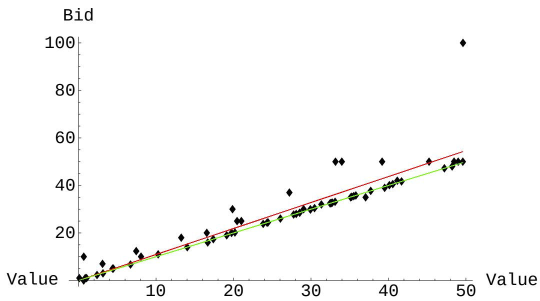

Figure 18: Second-price, $k = 1$, $o = Y$

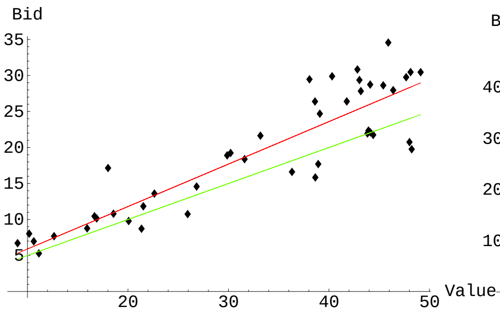

Figure 19: Dutch, $k = 1$, $o = Y$

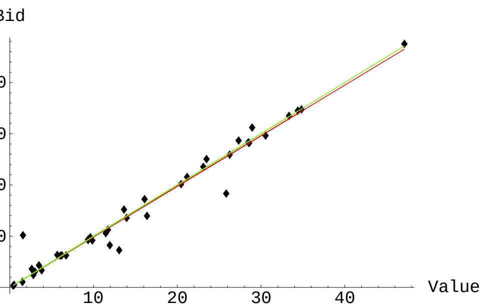

Figure 20: English, $k = 1$, $o = Y$

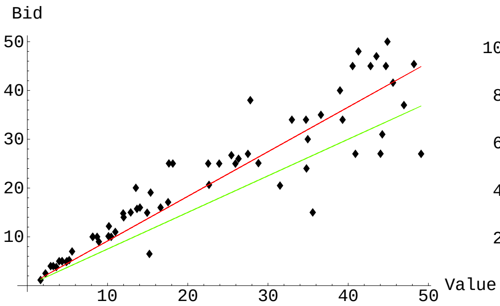

Figure 21: First-price, $k = 2$, $o = Y$

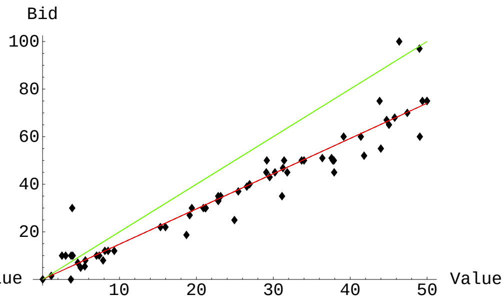

Figure 22: Second-price, $k = 2$, $o = Y$

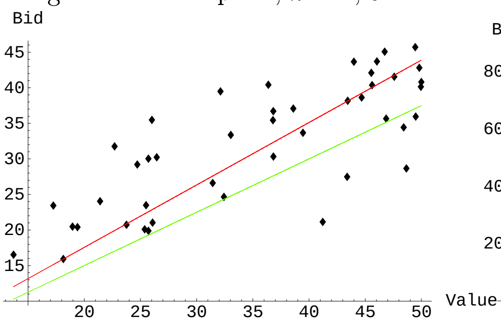

Figure 23: Dutch, $k = 2$, $o = Y$

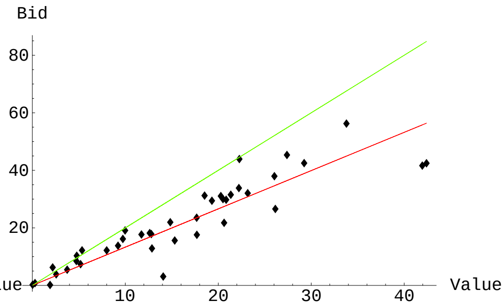

Figure 24: English, $k = 2$, $o = Y$

Buy or wait, that is the option

The buyer's option in sequential laboratory auctions*

Philippe Février[†], Laurent Linnemer[‡] and Michael Visser[§]

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Abstract

This paper reports the results from an experiment on two-unit sequential auctions with and without a buyer's option (which gives the first-auction winner the right to buy the second unit at the winning price). The 4 main auction institutions (first-price, Dutch, second-price, English) are studied. Observed bidding behavior is closer to risk-neutral Nash equilibrium bidding in the second auction than in the first auction. In Dutch and first-price auctions, the deviations from theory can be attributed to risk aversion among buyers; in the English and second-price auctions, they are a consequence of either myopic or punitive behavior. The revenue ranking of the 4 auction institutions is the same as in single-unit experiments. The buyer's option decreases (resp. increases) revenue in first-price (resp. second-price) auctions, but there is no significant effect in the "oral" auctions. The buyer's option causes declining price patterns in our experimental auctions.

Keywords: Experimental economics, sequential auctions, buyer's option.
***JEL* Classification:** C91; D44.

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1 Introduction

In sales of multiple units of a particular good, auction houses often choose to sell the items sequentially, i.e. the items are auctioned separately, one after the other. The advantage of a sequential auction is that it well fits the needs of both small and large buyers, whereas the alternative auction procedure that consists in selling all available units simultaneously, in one shot, typically excludes buyers who set low values on the items, thereby reducing competition at auction. The main disadvantage of a sequential method is that it can be very time-consuming, especially when the total number of units on sale is large. For this reason, auctioneers sometimes provide a so-called buyer's option, which gives the winner of the first auction the right to buy any number of units (1, 2, ..., or all units available). For each unit he/she must pay the winning price established at the first auction. If the winning bidder decides to purchase only part of the total quantity, the remaining items are reauctioned, in the same manner, through a second auction; and this scheme is repeated until all units are eventually sold.

The buyer's option thus clearly offers the best of both worlds: it allows the auctioneer to speed up sales, while keeping the auction mechanism sufficiently flexible to be of interest for different types of buyers. Not surprisingly therefore, the buyer's option is used in many auctions throughout the world. Cassady (1967) describes how the buyer's option is practiced in fur auctions in Leningrad and London, and fish auctions in English port markets. At the auction market in Aalsmeer, the Netherlands, huge quantities of flowers are sold through sequential descending auctions with a buyer's option (see van den Berg, van Ours, and Pradhan (1999)). Well-known auction houses such as Christie's and Sotheby's (see Ashenfelter (1989) and Ginsburgh (1998)) and Drouot (see Février, Roos, and Visser (2001)) systematically use the buyer's option in their sequential ascending auctions of fine wines.

Despite the practical importance of the buyer's option, little attention has been paid to the subject in the literature. The only theoretical article we are aware of is Black and De Meza (1992). They consider the Independent Private Value (IPV) paradigm, and derive optimal bidding strategies in two-unit sequential second-price auctions with and without the buyer's option. All buyers in their model have decreasing demand for the two units (the additional value of the second unit is less than the value of the first unit), or flat demand (both units are valued the same). Empirical studies are also rare. Ashenfelter (1989) and Ginsburgh (1998) report that the option is exercised by many buyers in ascending wine auctions at Christie's and Sotheby's.[1] Van den Berg, van Ours, and Pradhan (1999) study price patterns at sequential descending auctions of roses and argue that the presence of the option is the main determinant of the observed price decline. Finally, Février, Roos, and Visser (2001), using data on

[1] Ashenfelter (1989) claims that auctioneers feel uneasy and uncomfortable about revealing the declining price phenomenon (the fact that in sequential auctions of identical items successive prices tend to decline) to buyers, and use the option as a device for hiding it. The auctioneers with whom we have discussed at Drouot argue, however, that speed is the main reason for making the option available.

ascending auctions of wine held at Drouot, structurally estimate their optimal bidding model and use their estimations to analyze the impact of the option on revenue.

The main contribution of this paper is to study both theoretically and experimentally the role of the buyer's option in two-unit sequential auctions. We adopt the IPV paradigm and assume that the 2 units are sold to 2 risk-neutral buyers. Buyers desire both units, and their demand for the items is either decreasing, flat, or increasing (implying that the value of the second unit exceeds the value of the first unit). The 4 main auction institutions are considered: first-price, descending (Dutch), second-price (Vickrey), and ascending (English) auctions. Although there are apparently no field examples of first-price and second-price sequential auctions with or without a buyer's option,[2] it is nonetheless of interest to study these sealed-bid auctions. Like in standard one-unit auction theory, it is shown in this paper that first-price (resp. second-price) and Dutch (resp. English) sequential auctions with or without a buyer's option are theoretically isomorphic. Furthermore the 4 auction formats generate the same expected revenue. By analogy with experimental studies on single-unit auctions (see Kagel (1995)) for a survey), our experimental design thus allows us to test whether bidding behavior is identical and whether there is an equivalence in revenue.

Many other theoretical predictions are confronted with the experimental data. For each auction institution with and without buyer's option, and each form of the demand curve, we test if observed bidding behavior corresponds to risk-neutral Nash equilibrium bidding. We analyze to what extent the experimental subjects exercise their buyer's option. Referring to the title of the paper, we thus analyze to what extent winners of the first auction directly *buy* the second unit, or instead *wait* and attempt to obtain the additional unit (at a lower price!) in the second auction. Observed frequencies of buying/waiting are compared with optimal frequencies. Predictions on the degree of efficiency of auction outcomes are also tested. Finally we compare observed price patterns with their predicted counterparts. The design of our experiment is such that all types of price patterns are theoretically possible. Depending on the auction mechanism, the form of the demand function, and the presence or not of the buyer's option, theory predicts that successive prices are either decreasing, constant, or increasing.

Experimental work on sequential auctions is still very rare.[3] Burns (1985) considers sequential English auctions. The experiment is designed to mimic the Australian wool market, and the paper's main objective is to study the effect of market size on auction prices. The paper is essentially theory-free in that observed behavior is not confronted with any equilibrium bidding behavior.

[2]Cassady (1967, p. 197) describes the electronic auction market in Osaka, Japan, where lots of fruit and vegetables are sold via sequential first-price auctions, but he never explicitly mentions that the successive lots on sale are identical.

[3]Spurred by the recent FCC auctions, experimental papers on all sorts of *simultaneous* multi-demand auctions are, however, flourishing (see for example Kagel and Levin (2001) and the references therein, and the special issue of the *Journal of Economics & Management Strategy* (1997, Number 3)).

Keser and Olson (1996) consider sequential first-price auctions and suppose that buyers have single-unit demand functions. Their main objective is to compare observed price-sequences with the predicted patterns derived in Weber (1983), under different design parameters. Similarly as in Burns, the paper focuses on one particular auction mechanism, and no attempt is made to examine outcomes under alternative institutions. Robert and Montmarquette (1999) do consider several auction institutions, and also provide theoretical foundations for each of them. In their models, the number of items desired by each buyer is a random variable and demand functions are decreasing. They consider sequential Dutch, English and mixed auctions, and compare observed behavior with predicted behavior. None of these 3 experimental papers on sequential auctions analyzes the buyer's option.

The paper proceeds as follows. In the next section the theoretical background is presented. In deriving the risk-neutral Nash equilibrium bidding functions and the expected revenues in the different auction institutions, we partly draw on Black and De Meza (1992), Donald, Paarsch, and Robert (1997) and a recent paper by Février (2000). But most results in this section are actually new. Section 3 describes the experimental design, section 4 the experimental results, and section 5 concludes.

2 Theoretical background

Suppose that 2 units of a good are auctioned to 2 potential buyers. Each buyer is assumed to be risk-neutral and desires to purchase both units. Adopting the IPV paradigm, let v_i denote the value that buyer i places on the first unit. The value v_i and the value of i's opponent are independently drawn from a uniform distribution on the interval $[0, \overline{v}]$. It is assumed that the value that i places on the second unit is kv_i. The parameter k can take three values: $k \in \{\frac{1}{2}, 1, 2\}$. The value of k is common knowledge. Note that $k = \frac{1}{2}$ implies that the second unit is valued less than the first unit (decreasing demand), $k = 1$ that both units are valued the same (flat demand), and $k = 2$ that the second unit is valued more than the first (increasing demand).

