INSTITUT NATIONAL DE LA STATISTIQUE ET DES ETUDES ECONOMIQUES
Série des Documents de Travail du CREST
(Centre de Recherche en Economie et Statistique)

n° 2001-25

Optimal Incentives for Labor Force Participation

Ph. CHONE[1]
G. LAROQUE[2]

[1] INSEE and CNRS URA 2200.
[2] INSEE and CNRS URA 2200.

Optimal incentives for labor force participation

Philippe Choné and Guy Laroque*

August 2001

Abstract

Optimal taxation is analyzed under a Rawlsian criterion in an economy where the only decision of the agents is to participate, or not, to the labor force. The model allows for heterogeneity both in the agent's productivities and aversions to work. First best optimal schedules involve zero taxation at the margin: the marginal agent who decides to work pockets all of her productivity, while being just compensated for her work aversion. When the planner does not observe work aversion, financial compensation for work is lower than productivity. The theory provides a potential justification to (locally) negative marginal tax rates.

Résumé

Dans une optique Rawlsienne, on analyse la forme optimale de taxation dans une économie où la seule décision des agents est de travailler ou de ne pas prendre un emploi. Les agents diffèrent à la fois par leur productivité et leur goût pour le loisir. En information complète, un prélèvement nul à la marge est optimal : l'agent marginal qui décide de travailler reçoit tout le produit de son travail, ce qui le compense juste pour son effort. Quand le planificateur n'observe pas le goût pour le loisir des agents, la compensation financière est inférieure à ce que l'on trouve en information complète. La théorie peut fournir une justification à l'usage de taux marginaux d'imposition négatifs.

JEL classification numbers : D63, H21.
Keywords : Rawls, optimal taxation, participation.

Please address correspondence to Guy Laroque, INSEE-CREST J360, 15 boulevard Gabriel Péri, 92245 Malakoff Cedex, France.

*INSEE and CNRS URA 2200. We have benefited from the thoughtful comments of Bernard Salanié and from the remarks of the seminar participants at CREST.

Introduction

The equity efficiency dilemma is as old as economics. But it has been the object of renewed attention in the recent years in developed economies, confronted with the widening of the wage distribution, sometimes attributed to a skilled bias technical progress or trade globalization. Policy issues center around the type of benefits that should be given to people that are out of the labor force, and to the disincentives to work that such benefits might create.

Each country has a social minimum or safety net of its own. Some basic income support to all households (sometimes also called negative income tax) is provided in many European countries (the *Revenu Minimum d'Insertion* in France is an example). These programs often involve high marginal tax rates (100% for the RMI) in the range of incomes where the subsidy is phased out. To 'make work pay', some countries have implemented earning subsidies, which involve negative marginal tax rates: the Earned Income Tax Credit in the US and the Working Families Tax Credit are examples of such programs.

A number of descriptive studies are associated with these various policies, undertaking to measure the relevant participation elasticities and to provide a cost benefit analysis of the social minima. But, unfortunately, the normative approach has not been very fruitful. Indeed, the relevant framework of optimal taxation, which goes back to the seminal paper of Mirrlees (1971), seems too far from the tax-benefit systems observed in practice to be a useful guide for policy. When effort depends on financial incentives, at the *intensive* margin, the standard result has a zero marginal tax rate on the rich (which goes contrary to the common idea of equity, and is not observed, even in the US) and a zero marginal tax rate on the very poor, at least when they do work. The marginal tax rate is always non negative, which rules out pushing people to work through an earning subsidy, such as in the EITC.

A number of researchers have worked to bring the theory more in line with the facts. Diamond (1998) and Salanié (1998) show that the 'don't tax the rich' result does not hold when the distribution of wages has a fat enough upper tail. Piketty (1997) studies the optimal taxation program in the intensive framework with a Rawlsian criterion. The interesting work of Saez (2001) uses the available empirical evidence on the shape of the wage distribution and labor supply elasticities to compute optimal tax schedules in an intensive model *à la* Mirrlees. Although the schedules are not too far from what we see in practice, they do always have positive marginal tax rates.

More to the point that we are interested in, at the bottom of the distribution, Saez (2000) and Beaudry and Blackorby (1997) have worked on models of optimal income taxation with an *extensive* margin in order to look for properties of the optimal taxation schemes when agents may choose to participate or to stay out of the labor force. One purpose of Saez (2000) is to see whether one of the features of the EITC and of the WFTC, namely the local negative marginal tax rates which give large incentives to work to the concerned individuals, is compatible with optimal taxation in an extensive setup[1].

In the present work, we focus on the participation decision and work on a model where the only choice variable of the agents is to work or to stay out of the labor force. We further assume that there is a disutility to work, and that the financial compensation that makes an agent indifferent between working and not working,

[1]Besley and Coate (1995) address a similar issue, but in a quite different setup. They assume that the government aims for a minimum consumption level, not accounting for the disutility of work. This makes their results difficult to compare with the standard optimal taxation literature.

her *work aversion*, is non decreasing with her utility level : the richer you are, the more money you ask to go to work.

In this setup, we first study the properties of the first best allocations. A utilitarian welfare function *always* gives a higher utility to the agents with low productivity that do not work than to the ones that participate. But there are no clear ethical grounds to favor the less productive agents: we conclude that utilitarianism is not suited to study the kind of issues that we are interested in, even if a second best approach might correct the undesirable features of the first best. This leads us to turn to a Rawlsian criterion. Most of the time, under Rawls, all agents get the same utility level, so that the utilitarian paradox does not appear. The main feature of a first best allocation is easy to describe. Given the utility level which is aimed at, an agent can be on the dole, receive the social minimum and produce nothing, or work, generate an output equal to her productivity, and get the financial compensation necessary to make her indifferent between working or not working. The social planner then puts her to work whenever her productivity exceeds her work aversion. When an agent crosses the border line between unemployment and employment, her income is discontinuous and increases by her productivity, which just compensates her for the penibility of work: her utility does not change.

The income schedules implemented in practice, even the ones most favorable to work incentives, such as the EITC in the US, do not exhibit such discontinuities. We turn to second best situations, where the government is unable to observe the work aversions of the agents, to see whether the theory can get us closer to the observations. We use an additional assumption of a common utility function of all the agents when they do not work. A crucial tool for the analysis then turns out to be the distributions of work aversion in the economy, conditional on the productivity level, when productivity varies. In the special case where this distribution is constant, independent of the productivity level, we completely characterize the second best allocations. The income schedule is an increasing function of productivity; the financial compensation granted to the agents is always strictly smaller than their productivity (contrary to the first best); the unemployment rate decreases with productivity.

1 The model

We consider an economy made of a continuum of agents, indexed by an element a in some Euclidean space A endowed with a probability measure with c.d.f. F. The agents can produce an undifferentiated commodity, also called money or income. The productivity of a typical agent is denoted $w(a)$, where w is a continuous positive function on A. We assume that the function $w(a)$ is integrable with respect to the measure F. An agent may participate, or not, in the work force. We note the participation status of the agents in the economy with a measurable function s from A into $\{0,1\}$. When agent a participates ($s(a) = 1$), she produces $w(a)$ units of commodity, while she does not produce anything when she does not participate ($s(a) = 0$).

The agents do not differ only by their working abilities w, but also by their (lack of) taste for work. The utility function of a typical agent is a function of the non negative quantity c of commodity which she receives, and depends on the participation decision. The function $v(c;a)$ represents the utility of the non participating agent, while $u(c;a)$ is her utility when working. For every a, $u(.;a)$ and $v(.;a)$ are strictly increasing concave, twice continuously differentiable on $\mathbb{R}_+$.