The 2 units are sold sequentially. The first unit of the good is sold in the first auction. The manner in which it is auctioned depends on the auction institution. Let a indicate the auction institution, $a \in \{D, E, F, S\}$, where D stands for Dutch auction, E for English auction, F for First-price auction, and S for Second-price auction, and let p_1 denote the price the winner of the first auction has to pay for the first unit. When $a \in \{D, E\}$, the unit is auctioned using a clock. When $a = D$, the clock starts very high, and descends until one of the players stops the clock. This player wins the unit and p_1 equals the price at which the clock was stopped. When $a = E$, the clock starts at 0, and increases until one of the players stops the clock. Here the winner of the auction is the player who *did not stop* the clock. The price p_1 he/she has to pay for the first unit is again the amount at which the clock stopped. When $a \in \{F, S\}$, the unit is sold via sealed-bid auctions. Both players submit their sealed bid to the auctioneer who awards the unit to the highest bidder. When $a = F$ the winner pays his/her own bid, i.e. p_1 equals the highest submitted bid. When $a = S$ the winner pays the bid of his opponent, i.e. here p_1 equals the second highest submitted bid. For all institutions a, the price p_1 is revealed to both players once the first auction has ended.

The way in which the second unit is sold depends on whether the buyer's option is available or not. Let o be the indicator for the availability of the buyer's option, $o = N$ if it cannot be used, and $o = Y$ otherwise. For any auction institution a, if $o = N$ the second unit is auctioned under the prevailing rules of institution a. Let p_2 be the price paid for the second unit. If instead $o = Y$ the winner of the first auction has the option to buy 1 or 2 units, at the price of p_1 per unit. When he decides to purchase only 1 unit, a second auction is held under the conditions of institution a. When he/she exercises the buyer's option, no second auction is held. Note that in this case we automatically have $p_2 = p_1$.

The theoretical model presented here is essentially based on the framework built by Black and De Meza (1992). These authors, however, only considered the second price auction ($a = S$) and they do not analyze the case of increasing marginal valuation ($k = 2$). The hypothesis that each bidders' valuation for the second unit is connected, in a deterministic way, to the valuation of the first unit, is certainly restrictive, and might no necessarily reflect behavior at real auctions. The hypothesis on the number of players is also restrictive as real world auctions the number of participant is typically larger than two. These simplifying hypotheses are, however, needed to ease solving for the equilibrium strategies. Also, as mentioned in the introduction, this is the first experimental paper on the buyer's option, justifying a rather simple setup, that can be refined and generalized in future work.

For any given value of a, o, and k, let $G(a, o, k)$ denote the bayesian two-stage game described above. We are looking for perfect bayesian equilibria of the game $G(a, o, k)$ with pure and symmetric strategies in the first auction. Let $b_1(v)$ denote the equilibrium strategy of the bidders in the first auction. If $o = Y$, let $bo(p_1) \in \{0, 1\}$ indicate whether the winner exercises the buyer's option or not given the auction price p_1, with $bo(p_1) = 1$ meaning that he/she uses his/her option, and $bo(p_1) = 0$ that he/she does not. Finally, let $b_2^w(v, p_1)$ denote the second auction strategy of the winner of the first auction, and $b_2^l(v, p_1)$ the second auction strategy of the loser of the first auction. For practical reasons given in section 4.3, these strategies are only confronted with the data when the buyer's option is not available. In the following proposition, the strategies are therefore only given for $o = N$. But in the proof of the proposition (given in the appendix), explicit use is made of the strategies for $o = Y$.

Proposition 1. *A symmetric-first-auction perfect bayesian equilibrium of the game $G(a, o, k)$ is:*

1. *If $a \in \{E, S\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = kv$, $b_2^l(v, p_1) = v$, $b_2^w(v, p_1) = kv$.*

2. *If $a \in \{D, F\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1\right\}$, then no such equilibrium exists.*

3. *If $a \in \{D, F\}$, $o = N$, and $k = 2$, then $b_1(v) = \frac{1}{2}v$, $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.*

4. *If $a \in \{E, S\}$, $o = Y$, and $k = \frac{1}{2}$, then $b_1(v)$ is solution of $b_1(v) - \frac{v}{2} = 2\lambda(v - b_1(v))\, b_1'(v)$, with $\lambda = 0$ if $b_1(v) \geq \frac{1}{2}\overline{v}$ and $\lambda = 1$ otherwise; $bo(p_1) = 1$ if $p_1 \leq \frac{1}{2}v$ and $bo(p_1) = 0$ if $p_1 > \frac{1}{2}v$.*

5. *If $a \in \{E, S\}$, $o = Y$, and $k = 1$, then $b_1(v) = v$, $bo(p_1) \in [0, 1]$.*

6. *If $a \in \{E, S\}$, $o = Y$, and $k = 2$, then $b_1(v) = 2v$, $bo(p_1) = 0$.*

7. *If $a \in \{D, F\}$, $o = Y$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$, then $b_1(v) = \frac{1+k}{4}v$, $bo(p_1) = 1$.*

Let us first comment on the predictions for the English and second-price auctions. As mentioned in the introduction, the behavioral predictions are always the same for these 2 mechanisms. When $o = N$, theory requires bidders to bid kv in the first auction, that is they have to bid the value for the second unit. While this result is intuitive for flat demand, it is less so when demand is decreasing or increasing. With decreasing demand, bid shading is required because losing the first auction is not necessarily bad news, as it implies a weaker rival in the second auction. With increasing demand, over-bidding is required as the winner of the first auction is also going to be the winner of the second auction. In the second auction (still when $o = N$), it is a dominant strategy for each player to bid the value of the unit for which he/she is bidding. That is, the loser of the first auction should bid v, and the winner of the first auction kv.

When $o = Y$ and $k \in \{1, 2\}$, optimal first-auction bidding is the same as in the absence of the buyer's option. Put in other words, the buyer's option has no effect on first-auction bidding behavior. However, when $k = \frac{1}{2}$, first-auction bidding should be more aggressive than in the absence of the option. The optimal use of the buyer's option is fairly simple when $k = \frac{1}{2}$ or $k = 1$. In the former case it should be used if the first-auction price is lower than the second unit value, and in the latter case the first-auction winner is indifferent between exercising the option or not, which is the meaning of $bo(p_1) \in [0, 1]$. When $k = 2$ it is not optimal to use the option because the loser of the first auction is expected to bid less aggressively in the second auction, so the first-auction winner has a higher expected gain by waiting for the second auction.

Let us next comment on the predictions for the Dutch and first-price auctions. Again theory predicts that behavior is strictly identical under the 2 institutions. When $o = N$, there does not exist a symmetric pure strategy equilibrium for $k = \frac{1}{2}, 1$. An explanation for this result is the following. If such an equilibrium were to exist, the loser of the first auction would learn the valuation of the winner (since p_1 is revealed at the end of the first auction). The first-auction winner would then clearly be in an uncomfortable situation in the second auction. The equilibrium in the second auction would take the following form: the winner of the first auction would play a mixed strategy and the loser a pure strategy. However, this second-auction equilibrium is not compatible with a first-auction pure strategy, since we can show that there always exists a profitable deviation. This means that both players should hide their valuation by playing a mixed strategy in the first auction.

When $o = N$ and $k = 2$, a symmetric pure strategy equilibrium does exist for the Dutch and the first-price auctions. This equilibrium is not simple to compute and is not very intuitive as it implies a relatively low first-auction bid. At first sight one might indeed think that it should be rewarding for player 1 to deviate from equilibrium by bidding $\frac{x}{2}$ (with $x > v_1$) in the first auction in order to increase the probability to win the first unit, and thereby to enter the second auction with a stronger valuation $2v_1$. But this deviation is not profitable. Indeed, this deviation decreases the expected gain in the first auction (since bidding half of one's valuation is optimal in a single-unit auction), and, as can be shown, it does not affect the expected gain in the second auction. Note that the equilibrium given in the proposition is such that the winner of the first auction, say bidder 1, automatically wins the second auction: his/her valuation for the second unit is $2v_1$ while his/her opponent's valuation for the first unit is[4] $v_2 \leq v_1$, so by bidding v_1 he/she wins the second auction with probability one. Therefore, in equilibrium it is as if both bidders only compete for the first unit.

When $o = Y$, a symmetric pure strategy equilibrium does exist for all values of k. Note that in equilibrium, bidders behave exactly as in standard single-unit Dutch or first-price auctions. Indeed, in equilibrium each player bids $\frac{1+k}{4}v$ in the first auction and the winner *always* exercises his/her option. It is thus as if players submit a single bid equal to $\frac{1+k}{2}v$, for a "single good" with a value $(1+k)v$.[5] Note finally that for $k = 2$, first-auction bidding should be more aggressive when the option is available than when it is not available.

3 Experimental design

The experiment was conducted on 28 and 29 March 2001 at the *Ecole Nationale de Statistique et de l'Administration Economique* (ENSAE).[6] Students were recruited through personal emails, and fliers that we dispatched in their mailboxes. Seventy four students (out of roughly 360 students that studied at the time at ENSAE) actually participated in the experiment. We organized a total of 10 experimental sessions in the computer rooms at ENSAE, and each student took part in only one session. Only one type of auction mechanism was used per session. Table 1 lists for each session the type of auction mechanism that was studied and the number of participants. From Table 1 it can be seen

[4]Because the first-auction strategy is symmetric.

[5]Recall that, given our model assumptions, the optimal single-unit bid (in first-price and Dutch auctions) for a good valued at v is $\frac{1}{2}v$.

[6]The ENSAE is one of the leading French institutions of higher learning in the fields of statistics, economics, finance, and actuarial sciences. After completing the three-year curriculum of this school, graduates have a training comparable to the level attained by first-year Ph.D. students at a good North American university.

Table 1: Sessions

Session	Type of auction	Number of subjects
1	First-price	8
2	Second-price	8
3	Dutch	6
4	English	6
5	Dutch	10
6	English	8
7	First-price	8
8	Second-price	8
9	Dutch	6
10	English	6

that 22 students participated in the Dutch auctions, 20 in the English auctions, 16 in the first-price auctions, and 16 in the second-price auctions.

Each session was made up of two parts. The first part was devoted to sequential auctions without a buyer's option, and the second part to sequential auctions with a buyer's option.

We start by describing the first part of a session. We began by reading aloud the instructions about the auction's rules without a buyer's option. Written versions of the instructions were distributed to the participants and could be consulted at any time during the experimental session.[7] The first part had 12 periods. Since we focus in this paper on auctions with 2 buyers, participants were told that they were in competition with a single person. At the beginning of each period the computer randomly matched each student to another student present in the room (all sessions had an even number of participants), so participants were aware of the fact that their opponent differed from period to period. Participants were also told that in each period 2 units of a fictitious good were sold at auction to each couple.

At the start of each period, valuations were independently drawn from a uniform distribution on $[0;\overline{v}]$=[0;FFr50.00]. On the computer screen of participant i appeared his/her valuation for the first unit of the good v_i, the prevailing value of k, and his/her valuation for the second unit kv_i. The value of k changed every 4 periods ($k = \frac{1}{2}$ in periods 1-4, $k = 1$ in periods 5-8, and $k = 2$ in periods 9-12). Participants could observe this information for 30 seconds, after which the first auction started (but the information remained on the screen even during the auction). The manner in which participants could bid depended on the type of auction mechanism that was used during the session. The auction-specific bidding devices will be described later on.

Once the first auction was over, some information concerning the first auction was added to the screen of each subject i. It indicated whether i was the winner or not, his/her own bid (if any), the winning price p_1, i.e. the price he/she or

[7]The instructions can be obtained from the authors upon request.

his opponent had to pay for the first unit, and his/her gain associated with the auction ($v_i - p_1$ if i was the winner, 0 otherwise). Since the identity of the winner of the first auction is crucial knowledge in our experiment, we emphasized this by coloring the box marked "Winning bid" blue if i had won the first auction, and red otherwise. Note that the exact nature of information released between the two auctions differed slightly with the type of auction mechanism. For instance, for the winner of an English auction the box marked "Your bid" remained empty, while for the winner of a Dutch auction this box indicated the price at which he had stopped the clock.

Before the start of the second auction, participants again had a thirty-seconds reflection period during which they could, if they wished, consult all information on their screen (again, all information remained visualized during the second auction). The second auction functioned in the same way as the first auction. We stressed the fact that the gain associated with the second auction depended on the outcome of the first auction. Thus, winner i of the second auction had a gain of $kv_i - p_2$ if he had also won the first auction, and a gain equal to $v_i - p_2$ if he had lost the first auction. Once the second auction was terminated for all couples in the room, we proceeded with the next period.

The 12 periods of the first part of each experimental session were preceded by 6 "dry" periods (2 for each value of k). This gave participants the opportunity to familiarize themselves with the bidding method, determine their strategy for the different values of k, and ask questions to the experimenter.

Next we describe the second part of the session, the one that was designed to study the buyer's option. We began by reading aloud the instructions about this part of the experiment. Like the first part it consisted of 12 periods. Each period started exactly like in the first part of the experiment: the valuations and the value of k (the values of k alternated as in the first part) showed up on the screen, the first auction started after 30 seconds, and once the first auction was over for player i and his/her rival, their screens updated them on the relevant auction results. Unlike the first part of the session, subjects were told that the winner of the first auction could, if he/she desired, use the buyer's option. If winner i chose to execute his/her option, the period was over for him and his/her opponent, and his/her total gain in the period was $(v_i - p_1) + (kv_i - p_1) = (1 + k)v_i - 2p_1$. If he/she choses not to do so, his/her gain associated with the first auction was $v_i - p_1$, and a second auction was held after the thirty-seconds pause. The second auction was in all respects identical to the second auction conducted in the first part of the experiment.

The 12 "wet" periods of the second part of each experimental session were again preceded by dry periods, but now just 3 of them (1 for each value of k) since, at least from a practical point of view, the second part differed little from the first.

As mentioned above, the way in which participants had to submit their bids depended on the auction format. In the first-price and second-price auctions participants could submit their bid by entering a number in a box marked "Submit your bid here". The number could be any positive real integer, i.e. we did not forbid subjects to bid in excess of their valuations.

In the Dutch and English auctions bidding took place via numerical clocks. After the 30-seconds reflection period, the clock appeared on the screens of the participants. In the English auctions the clock started at 0.00FFr, augmented continuously at a rate of 50.00FFr per minute, and stopped automatically at FFr120.00. The clock started and operated simultaneously on the screens of participant i and his/her rival. They could stop the clock at any time by pressing the "Enter" key or "Space bar", or click on a window marked "Stop the clock". If neither i nor his rival had stopped the clock before it reached FFr120.00, the computer randomly selected i or his/her rival as the winner (actually this never happened during our experiments). In the Dutch auctions the clock started at FFr60.00 (if $k = \frac{1}{2}$ or $k = 1$) or FFr120.00 (if $k = 2$), descended continuously at the speed of 50.00FFr per minute, and stopped automatically at FFr0.00. The Dutch clock started and operated simultaneously for subject i and his/her opponent and they could stop it, at any time, as the English clock. If neither i nor his/her rival had stopped the clock before it reached FFr0.00, there was no auction winner (again, this never occurred during our experiments). Note that as in the sealed-bid auctions, subjects could bid above their valuations (up to a reasonable limit) in the clock auctions.

At the start of an experimental session, i.e. at the beginning of the first period, all participants were given a capital balance of FFr50.00. At the end of each period, the gains made during the period were added to the balance, and losses were subtracted from it. We informed the experimental subjects that if the end-of-period balance of a participant was negative (as a result of his/her bidding behavior in the period), the balance would immediately be readjusted to 0. We stressed that balances would only be readjusted at *the end* of a period, in view of the end-of-period balance, and not at some point *during* a period. The reason for censoring the start-of-period balances at 0 is to incite subjects to play well all along the experiment.[8] As it turned out, for none of the experimental subjects the capital balance went negative, so it was never necessary to implement the readjustment procedure.

At the end of the session participants were paid in cash their final capital balance divided by two. This 50% cut does not affect bidding behavior and could be interpreted by the participants as a tax due to the auctioneer. On average we paid FFr229 to the students, the minimum payment was FFr60, and the maximum payment FFr360. Experimental sessions lasted between 1.5 and 2 hours.

Before turning to the experimental results, we want to comment on the number of treatment levels in our experiment. In each session subjects went through 2×3 different treatments (with or without a buyer's option and three forms of the demand function). A drawback of our design is this high number of treatment levels as it might have made the subjects susceptible to hysteresis effects. However, we do not think that this occurred. Each change in the value of k was clearly indicated both on the screen and orally by the experimenter.

[8] Had we not done this, a subject with a balance of say minus FFr300 at the beginning of period 24, would clearly not have been incited to behave optimally in this last period.

Moreover, the introduction of the buyer's option was made very clear since we began the second part of the experiment by oral instructions about the rules of this mechanism. Therefore, subjects have not been confused nor by the shifts in the value of k nor by the introduction of the buyer's option. Instead, the frequent changes in the treatment helped to keep the subjects alert and attentive. On the other hand, the advantage of having several treatments within a session is that the estimation of treatment effects is facilitated since it is not necessary to control for inter-individual differences.

4 Experimental results

4.1 Bidding behavior in the first auction

Figures 1-12 show all first-auction bids for the different values of k for the 4 auction formats without buyer's option. The figures thus graph all first-auction bids submitted during the first part of the experiment, that is during periods 1-12. They depict the losing bids for the English auctions, the winning bids for the Dutch auctions, and both winning and losing bids for the sealed bid auctions. Whenever there is a theoretical prediction (see Proposition 1), the optimal equilibrium bid function $b_1(.)$ is drawn in green. For instance, in Figure 2 (second-price auction, $k = \frac{1}{2}$) the green line is the function $b_1(v) = \frac{1}{2}v$, but in Figure 1 (first-price auction, $k = \frac{1}{2}$) no green line is drawn since no prediction is available. Since all optimal bidding strategies are linear functions of the valuations, equal to zero when $v = 0$, each figure also shows, in red, the fitted line $\hat{\beta}v$ where $\hat{\beta}$ is the OLS estimate of the coefficient in the regression $b_{1it} = \beta v_{it} + \varepsilon_{it}$ where b_{1it} and v_{it} are i's bid and valuation in period t, and ε_{it} an error term that is assumed independent over i and t. A comparison of the green and red lines is therefore a quick eyeball test of the theoretical predictions.

Looking at Figures 1-12, one can distinguish, roughly speaking, three types of graphs. First there are graphs where the fitted and predicted lines more or less coincide, suggesting that observed bidding behavior is coherent with theory. This is the case for Figures 6 and 8, i.e. the English and second-price auctions with flat demand. In the second category of graphs, the red and green lines are distinct and the large majority of bids is closely concentrated around the red line, suggesting that most subjects deviate, *in the same manner*, from optimal behavior. This is the case for Figures 9, 10, 11 and 12. In the third category of graphs the red and green lines are again distinct, but part of the bids is now closely concentrated around the green line. Apparently there is a group of subjects that bid according to theory. Another part of the bids is clearly not located near the optimal bidding line. The bids are closely scattered however, so there is again the impression that the subjects who deviate behave quite similarly. There is not a "continuum" of types of observed behavior, but instead the number of different behavioral strategies that can be observed seems limited. Figures 2 and 4 fit in this category.

Table 2 reports the OLS results of the bid regressions, that is the red lines in

Table 2: First auction without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	64	0.46 (0.03)	0.78	$\emptyset$	$\emptyset$
Second-price	64	0.83 (0.04)	0.87	0.5	No
Dutch	44	0.49 (0.02)	0.93	$\emptyset$	$\emptyset$
English	40	0.64 (0.04)	0.85	0.5	No
				$k = 1$	
First-price	64	0.55 (0.03)	0.8	$\emptyset$	$\emptyset$
Second-price	64	1.03 (0.02)	0.98	1	Yes
Dutch	44	0.54 (0.01)	0.97	$\emptyset$	$\emptyset$
English	40	1.00 (0.02)	0.98	1	Yes
				$k = 2$	
First-price	64	1.01 (0.04)	0.91	0.5	No
Second-price	64	1.36 (0.05)	0.93	2	No
Dutch	44	0.90 (0.04)	0.94	0.5	No
English	40	1.30 (0.06)	0.92	2	No

Figures 1-12. The table also reports the predicted slopes of the optimal bidding strategies and test results of the hypothesis that observed behavior is in line with predicted behavior. The null hypothesis can be tested simply by testing whether the coefficient β equals some specific value. For example, the OLS estimate $\hat{\beta}$ equals 0.83 for the second-price auctions with decreasing demand, the estimated standard error is 0.04, the number of observations in the regression is 64 (16 subjects $\times 4$ periods), the R^2 is 0.87 (defined for a regression model without a constant), the predicted slope is $\frac{1}{2}$, and the null hypothesis that $\beta = \frac{1}{2}$ is rejected at the 5% level.

As Table 2 shows, the null is accepted just 2 out of 8 times. Theory is accepted precisely in the two cases where the optimal strategies are relatively transparent; subjects have indeed understood that in second-price and English auctions with flat demand it is optimal to bid the value v in the first auction. However, in second-price and English auctions with decreasing (resp. increasing) demand, subjects had a tendency to over-bid (resp. under-bid); although subjects have understood that optimal behavior calls for bid shading (resp. bidding above value), the extent to which they did this was too modest. In first-price and Dutch auctions with increasing demand, observed bidding is on average close to v, whereas optimal behavior requires agents to bid $\frac{1}{2}v$. Note that as predicted in Proposition 1, observed behavior in respectively the first-price and Dutch auctions, and second-price and English auctions, is very similar.

Figures 13-24 show all first-auction bids for the different values of k for the 4 auction formats with buyer's option, i.e. they are based on all first-auction bids submitted during periods 13-24 of the experiment. Table 3 presents the corresponding estimation and test results. For second-price and English auctions with decreasing demand the solution (approximated using simulations) of the differential equation given in Proposition 1 is $b_1(v) = 0.99v - 0.009v^2$. This explains why the green line in Figures 14 and 16 is curved. The red line in each of these pictures is the fitted line $\hat{\beta_1}v + \hat{\beta_2}v^2$ where $\hat{\beta_1}$ and $\hat{\beta_2}$ are the OLS estimates of the coefficients in the regression $b_{1it} = \beta_1 v_{it} + \beta_2 v_{it}^2 + \varepsilon_{it}$. The test of the theory in these 2 cases amounts to testing, using a standard Fisher-test, the joint hypothesis that $\beta_1 = 0.99$ and $\beta_2 = 0.009$.

The results for the first auctions with buyer's option are quite similar to those obtained for the first auctions without the option. Indeed, as Table 3 shows, the theoretical predictions are again rejected in most cases: the null is accepted only 2 out of 12 times. Theory is again accepted for the English auction with flat demand, and now also for the English auction with decreasing demand (not at the 5% level, but at the 1% level, which is indicated by "Yes*" in the table). Unlike the first-auction results without the option, observed bidding behavior is not in line with theory for the second-price auction with flat demand. Observed bidding in this case is on average slightly above the value v. As in the first auctions without option, the degree of bid shading is too modest in the second-price auctions with decreasing demand; the degree of over-bidding is too modest in the English and second-price auctions with increasing demand. Table 3 shows that in all Dutch and first-price auctions, observed bidding is on average above equilibrium bidding. Note finally that there is again much behavioral similarity between Dutch and first-price auctions on the one hand, and English and second-price auctions on the other.

The impact of the buyer's option on first-auction bidding behavior can be studied by comparing the results of Table 2 and Table 3. Let us first consider the Dutch and first-price auctions. Here the theoretical effect of the buyer's option can be confronted with the data only for $k = 2$. With increasing demand, theory predicts that bidders should be more aggressive when the option is available (the predicted slope is 0.75 with the option, and 0.50 without). However, running an appropriate regression model and testing for a buyer's option effect,[9] it turns out that bidding behavior in the Dutch auction is *not* affected by the presence of a buyer's option, whereas bidding in the first-price auction is actually significantly *less* aggressive when the option is available.

Concerning the English and second-price auctions, we only test for an impact of the buyer's option when $k = 1$ and $k = 2$. In these cases, theory predicts that the buyer's option does not modify first-auction bidding behavior. Performing the same kind of tests as described just before, we find that for both auction institutions the theory is verified when demand is flat. When demand is increasing, the data are again in line with theory for the English auction but not for the second-price auction. In the latter case, first-auction bidding is actually more aggressive when the buyer's option is available.

[9]Pooling all first-auction bids submitted in the Dutch (resp. first-price) auctions, we consider the model $b_{1it} = \beta v_{it} + \gamma v_{it} 1\{o_{it} = Y\} + \varepsilon_{it}$, with $1\{.\}$ the indicator function and o_{it} the option indicator for individual i in period t, and check for the buyer's option effect in Dutch (resp. first-price) auctions by testing whether $\gamma = 0$.

Table 3: First auction with buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	64	0.51 (0.02)	0.92	0.375	No
Second-price	64	1.25 (.09) -0.008 (.002)	0.95	0.99 -0.009	No
Dutch	44	0.48 (0.02)	0.95	0.375	No
English	40	1.02 (.06) -0.008 (.001)	0.94	0.99 -0.009	Yes*
				$k = 1$	
First-price	64	0.61 (0.01)	0.97	0.5	No
Second-price	64	1.09 (0.03)	0.95	1	No
Dutch	44	0.59 (0.02)	0.97	0.5	No
English	40	0.99 (0.02)	0.99	1	Yes
				$k = 2$	
First-price	64	0.91 (0.03)	0.95	0.75	No
Second-price	64	1.48 (0.03)	0.97	2	No
Dutch	44	0.88 (0.03)	0.96	0.75	No
English	40	1.33 (0.05)	0.94	2	No

4.2 The buyer's option

In this subsection we study to what extent the buyer's option has been exercised by the subjects in our experiment. The results can be found in Table 4. The second column reports the relative number of times the buyer's option has been used for each auction institution. For ease of presentation, the third column recalls the optimal frequencies given in Proposition 1. Let us first discuss the results for the Dutch and first-price auctions. For each value of k, Nash equilibrium behavior requires that the first-auction winner should always use the buyer's option. Table 4 shows that observed behavior is quite well in line with this prediction for flat and increasing demand. However, when the demand function is decreasing, the observed frequency of exercising the option is much too low (only 32% in the Dutch auctions and 41% in the first-price auctions). Given that in both these auction institutions many first-auction bids are clearly out of equilibrium (see Figures 13 and 15), the question arises whether the deviating bidders are responsible for the low frequency observed in our data. To answer this question, we have run the logit model $Prob(bo_{it} = 1) = 1/(1 + \exp[\beta_1 + \beta_2 1\{b_{1it}$ "close" to $b_1(v_{it})\}])$ with $bo_{it} = 1$ if i uses the buyer's option in period t and 0 otherwise, and the observed bid b_{1it} is "close" to the optimal bid $b_1(v_{it})$ if the relative difference between the two is smaller than 30%. The ML estimate $\hat{\beta}_2$ (standard error) is -1.70 (0.75) for the Dutch auction and -2.61 (0.94) for the first-price auction. In both cases the coefficient is negative and significant, suggesting that subjects whose first-auction bids are at or close to equilibrium are more likely to exercise the buyer's option, and are thus more likely to remain coherent with theory. See also the discussion

Table 4: The buyer's option

Auction	Relative frequency[1]	Prediction
		$k = 0.5$
First-price	41%	$bo(p_1) = 1$
Second-price	93%	$bo(p_1) = 1$ if $p_1 \leq 0.5v$
	0%	$bo(p_1) = 0$ if $p_1 > 0.5v$
Dutch	32%	$bo(p_1) = 1$
English	96%	$bo(p_1) = 1$ if $p_1 \leq 0.5v$
	0%	$bo(p_1) = 0$ if $p_1 > 0.5v$
		$k = 1$
First-price	91%	$bo(p_1) = 1$
Second-price	78%	$bo(p_1) \in [0, 1]$
Dutch	84%	$bo(p_1) = 1$
English	68%	$bo(p_1) \in [0, 1]$
		$k = 2$
First-price	81%	$bo(p_1) = 1$
Second-price	69%	$bo(p_1) = 0$
Dutch	98%	$bo(p_1) = 1$
English	70%	$bo(p_1) = 0$

[1] Relative number of times the buyer's option is used.

in section 4.5.1.

Next consider the English and second-price auctions. Under decreasing demand the observed frequencies are very close to the frequencies predicted by the theory. Subjects have well understood that the option should be exercised when $p_1 \leq \frac{1}{2}v$, and inversely that it should not be used when $p_1 > \frac{1}{2}v$. That the last prediction is verified may seem somewhat obvious (exercising the option while $p_1 > \frac{1}{2}v$ implies a loss on the second unit acquired) but is nonetheless reassuring since it means that subjects have apparently well understood the rules of the game. Under flat demand the theory predicts that the winner of the first auction is indifferent between using or not using the buyer's option, so in this case there is no testable implication of the theory. Under increasing demand the theory predicts that the winner should never use the option, but should instead wait and try to win the second unit of the good in the second auction. This prediction is mostly rejected by the data since 70% (resp. 69%) of the winners in the English (resp. second-price) auctions *did* use the buyer's option.[10]

[10]Figures 22 and 24 show that for the English and second-price auctions with increasing demand practically all bids appear out of equilibrium. Performing the logit analysis would therefore not make sense in these 2 cases.

Table 5: Second auction (for loser of first auction) without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	32	0.57 (0.03)	0.93	$\emptyset$	$\emptyset$
Second-price	32	0.95 (0.03)	0.96	1	Yes
Dutch	18	0.55 (0.03)	0.95	$\emptyset$	$\emptyset$
English	26	0.97 (0.03)	0.97	1	Yes
				$k = 1$	
First-price	32	0.76 (0.03)	0.96	$\emptyset$	$\emptyset$
Second-price	32	1.07 (0.03)	0.97	1	Yes*
Dutch	15	0.74 (0.03)	0.74	$\emptyset$	$\emptyset$
English	38	0.99 (0.03)	0.99	1	Yes
				$k = 2$	
First-price	32	1.04 (0.06)	0.91	1	Yes
Second-price	32	1.21 (0.08)	0.88	1	No
Dutch	5	1.12 (0.14)	0.94	1	Yes
English	40	1.29 (0.06)	0.92	1	No

4.3 Bidding behavior in the second auction

We only discuss the second-auction results for the 4 auction formats without buyer's option, that is we focus on second-auction bids submitted during periods 1-12 of the experiment. The reason is that the buyer's option has been frequently used by our experimental subjects, so relatively few second auctions were actually held during periods 13-24, leaving us, in most cases, with too few data to reliably estimate the second-auction bidding strategies.[11] Since winners and losers of the first auction should generally behave differently in equilibrium, the results for the 2 groups are presented separately.

Let us first describe second-auction bidding behavior for the losers of the first auction. They are presented in Table 5.[12] Unlike the first-auction results, the majority of predictions are now verified by the data: the null hypothesis is accepted 6 out of 8 times. In the Dutch and first-price auctions with increasing demand, we again find the by now familiar pattern, namely that observed bidding in these 2 auction formats is on average in excess of equilibrium bidding. However, the deviations from optimality are not significant here.

In English and second-price auctions, and for each value of k, it is a dominant strategy for the first-auction loser to bid v in the second auction. Thus, like in standard single-unit English and second-price auctions, it is optimal for bidders to reveal their valuation. Because of this theoretical equivalence, it is of interest to compare our results with those obtained in the experimental literature on single-unit English and single-unit second-price auctions (see Coppinger, Smith,

[11]This is especially true for the auction periods with flat or increasing demand.

[12]To economize on space, the figures depicting the second-auction bids are not given here.

and Titus (1980), Cox, Roberson, and Smith (1982), Kagel, Harstad, and Levin (1987), and Kagel and Levin (1993)).[13] It should be stressed however that bidding in our experiment and bidding in the single-unit experiments took place in slightly different contexts. First because our experimental subjects were more informed about their opponents (when subjects in our experiment submitted their second-auction bid, they knew the first-auction price p_1) than subjects in the single-unit experiments. And also because, under decreasing and increasing demand, bidders in the second auction are no longer symmetric as in the single-unit experiments. Therefore, any differences between the results obtained in the single-unit auction literature and our's can be attributed to these contextual differences.

For $k = \frac{1}{2}$ and $k = 1$ we find the same results as in the experimental single-unit literature: in English and second-price auctions bidders play the dominant strategy as predicted by the theory. For $k = 2$ we find that bidding in English auctions is significantly above value, contradicting both the theory and the results from the single-unit experiments. Bidding in the second-price auctions is also significantly above value, which is contradictory to theory but coherent with the single-unit experiments. Kagel, Harstad, and Levin (1987) explain over-bidding in their single-unit second-price auctions by arguing that bidding above value does not necessarily entail losses and increases the probability of winning, so that subjects can have the illusion that such a strategy increases expected profits. It is unlikely that this explanation also holds in our experiment since it is incompatible with the fact that for second-price auctions with $k = \frac{1}{2}$ or $k = 1$ we *do not* observe over-bidding. A more plausible explanation for overbidding in English and second-price auctions is given in section 4.5.

Next we describe second-auction bidding behavior for the winners of the first auction. They can be found in Table 6. Observed behavior is again in line with theory in the majority of cases: the null is accepted 5 out of 7 times. In English and second-price auctions, and for all values of k, the dominant strategy for the first-auction winner is to reveal his/her value for the second unit, i.e. $2v$. In all English and second-price auctions, observed behavior is in line with theory and the single-unit literature (except the English auction with $k = 1$, but here there are just 2 observations, and the result is probably not very reliable; for $k = 2$ there are no observations at all for the English auction, so theory can not be tested). In the first-price auction with increasing demand, the data are in line with theory at the 1% level, but in the Dutch auction observed bidding is below equilibrium bidding.

[13]This literature shows that subjects bid according to equilibrium behavior in single-unit English auctions. Regarding the single-unit second-price auctions, Coppinger, Smith, and Titus (1980) and Cox, Roberson and Smith (1982) find that average bidding is below (but not always significantly) value; Kagel, Harstad, and Levin (1987) and Kagel and Levin (1993) find, however, that the subjects in their study bid significantly above value. They point out that a likely explanation for these conflicting findings is that, unlike their experiments (and our's!), the designs of Coppinger, Smith, and Titus (1980) and Cox, Roberson and Smith (1982) *did not allow* subjects to bid in excess of their valuation.

Table 6: Second auction (for winner of first auction) without buyer's option

Auction	#obs.	Estim. (Std. Err.)	R^2	Prediction	Accepted
				$k = 0.5$	
First-price	32	0.41 (0.01)	0.98	$\emptyset$	$\emptyset$
Second-price	32	0.53 (0.02)	0.94	0.5	Yes
Dutch	26	0.34 (0.02)	0.95	$\emptyset$	$\emptyset$
English	14	0.54 (0.03)	0.96	0.5	Yes
				$k = 1$	
First-price	32	0.61 (0.02)	0.97	$\emptyset$	$\emptyset$
Second-price	32	1.00 (0.01)	0.99	1	Yes
Dutch	29	0.57 (0.02)	0.96	$\emptyset$	$\emptyset$
English	2	0.98 (0.00)	0.99	1	No
				$k = 2$	
First-price	32	1.05 (0.02)	0.99	1	Yes*
Second-price	32	2.34 (0.23)	0.77	2	Yes
Dutch	39	0.93 (0.03)	0.97	1	No
English	0	- (-)	-	2	$\emptyset$

4.4 Efficiency, price patterns, and revenue comparisons

We start this subsection by comparing observed revenues with their theoretical counterparts. Results are given separately for auctions without a buyer's option (Table 7) and auctions with a buyer's option (Table 8). The third column in these tables gives the revenues as predicted by the theory. These predictions follow from Proposition 1. In the absence of a buyer's option, and for each value of k, the 4 auction institutions are equivalent in terms of the expected revenue they generate. When the buyer's option is available, there is again revenue-equivalence when $k = 1$ and $k = 2$. However, when $k = \frac{1}{2}$, the first-price and Dutch auctions generate more revenue than the English and second-price auctions. Comparing Table 7 and Table 8, it can be seen that in theory the buyer's option has no effect on expected revenue. The only exceptions are the English and second-price auctions with decreasing demand. In these cases the buyer's option increases expected revenue.

To test the revenue predictions, we define for each couple and for each period the seller's revenue $REV = p_1 + p_2$. For each value of k and each auction mechanism the empirical average of REV is calculated, and using a T-test we test the hypothesis that REV has a mean equal to the predicted revenue. For instance, in Table 7, the average revenue in English auctions without a buyer's option when $k = 2$ (the average is thus calculated over 10 couples $\times 4$ periods $=$ 40 observations) equals 40.95 (standard error equal to 5.45), and the hypothesis that the mean of REV equals the predicted value 50.00 is accepted at the 5% level.

As Table 7 and Table 8 show, the results for all the English auctions are

Table 7: Seller's revenue without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	31.41 (2.00)	$\emptyset$	$\emptyset$
Second-price	27.52 (2.45)	20.83	No
Dutch	29.81 (1.75)	$\emptyset$	$\emptyset$
English	20.12 (2.46)	20.83	Yes
		$k = 1$	
First-price	46.46 (2.80)	$\emptyset$	$\emptyset$
Second-price	33.22 (3.75)	33.33	Yes
Dutch	37.82 (1.74)	$\emptyset$	$\emptyset$
English	31.10 (3.33)	33.33	Yes
		$k = 2$	
First-price	70.91 (4.79)	50.00	No
Second-price	38.91 (4.13)	50.00	No
Dutch	64.43 (3.81)	50.00	No
English	40.95 (5.45)	50.00	Yes

Table 8: Seller's revenue with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	33.98 (2.09)	25.00	No
Second-price	29.15 (3.14)	23.65	Yes
Dutch	27.07 (1.93)	25.00	Yes
English	24.56 (2.70)	23.65	Yes
		$k = 1$	
First-price	39.85 (2.69)	33.33	Yes*
Second-price	37.48 (4.93)	33.33	Yes
Dutch	37.86 (2.51)	33.33	Yes
English	30.27 (3.69)	33.33	Yes
		$k = 2$	
First-price	59.49 (4.48)	50.00	Yes*
Second-price	45.49 (5.94)	50.00	Yes
Dutch	63.87 (2.63)	50.00	No
English	39.84 (4.19)	50.00	Yes*

in line with the theoretical predictions. Concerning the Dutch, first-price and second-price auctions, the null is generally accepted when the buyer's option is available, but rejected when it is not. In most cases where the null hypothesis is rejected, mean revenue is significantly above the predicted revenue. These deviations from theory are sometimes considerable. For instance, when a buyer's option is not proposed by the auctioneer, a first-price auction ($k = 2$) generates almost FFr21 more per period than predicted by the theory. Table 7 and Table 8 also indicate that generally the first-price auction generates the highest revenue, followed by the Dutch auction, then the second-price auction, and lastly the English auction. Note that our revenue-ranking of auction formats is exactly identical to the ordering found by Cox, Roberson and Smith (1982) in their experimental study on one-unit auctions.[14] Two-sample T-tests on the equality of mean revenues (not reported in the Tables) suggest that the difference between respectively the Dutch and first-price auctions and the first-price and second-price auctions are significant at the 5% level, but the difference between the second-price and English auctions is generally not significant. Two-sample T-tests also suggest that the buyer's option significantly decreases (resp. increases) revenue in first-price (resp. second-price) auctions; the buyer's option does not significantly affect expected revenue in Dutch or English auctions.

Table 9 and Table 10 report for each value of k and each auction mechanism the mean and standard deviation of the difference in prices $p_2 - p_1$. The Tables also indicate the theoretical predictions on the expected value of $p_2 - p_1$ and whether these predictions are rejected by the data or not. The predicted price variations follow immediately from Proposition 1.

As shown in Table 10, the winning price p_1 is expected to be equal to the winning price p_2 when the buyer's option is available. The only exceptions are the second-price and English auctions for $k = \frac{1}{2}$ and $k = 2$ where on average the sequence of prices is expected to be declining. Note that expected price variations in second-price and English auctions for $k = 2$ are almost 19 times larger compared to the predicted variations for $k = \frac{1}{2}$. Note also that in the case of first-price and Dutch auctions with buyer's option the prediction of constant prices is not surprising since, according to Proposition 1, winners of the first auction should always execute their buyer's option. Table 9 shows that in the absence of a buyer's option the theoretical predictions vary considerably with the auction format and the value of k. The sequence of prices in second-price and English auctions is expected to be increasing when $k = \frac{1}{2}$, and constant when $k = 1$ (no predictions for first-price and Dutch auctions for these values of k). It is quite striking that for $k = 2$ the predicted patterns in the first-price and Dutch auctions are completely opposite to those of the second-price and English auctions: in the former two auction formats the theory predicts a price increase of FFr16.67, while in the latter two auction types a price decline of FFr16.67 is expected.

Table 9 and Table 10 show that for $k = 1$ the results are in line with the

[14] Cox et al. experimentally study first-price, second-price, and Dutch auctions, but not the English auctions.

Table 9: Price variation ($p_2 - p_1$) without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	-1.09 (1.17)	$\emptyset$	$\emptyset$
Second-price	-1.18 (1.28)	4.17	No
Dutch	-3.53 (0.81)	$\emptyset$	$\emptyset$
English	0.57 (0.75)	4.17	No
		$k = 1$	
First-price	0.99 (1.49)	$\emptyset$	$\emptyset$
Second-price	0.75 (0.83)	0.00	Yes
Dutch	1.40 (0.43)	$\emptyset$	$\emptyset$
English	-0.39 (0.49)	0.00	Yes
		$k = 2$	
First-price	2.02 (2.24)	16.67	No
Second-price	0.41 (1.28)	-16.67	No
Dutch	1.95 (1.03)	16.67	No
English	0.11 (1.50)	-16.67	No

Table 10: Price variation ($p_2 - p_1$) with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	-3.74 (1.00)	0.00	No
Second-price	-5.29 (1.11)	-0.88	No
Dutch	-3.31 (0.64)	0.00	No
English	-1.00 (0.45)	-0.88	Yes
		$k = 1$	
First-price	-0.18 (0.14)	0.00	Yes
Second-price	-0.07 (0.13)	0.00	Yes
Dutch	-0.94 (0.40)	0.00	Yes*
English	-0.72 (0.77)	0.00	Yes
		$k = 2$	
First-price	-0.71 (0.34)	0.00	Yes*
Second-price	-0.30 (0.89)	-16.67	No
Dutch	-0.07 (0.07)	0.00	Yes
English	-3.77 (1.73)	-16.67	No

theoretical predictions for all auction institutions, with and without buyer's option: when demand is flat, observed price differences are indeed not significantly different from 0. For $k = \frac{1}{2}$ and $k = 2$ the results are somewhat less satisfactory. When the buyer's option is not available, theory predicts (strong) decreasing or increasing price patterns, but the hypothesis that prices remain constant can never be rejected; when the buyer's option is available, the observed price patterns are in line with theory for English auctions with decreasing demand, and Dutch and first-price auctions with increasing demand.

Table 9 and Table 10 show that observed price sequences are mostly constant when the buyer's option is not available, but significantly decreasing when it is available. These findings are compatible with the field-data studies mentioned in the introduction: they are in support of Van den Berg, Van Ours, and Pradhan (1999), who think that the buyer's option is responsable for the price declines in their Dutch auctions of flowers; and they are in line with the studies on sequential English auctions of wine at Christie's, Drouot and Sotheby's, (see Ashenfelter (1989), Ginsburgh (1998), and Février et al. (2001)), where successive prices are generally found to be declining.

In the last part of this subsection we study auction efficiency. The results can be found in Table 11 and Table 12. For each k and auction mechanism, the first column reports the mean and standard deviation of the relative efficiency $RE = \frac{1}{2}(RE_1 + RE_2)$, where RE_j is the value that the j-th unit winner places on unit j, divided by the maximum of this value and his/her rival's value. For example, if bidder 1 wins the first unit, and bidder 2 the second unit, $RE = \frac{1}{2}(\frac{v_1}{\max(v_1,v_2)} + \frac{v_2}{\max(kv_1,v_2)})$. We also report the predicted values of RE (these predictions follow from Proposition 1), and whether the predictions are accepted or not in the data.

As Table 11 and Table 12 show, all auction institutions are, in theory, efficient mechanisms. The only exceptions are the auctions with buyer's option and decreasing demand. The auction institutions are slightly inefficient in these cases since the buyer's option allows the first-auction winner to buy the second unit while having a lower valuation than his opponent. Actual efficiency is generally remarkably close to predicted efficiency, and theory is accepted in most cases. In spite of the high degree of out-of-equilibrium behavior observed in the data, the 4 auction institutions are highly efficient in our experiments.

4.5 Understanding deviations from optimal bidding behavior

The purpose of this subsection is to understand and interpret the deviations from optimal behavior described in sections 4.1-4.3. Depending on the type of auction mechanism, we find different explanations for the deviations. In the Dutch and first-price auctions, observed bidding behavior turns out to be compatible with risk-averse Nash equilibrium theory. As in single-unit auction experiments (see Kagel (1995) for a survey), the deviations from theory can thus be attributed to risk aversion among experimental subjects. Risk aversion is modeled as in

Table 11: Relative efficiency without buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	0.93 (0.02)	$\emptyset$	$\emptyset$
Second-price	0.98 (0.01)	1.00	Yes
Dutch	0.96 (0.01)	$\emptyset$	$\emptyset$
English	0.98 (0.01)	1.00	Yes*
		$k = 1$	
First-price	0.95 (0.02)	$\emptyset$	$\emptyset$
Second-price	0.99 (0.01)	1.00	Yes
Dutch	0.94 (0.01)	$\emptyset$	$\emptyset$
English	0.99 (0.01)	1.00	Yes
		$k = 2$	
First-price	0.97 (0.02)	1.00	Yes
Second-price	1.00 (0.00)	1.00	Yes
Dutch	0.96 (0.01)	1.00	Yes*
English	1.00 (0.00)	1.00	Yes

Table 12: Relative efficiency with a buyer's option

Auction	Avg. (Std. Err.)	Prediction	Accepted
		$k = 0.5$	
First-price	0.94 (0.03)	0.92	Yes
Second-price	0.98 (0.01)	0.98	Yes
Dutch	0.92 (0.03)	0.92	Yes
English	0.96 (0.02)	0.98	Yes
		$k = 1$	
First-price	0.98 (0.01)	1.00	Yes*
Second-price	0.99 (0.01)	1.00	Yes
Dutch	0.97 (0.01)	1.00	No
English	1.00 (0.00)	1.00	Yes
		$k = 2$	
First-price	0.99 (0.01)	1.00	Yes
Second-price	0.99 (0.01)	1.00	Yes
Dutch	1.00 (0.00)	1.00	Yes
English	1.00 (0.00)	1.00	Yes

the single-unit experiments (see for example Cox, Roberson and Smith (1982) and Kagel, Harstad, and Levin (1987)).[15] All agents are thus assumed to have the same concave utility function $u(\cdot)$ over money income. Furthermore, the utility function is assumed to be of the form $u(x) = x^{\alpha}$ with $\alpha \in [0, 1)$. This is a constant relative risk-aversion (CRRA) model. In analyzing the deviations, it is assumed that the participants in our experiment have a coefficient of relative risk aversion α equal to 0.6.[16]

In the English and second-price auctions, most of the Nash equilibrium strategies (the only exceptions are the auctions with buyer's option and decreasing demand) stated in Proposition 1 are robust to the form of risk aversion that we consider; put in other words, risk-neutral Nash equilibrium bidding behavior in English and second-price auctions remains optimal under the CRRA assumption. The implication of this invariance property is that something other than risk aversion is responsible for the observed deviations in English and second-price auctions. As will be seen below, the deviations from theory are a consequence of either myopic behavior or punitive behavior. By myopic behavior is meant that agents' bidding behavior in the first auction is identical to bidding behavior in a single-unit auction. Although agents fully understand that 2 units are on sale instead of 1 unit, their first-auction behavior does not reflect this crucial difference. By punitive behavior is meant that first-auction losers attempt to harm their opponents by bidding above their value v in the second auction (the dominant strategy), thereby reducing the second-auction profits of their opponents.

4.5.1 Dutch and first-price auctions

In sections 4.1-4.3 we have seen that observed bidding behavior in Dutch and first-price auctions is generally above risk-neutral equilibrium behavior. Furthermore, experimental subjects have used the buyer's option much too rarely when demand is decreasing. In this subsection it is shown that these deviations can be explained once agents are allowed to be risk-averse. We can thus rationalize all observed behavior in terms of risk aversion. We give the risk-averse Nash equilibrium bidding functions in all relevant cases but omit the proofs of their derivations (obtainable from the authors).

In the absence of a buyer's option and when demand is decreasing, there again does not exist a symmetric pure Nash equilibrium. When the option is available an equilibrium does exist, and the first-auction optimal bidding function under CRRA is $b_1(v, \alpha) = \frac{3}{4(1+\alpha)} v$ (note that when $\alpha = 1$ we find

[15] See Harrison (1989) and the subsequent debate in the *American Economic Review*, Vol. 82 No 5, pp. 1374-1443, for an alternative explanation for the overbidding phenomenon. In particular, see Cox, Smith, and Walker (1992) (and the references herein) for an extensive discussion of risk-aversion models.

[16] This value is obtained in a somewhat ad hoc way by minimizing, over α, the sum of squared deviations between observed bids and optimal bids under risk aversion. Our estimate is a bit higher than the one found in Kagel, Harstad, and Levin (1987). They report an average estimate of α equal to 0.49, suggesting that students in our experiment were slightly less risk-averse than their North American counterparts.

the risk-neutral bidding function $b_1(v) = \frac{3}{8}v$ given in point 7 of Proposition 1). Taking $\alpha = 0.6$, we have $b_1(v, \alpha) = 0.47v$, and performing the same tests as in section 4.1 (i.e. we test whether $\beta = 0.47$, etc.), we accept, at the 5% level, for both the Dutch and first-price auctions, the hypothesis that observed bidding is in accordance with risk-averse bidding behavior.

The question that is still unanswered is whether risk-aversion provides a rationale for the fact that the option is exercised too rarely under decreasing demand? Given that under risk aversion it is still optimal to always use the buyer's option, the answer is no. Note however that the optimal bidding function under risk aversion $b_1(v, \alpha) = 0.47v$ is very close to the threshold curve $0.50v$ above which it is not profitable to use the buyer's option. The fact that risk-neutral equilibrium behavior is accepted of course only means that *average* bidding is according to the function $b_1(v, \alpha)$, and does obviously not exclude that part of the observations are located above the nearby located threshold curve. In our data all persons with bids above the threshold curve did indeed not use the option (and inversely, those with bids under the threshold curve did exercise the option), explaining why the observed frequency of using the option is lower than predicted by optimal behavior under CRRA.

Let us next consider consider the auctions under flat demand. Again, an equilibrium only exists when the buyer's option is available, and the first-auction optimal bidding function under CRRA is now $b_1(v, \alpha) = \frac{1}{(1+\alpha)}v$. Given $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.625v$, and again risk-averse equilibrium theory is accepted for both the Dutch and first-price auction. Under CRRA it remains optimal to always use the option, so regarding the use of the buyer's option, the data remain in line with theory. Note that the problem that was mentioned above does not play a role here since the threshold curve here is v, i.e. well above the optimal function.

Finally consider the auctions under increasing demand. As in the case with risk-neutrality, equilibria exist for the auctions with and without the buyer's option. For the auctions without buyer's option, we have $b_1(v, \alpha) = \frac{2-\alpha}{(1+\alpha)}v$, and taking $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.875v$. The hypothesis that bidding behavior is according to equilibrium behavior under CRRA is only accepted for the Dutch auction. The second-auction strategies are not affected by risk-aversion, so the data remain coherent with the theory except for first-auction winners in the Dutch auction (see Table 5 and Table 6). When the buyer's option is available, we obtain $b_1(v, \alpha) = \frac{3}{2(1+\alpha)}v$, and taking $\alpha = 0.6$ we get $b_1(v, \alpha) = 0.938v$, implying that equilibrium behavior under CRRA is once again accepted for both auctions institutions. The optimal use of the buyer's option is not affected by risk-aversion, so observed frequencies remain coherent with predicted frequencies.

4.5.2 English and second-price auctions

In sections 4.1-4.3 it was shown that subjects' behavior is quite well in line with the predictions of Proposition 1 when the demand function is flat. Important deviations are however observed when the demand function is decreasing or increasing: in the first auctions the degree of bid shading (resp. over-bidding) observed in the data is clearly too small under decreasing (resp. increasing) demand; in the second auctions with increasing demand, first-auction losers are found to be bidding significantly above the dominant strategy which consists in revealing the value v; the final deviation that needs our attention concerns the higher-than-optimal use of the buyer's option when demand is increasing. As in the previous subsection, we discuss each of these deviations separately.

First consider the deviations under decreasing demand. In analyzing the first-auctions without buyer's option, it is helpful to look again at Figures 2 and 4. In both figures there is evidence of there being 2 groups of bids: one group of bids closely scattered around the optimal bidding line $b_1(v) = \frac{1}{2}v$, and another group of bids concentrated around the line v. Apparently part of the subjects play the optimal strategy, while others bid in a myopic way.[17] This can be checked more formally by running the following switching regression model

$$b_{1it} = \beta_1 v_{it} + \varepsilon_{1it} \text{ with probability } \pi$$
$$b_{1it} = \beta_2 v_{it} + \varepsilon_{2it} \text{ with probability 1-}\pi.$$

The estimates (standard error) of β_1 and β_2 are 1.04 (0.04) and 0.52 (0.06) for the second-price auction, and 0.98 (0.01) and 0.49 (0.04) for the English auction. The estimate (standard error) of the probability π is 0.62 (0.07) for the second-price auction, and 0.43 (0.09) for the English auction. Our switching regression estimates thus confirm that there are 2 groups of agents, one made up of rational bidders and the other of myopic bidders, and that the proportion of myopic agents is quite important in the data. The fact that the prevalence of myopic agents is so high explains the rejection of the theory in section 4.1.

The only strategies that are sensitive to the introduction of CRRA are the English and second-price auctions with buyer's option (still $k = \frac{1}{2}$). Under CRRA, the optimal bid function is the solution of $-(v - b_1(v,\alpha))^\alpha + (\frac{v}{2})^\alpha = 2(v - b_1(v,\alpha))^\alpha b_1'(v,\alpha)$. As in section 4.1, the solution can be precisely approximated by a second-order polynomial in v. Taking $\alpha = 0.6$, and performing the same tests as in section 4.1, the results are in support of risk-averse Nash equilibrium behavior, at the 5% level for the second-price auction, and at the 1% level for the English auction. CRRA does not affect the optimal use of the buyer's option, so observed frequencies remain in line with predictions.

Next consider the deviations under increasing demand. To understand the deviations in the auctions without buyer's option, we first look at the second-auction results. As Tables 5 and 6 show, winners of the first auction bid their valuation for the second unit, as predicted by theory, but losers of the first auction bid significantly above the dominant strategy v. As pointed out in

[17] Subjects are quite consistent in their behavior over the 4 periods for which $k = \frac{1}{2}$ and $o = N$: most optimal bidders are optimal in all 4 periods, and similarly, most deviators persistently deviate in all 4 periods.

section 4.3, the latter result contrasts with the experimental literature on single-unit auctions. However, unlike single-unit experiments, first-auction losers have information about the value of their opponent (the contextual difference mentioned in section 4.3), which allows them, without taking any personal risk, to punish their competitor by bidding above the dominant strategy. This punitive behavior in the second auction can also explain the deviation observed in the first auction. It can be formally shown that, in anticipation of punitive behavior in the second auction, players should bid somewhere between v and $2v$ (depending on the degree of punitive behavior among subjects) in the first auction to compensate for the smaller gain in the second auction. As Figures 10 and 12 show, this line of reasoning is well supported by the data, since practically all first-auction bids are indeed between v and $2v$ (with some exceptions for the second-price auction without buyer's auction).

The deviations that are observed in the auctions with buyer's option can be explained in the same manner. In the first auctions, player's anticipate future punitive behavior by bidding between v and $2v$ (Figures 22 and 24 show that this is indeed the case). Furthermore, under punitive behavior, it can be shown that it is optimal for first-auction winners to always exercise the buyer's option, justifying why the subjects in our experiment do not wait but buy instead.

5 Conclusion

This paper experimentally studies two-unit sequential auctions with and without the buyer's option. The 2 identical units are sold to 2 potential buyers. Each buyer desires both units, and their demand function is either decreasing, flat, or increasing. The four best known auction mechanisms are considered: Dutch, English, first-price and second-price auctions. Experimental papers on sequential auctions are still very rare and none analyzes the buyer's option despite its practical importance.

Observed bidding behavior in English and second-price auctions is closer to risk-neutral Nash equilibrium bidding in the second auction than in the first auction. This is not surprising since the first-auction strategies are more subtle and less transparent than the second-auction strategies. In the first auction, buyers face a complex situation because they need to anticipate that the first-auction winning price is revealed, that a second unit is going to be sold, and that the winner has the right to exercise the buyer's option (if the option is available). The inexperienced subjects in our experiment have nonetheless understood the basic strategic effects called for by optimal bidding behavior. Subjects have indeed understood that under flat demand it is optimal to bid their valuation in the first auction; under decreasing (resp. increasing) demand, subjects have understood that optimal behavior requires bid shading (resp. over-bidding), but the extent to which they did this was too modest. In the second auction, buyers face a relatively simple situation. As in single-unit English and second-price auctions, it is a dominant strategy for bidders to reveal their valuation. Apart from the first-auction losers under increasing demand, who bid in excess of their valuation, we find, as in the experimental literature on single-unit auctions, that our subjects play according to the dominant strategy. Bidders in our experiment have exercised the buyer's option quite adequately under decreasing and flat demand, but not when demand is increasing. In the latter case the bidders have made too much use of the option.

For the Dutch and first-price auctions without buyer's option, there only exists an equilibrium when the demand function is increasing. Compared to the English and second-price auctions, there are therefore less theoretical predictions that can be tested. As in the English and second-price auctions, risk-neutral Nash equilibrium behavior organizes the data better in the second auction than in the first auction. Practically all deviations that we observe share a common feature which is that bidding behavior is above optimal bidding behavior. This is a phenomenon that is also observed in experiments on single-unit first-price and Dutch auctions. Bidders in our experiment have exercised the buyer's option very often under flat and increasing demand, as theory predicts them to do, but too little when demand is decreasing.

Depending on the type of auction mechanism, we find different explanations for the deviations. In the Dutch and first-price auctions, observed bidding behavior turns out to be compatible with risk-averse Nash equilibrium theory. As in the single-unit auction experiments, the deviations from theory can thus be attributed to risk aversion among experimental subjects. In the English and second-price auctions, most of the Nash equilibrium strategies are robust to the form of risk aversion that we consider. For these auction institutions, the deviations from theory can be explained by either myopic or punitive behavior.

The paper also looks at the revenue and the price patterns in the different auction mechanisms with and without buyer's option. It is quite remarkable that the revenue ranking of the 4 auction institutions is the same as in single-unit experiments. We also find that the buyer's option decreases (resp. increases) revenue in first-price (resp. second-price) auctions, but that there is no significant effect in the clock auctions. Successive prices are found to be declining in the auctions with buyer's option, but are constant when the option cannot be used. This result, in conjunction with the fact that subjects in our experiment are found to be risk-averse, suggests that the buyer's option, and not risk-aversion, is responsable for the declining price anomaly.

In future work we plan to study the effect of an increase in the number of buyers at auction. We also plan to investigate the role of the buyers' option in the common value paradigm.

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Appendix

A Proof of proposition 1

A.1 $a \in \{E, S\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1, 2\right\}$

The second auction strategies are obtained by the standard dominated strategies argument. Therefore, both the loser and the winner of the first auction bid their valuation:

$$b_2^l(v, p_1) = v \text{ and } b_2^w(v, p_1) = kv.$$

To derive the first auction equilibrium strategies, we have to distinguish the English auction from the second-price auction as the available information is not the same in these two auction institutions.

A.1.1 $a = E$ and $k = \frac{1}{2}, 1$

See Donald, Paarsch, and Robert (1997).

A.1.2 $a = E$ and $k = 2$

Let $b_1(v)$ be the first auction equilibrium strategy and v_1 the value of player 1. Suppose the clock has reached p (close to $b_1(v_1)$) and player 1 has to decide to continue or to stop bidding. Let $G(\varepsilon, p)$ denote the expected total gain (for the first and second auctions) for player 1 if he decides to continue with bidding until $p + \varepsilon$:

$$G(\varepsilon, p) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w) + 2v_1 - w) \frac{dw}{1 - b_1^{-1}(p)}.$$

The above expression follows because player 1 can only win the first auction if $p \leq b_1(v_2) \leq p + \varepsilon$, with v_2 being the valuation of player 2. If he wins the first auction, he also wins the second auction because, since p is close to $b_1(v_1)$, we have that $2v_1$ is larger than v_2. On the contrary, if he loses the first auction he also loses the second one. Note that the density in the integral is the conditional density of v_2 given $v_2 \geq b_1^{-1}(p)$. Derivation with respect to ε gives:

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon, p) = \left(b_1^{-1}(p+\varepsilon)\right)'(p+\varepsilon) \frac{v_1 - (p+\varepsilon) + 2v_1 - b_1^{-1}(p+\varepsilon)}{1 - b_1^{-1}(p)}.$$

The equilibrium condition is:

$$\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = b_1(v_1)) = 0,$$

which leads to

$$b_1(v_1) = 2v_1.$$

To prove that it is indeed a Nash equilibrium, assume that player 2 follows the strategy $b_1(v_2) = 2v_2$ and assume that player 1 deviates from this strategy and stops at $p < v_1$. In that case he loses the first auction and can only win the second if $v_2 \in \left[\frac{p}{2}, \frac{v_1}{2}\right]$ which would lead to a gain of $v_1 - 2v_2$. But in that case, by bidding until $p = v_1$ player 1 would win both the first and the second auction (because $v_2 < v_1$), which leads to a larger profit of $v_1 - 2v_2 + 2v_1 - v_2$. Assume, now, that player 1 stops at p with $v_1 < p < 2v_1$. In that case losing the first auction also means losing the second because $v_1 < p < 2v_2$ (here in the second auction player 2 bids $2v_2$), therefore it is optimal to bid until $2v_1$. Finally, a deviation $p > 2v_1$ is weakly dominated: it does not improve the gain when $v_2 < v_1$ while it implies a loss when $v_1 < v_2 < \frac{p}{2}$ as the revenue of player 1 is then $3v_1 - 3v_2 < 0$.

A.1.3 $a = S$ and $k = \frac{1}{2}, 1$

See Black and De Meza (1992).

A.1.4 $a = S$ and $k = 2$

In order to characterize the equilibrium strategy $b_1(.)$, assume that player 1 deviates from $b_1(v_1)$ by bidding $b_1(x)$, with x close to v_1. If he loses the first auction while bidding $b_1(x)$, he is sure to lose the second auction as well. On the contrary, if he wins the first auction he is also sure to win the second auction. That is, the expected total gain of player 1 playing $b_1(x)$ is:

$$G(x) = \int_0^x [v_1 - b_1(w) + 2v_1 - w]\, dw.$$

In equilibrium such a deviation cannot be profitable which means that:

$$G'(x = v_1) = 0,$$

which leads to

$$b_1(v_1) = 2v_1.$$

To prove that it is a Nash equilibrium, assume that player 2 bids $2v_2$. It is then obvious that a bid equal to $2x$, $x < v_1$, gives player 1 a lower expected gain than a bid equal to $2v_1$ as it does not increase the gain when player 1 wins both auctions but it reduces the probability of winning. Next, a bid equal to $2x$, $v_1 < x$, also reduces the expected gain of player 1 because when player 1 wins the first auction with $2x$ but not with $2v_1$ he has a negative total gain.

A.2 $a \in \{D, F\}$, $o = N$, and $k \in \left\{\frac{1}{2}, 1\right\}$

The non-existence of a Nash equilibrium with symmetrical pure strategies in the first auction is proved in Février (2000).

A.3 $a \in \{D, F\}$, $o = N$, and $k = 2$

We first study the second auction assuming that the players bid according to $b_1(.)$ in the first auction. Suppose that player 1 with valuation v_1 won the first auction and let v_2 denote the valuation of player 2. Therefore the value of $b_1(v_1)$ is revealed before the second auction starts and both players know that $v_2 < v_1 < 2v_1$. In equilibrium the second player knows that he cannot win the second auction and his (weakly dominant) strategy is to bid v_2 in the second auction.

By bidding $x \leq v_1$ the expected gain of player 1 in the second auction is $\text{Prob}(x > v_2 | v_2 < v_1)(2v_1 - x) = \min\left\{\frac{x}{v_1}\ ;\ 1\right\}(2v_1 - x)$, which is maximized for $x = v_1$. Of course, it is not profitable to bid more than v_1. Consequently, both players bid their first-unit valuation in the second auction, i.e. $b_2^l(v, p_1) = b_2^w(v, p_1) = v$.

We now study the first auction. Suppose player 2 bids $b_1(v_2)$ in the first auction and player 1 bids $b_1(x) > b_1(v_1)$. If he wins the first auction he learns that $v_2 < x$, and he maximizes over y (in the second auction) $\frac{y(2v_1 - y)}{x}$. On the other hand, if he loses the first auction then $v_2 > x > v_1$ and he loses the second auction as well. Therefore, his expected total gain is $x\left(v_1 - b_1(x) + \max_y \min\left\{\frac{y}{x}\ ;\ 1\right\}(2v_1 - y)\right)$. This expected gain must be maximized in equilibrium for $x = v_1$. The first order condition leads to $b_1(v_1) \geq \frac{v_1}{2}$.

Suppose, now, that player 1 bids $b_1(x) < b_1(v_1)$. If he wins the first auction he learns that $v_2 < x < v_1$ and he maximizes his second auction gain $\max_y \min\left\{\frac{y}{x}\ ;\ 1\right\}(2v_1 - y)$ by bidding x. On the other hand, if he loses the first auction he learns the value of v_2. If $v_2 > v_1$, he also loses the second auction. If $v_2 < v_1$, he wins the second auction by bidding just above v_2. Therefore the expected gain is:

$$G(x) = x[v_1 - b_1(x) + 2v_1 - x] + \int_x^{v_1} (v_1 - w)\, dw.$$

The first order condition leads to $b_1(v_1) \leq \frac{v_1}{2}$.

Therefore, the equilibrium first auction strategy is $b_1(v_1) = \frac{v_1}{2}$.

A.4 $a \in \{E, S\}$, $o = Y$, and $k = \frac{1}{2}, 1, 2$

The second auction strategies are obtained by the standard dominated strategies argument. Therefore, each player bids his valuation

$$b_2^l(v, p_1) = v \text{ and } b_2^w(v, p_1) = kv.$$

To derive the first auction equilibrium strategies, the English auction has to be distinguished from the second-price auction as the available information are not the same in the two auction mechanisms.

A.4.1 $a = E$ and $k = \frac{1}{2}$

We start with the buyer's option. If $\frac{v_1}{2} \geq p_1 = b_1(v_2)$, it is profitable to use the option because if he does not execute the option his gain in the second auction is $\max\left\{0, \frac{v_1}{2} - v_2\right\}$, which is lower than $\frac{v_1}{2} - b_1(v_2)$. On the contrary, if $\frac{v_1}{2} < p_1 = b_1(v_2)$, it is clear that the winner must not use the option.

We study now the first auction. As it will become clear later we can restrict ourselves to the search of a first auction equilibrium $b_1(v) \geq \frac{v}{2}$. Suppose the clock has reached p and player 1 has to decide to continue or to stop bidding. It is important to remark that as player 2 is still active at p, his valuation is greater than $b_1^{-1}(p)$.

To derive the equilibrium necessary conditions, we assume that p is close to $b_1(v_1)$. Let $G(\varepsilon, p)$ denote the expected total gain if player 1 decides to continue with bidding until $p + \varepsilon$.

If player 2 withdraws between p and $p + \varepsilon$, player 1 wins the first auction. As we have assumed that $b_1(v_1) \geq \frac{v_1}{2}$, and that p is close to $b_1(v_1)$, it is not profitable to use the buyer's option. Furthermore, player 1 loses the second auction (indeed, his valuation is divided by two, while player 2 valuation remains around v_1). The expected gain in this case is: $\int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)}$

If player 2 remains active at $p + \varepsilon$, player 1 loses the first auction. As seen before, player 2 uses his option if and only if $p + \varepsilon \leq \frac{v_2}{2}$. In case player 2 does not use the option, we have $\frac{v_2}{2} < p + \varepsilon \simeq b_1(v_1) < v_1$ which means that player 1 wins the second auction. The expected gain in this case is: $\int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon),\overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)}$.

Finally

$$G(\varepsilon, p) = \int_{b_1^{-1}(p)}^{b_1^{-1}(p+\varepsilon)} (v_1 - b_1(w)) \frac{dw}{\overline{v} - b_1^{-1}(p)} + \int_{b^{-1}(p+\varepsilon)}^{\min(2(p+\varepsilon),\overline{v})} (v_1 - \frac{w}{2}) \frac{dw}{\overline{v} - b_1^{-1}(p)},$$

The equilibrium condition is $\frac{\partial G}{\partial \varepsilon}(\varepsilon = 0, p = b_1(v_1)) = 0$. Under the assumption that $b_1(v_1) \leq \frac{\overline{v}}{2}$ this leads to:

$$\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) + 2(v_1 - b_1(v_1)) = 0.$$

On the contrary, if $b_1(v_1) \geq \frac{\overline{v}}{2}$ we obtain:

$$\frac{1}{b_1'(v_1)}(\frac{v_1}{2} - b_1(v_1)) = 0.$$

This second differential equation combined with the assumption $b_1(v) \geq \frac{v}{2}$ implies that $b_1(\overline{v}) = \frac{\overline{v}}{2}$. The first differential equation and this terminal condition define a unique bidding function which verifies $b_1(v) \geq \frac{v}{2}$.

To end the proof, it is necessary to show that this function constitutes indeed a Nash equilibrium of the game by checking that there is no profitable deviation which is straightforward.

A.4.2 $a = S$ and $k = \frac{1}{2}, 1$

See Black and De Meza (1992).

A.4.3 $a = S$ and $k = 2$ or $a = E$ and $k = 1,\ 2$

The proof is identical to the proof without buyer's option because it is optimal not to use the buyer's option. Indeed, assume that both players bid $2v$ and that $v_1 > v_2$. Player 1 wins the first auction and the price $p_1 = 2v_2$. If player 1 uses the option he pays the second unit $2v_2$, while if he waits he only have to pays v_2.

A.5 $a \in \{D, F\}$, $o = Y$, and $k \in \frac{1}{2}, 1, 2$

The second auction strategies are obtained from Février (2000) (proposition 4.5). The second auction strategy for the loser of the first auction is

$$b_2^l(v, p_1) = \begin{cases} v & \text{if } v \leq \frac{2k}{1+k}p_1 \\ \frac{kp_1}{1+k}\left(1 - \frac{4kp_1}{(1+k)v}\right) & \text{if } v \geq \frac{2k}{1+k}p_1 \end{cases}$$

The winner of the first auction plays the following strategy: If $p_1 = b(v)$ (that is if he played in the first auction according to the equilibrium strategy but he did not use the option), he plays a mixed strategy, such that he bids x, $x \in \left[\frac{kv}{2}, \frac{k - \frac{k^2}{4}}{v}\right]$, with x having the distribution function

$$F(x) = \frac{1 - k + \frac{k^2}{4}}{1 - \frac{k}{2}} \frac{kv}{2x - kv} \exp\left[\frac{4x - 2\left(2k - \frac{k^2}{2}\right)v}{(2x - kv)(2 - k)}\right].$$

If $p_1 > b(v)$ (that is he played in the first auction a bid above the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{k}{2}v$.

If $p_1 < b(v)$ (that is he played in the first auction a bid below the equilibrium strategy, won the auction and did not use the option) then $b_2^w(v, p_1) = \frac{4k - k^2}{1+k}p_1$.

Consider now the first auction. Assume that player 2 bids $b_1(v_2)$. If player 1 bids $b_1(x)$ and uses his option, then his expected gain is:

$$G(x) = x\left[(1 + k)v - 2b_1(x)\right].$$

The first-order condition is

$$G'(v) = 0 \Leftrightarrow \frac{1 + k}{4}v^2 = (vb_1(v))',$$

which leads to

$$b_1(v) = \frac{1 + k}{4}v.$$

If both players bid according $b_1(v)$ the expected gain of a player with a valuation v is $\frac{1+k}{2}v^2$. See Février (2000) for the proof that given the strategies described above, it is not profitable to deviate in the first auction and to abstain from using the buyer's option.

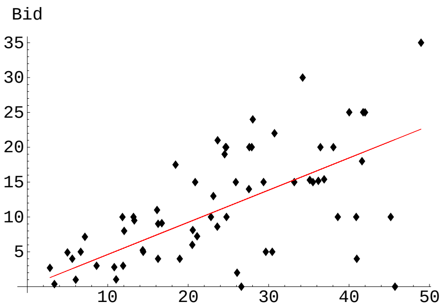

Figure 1: First-price, $k = \frac{1}{2}$, $o = N$

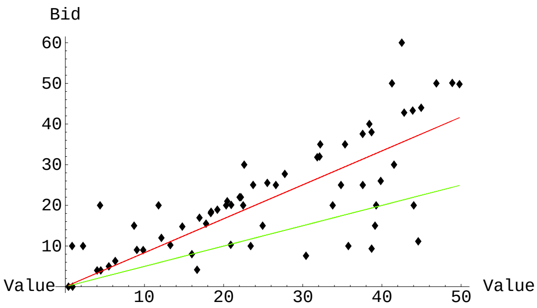

Figure 2: Second-price, $k = \frac{1}{2}$, $o = N$

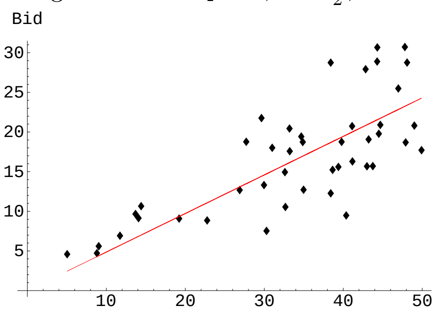

Figure 3: Dutch, $k = \frac{1}{2}$, $o = N$

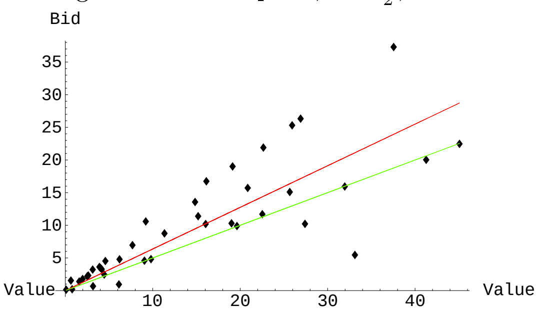

Figure 4: English, $k = \frac{1}{2}$, $o = N$

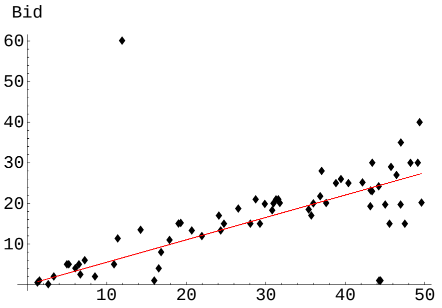

Figure 5: First-price, $k = 1$, $o = N$

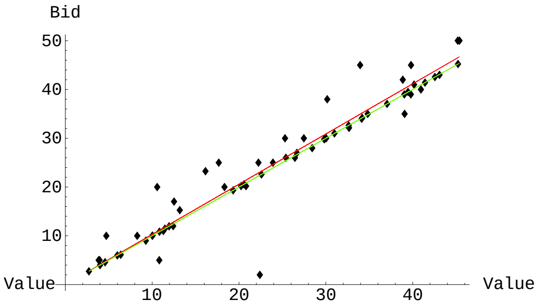

Figure 6: Second-price, $k = 1$, $o = N$

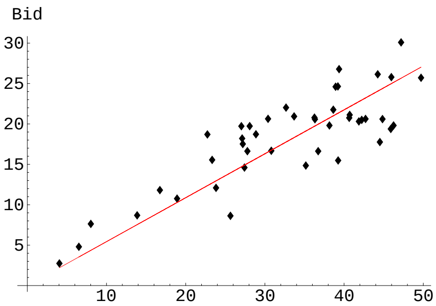

Figure 7: Dutch, $k = 1$, $o = N$

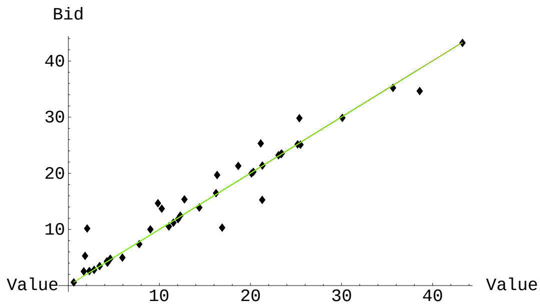

Figure 8: English, $k = 1$, $o = N$

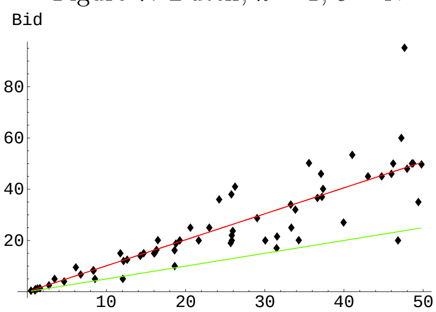

Figure 9: First-price, $k = 2$, $o = N$

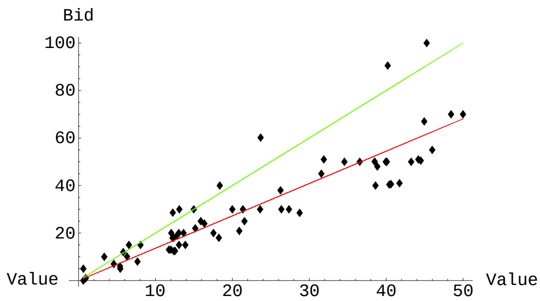

Figure 10: Second-price, $k = 2$, $o = N$

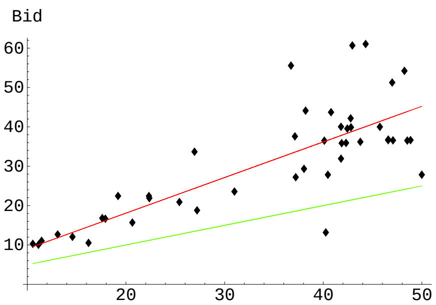

Figure 11: Dutch, $k = 2$, $o = N$

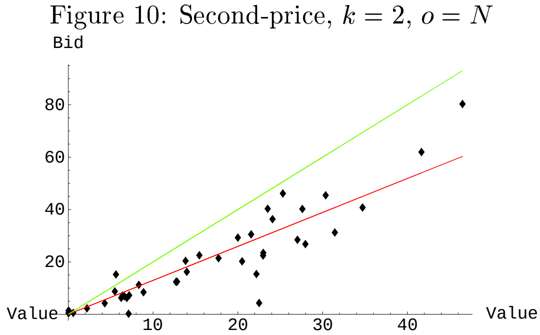

Figure 12: English, $k = 2$, $o = N$

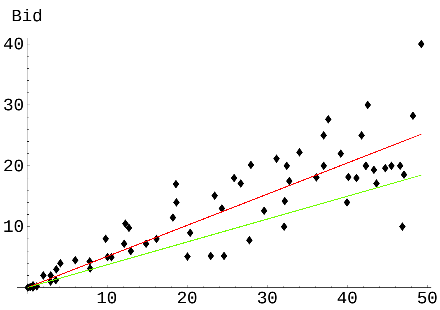

Figure 13: First-price, $k = \frac{1}{2}$, $o = Y$

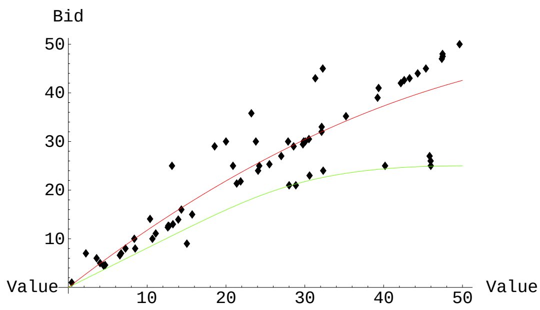

Figure 14: Second-price, $k = \frac{1}{2}$, $o = Y$

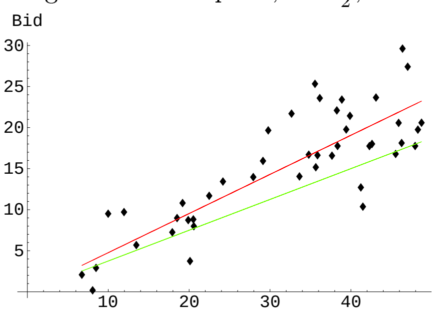

Figure 15: Dutch, $k = \frac{1}{2}$, $o = Y$

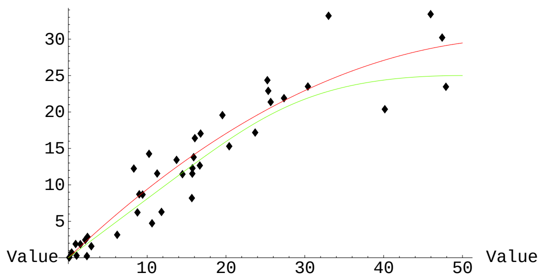

Figure 16: English, $k = \frac{1}{2}$, $o = Y$

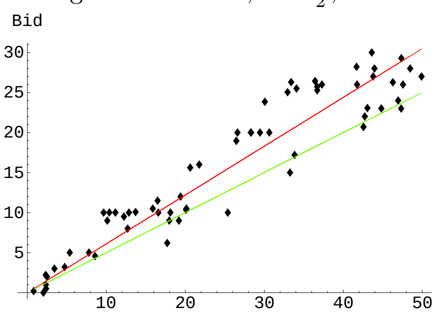

Figure 17: First-price, $k = 1$, $o = Y$

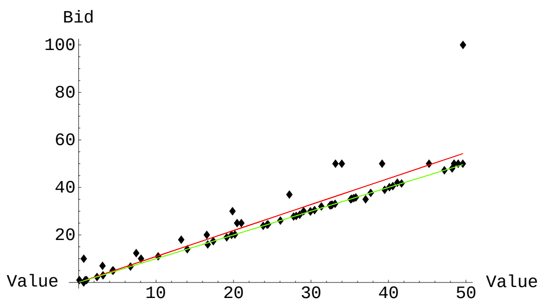

Figure 18: Second-price, $k = 1$, $o = Y$

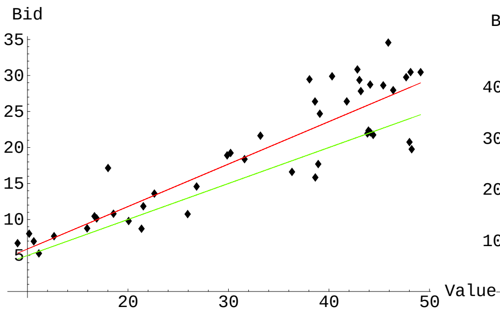

Figure 19: Dutch, $k = 1$, $o = Y$

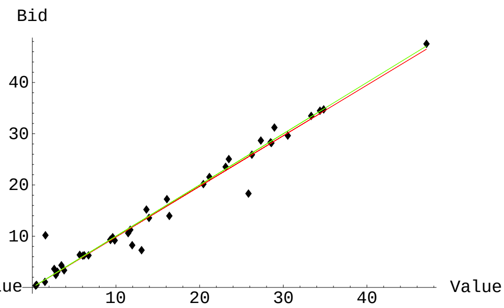

Figure 20: English, $k = 1$, $o = Y$

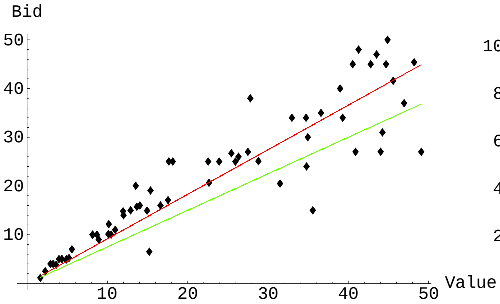

Figure 21: First-price, $k = 2$, $o = Y$

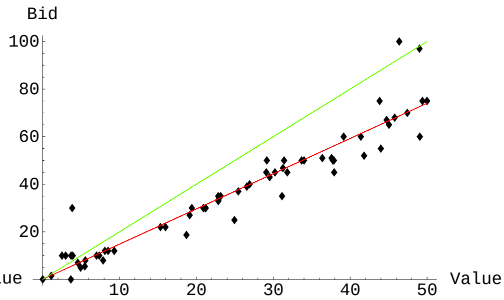

Figure 22: Second-price, $k = 2$, $o = Y$

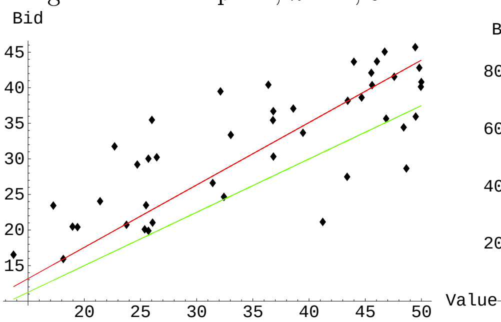

Figure 23: Dutch, $k = 2$, $o = Y$

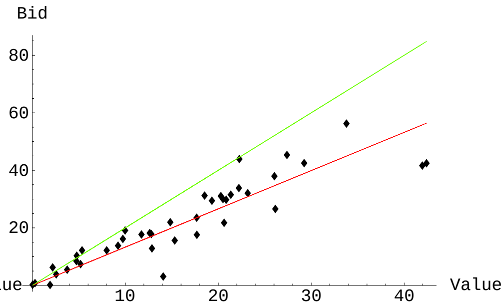

Figure 24: English, $k = 2$, $o = Y$