A parameterization is particularly convenient. Let the first component of a be the wage $w(a)$, while the other components represent the agent's type $x(a)$, so that $a = (w(a), x(a))$. In the first best situations, the social planner knows both $w(a)$

and $x(a)$, while in the second best, the planner only observes $w(a)$ when agent a works. In the latter situation, the marginal distribution of w in the population, whose c.d.f. is noted $\tilde{F}$, is of interest. Furthermore, it will be convenient, without great loss of generality, to assume that A is equal to $\mathbb{R}_+ \times X$ (for each type x, there possibly are agents of any positive productivity), where X is compact, that u and v are differentiable with respect to c, and that the derivatives u'_c and v'_c are continuous on $\mathbb{R}_+ \times A$.

We assume that there is a disutility to work, i.e.

Assumption 1: *For all agents a in A, for all c in $\mathbb{R}_+$,*

$$u(c;a) \leq v(c;a).$$

The analysis uses a measure of the disutility of work, which we call *work aversion*, and note $\Delta(c;a)$. The work aversion is the minimum income supplement which makes agent a indifferent between working or living on resources c without working, i.e. the unique solution in Δ of the equation

$$u(c+\Delta;a) = v(c;a),$$

when such a solution exists or $+\infty$ otherwise, if the agent does not want to work, whatever the wage. Note that, by the implicit function theorem, the function Δ is continuously differentiable with respect to c (when it is finite). We postulate

Assumption 2 : *Whenever defined, $\Delta(c;a)$ is a nondecreasing (and continuously differentiable) function of c.*

The larger the income when unemployed, the larger the required income *supplement* to make it worthwhile to take a job. Under concavity of the utility function, it is easy to check that this property is satisfied for instance under Assumption 1 when $u(c;a) = v(c;a) - x(a)$, for some positive real valued function x on A.

An allocation is defined as a pair of integrable functions $s(a)$ and $c(a)$ with values respectively in $\{0,1\}$ and $\mathbb{R}_+$. It is feasible when

$$\int c(a)\mathrm{d}F(a) = \int_{s(a)=1} w(a)\mathrm{d}F(a). \tag{1}$$

The above equality is the resource constraint, which says that total consumption is equal to total production.

The *laissez-faire* allocation is easy to describe in this framework. Each agent decides to work when her productivity makes it worthwhile, in comparison with a zero income when non participating, i.e. when

$$u(w(a);a) \geq v(0;a),$$

with indifference when there is equality.

Such an allocation can be very unequal, and it is of interest to look at redistribution schemes that tax the rich workers, with high w's, and give the proceeds to the unemployed. Any redistribution scheme reduces the incentives to work. Indeed if $R(a)$, $R(a) \leq w(a)$, is the income given to worker a, and r, $r \geq 0$, the subsistence level attributed to the unemployed, the decision to work under the redistribution scheme is associated with the inequality

$$u(R(a);a) \geq v(r;a),$$

which is always more stringent than under *laissez-faire*. The purpose of the paper is to look at the tradeoff between equity (more equal utility levels) and efficiency (loss of output due to non participation generated by redistribution) depending on the government objective and to see whether the optimal taxation schemes, or equivalently the optimal functions $R(a)$, exhibit some general properties.

There are a number of possible ways to represent society's preferences among equity-efficiency tradeoffs. The one most used in the optimal taxation literature, following the seminal work of Mirrlees (1971), is close to utilitarianism. There is an increasing concave function Ψ, whose concavity is an indicator of society's desire for equality, such that, when $c(a)$ is allocated to agent a, welfare can be written as

$$W_U(c,s) = \int_{s(a)=1} \Psi[u(c(a),a)]\mathrm{d}F(a) + \int_{s(a)=0} \Psi[v(c(a),a)]\mathrm{d}F(a).$$

We shall argue that this kind of criterion is not well suited to the study of participation and equity. Instead we argue for the use of the Rawlsian criterion, which here takes the form

$$W_R(c,s) = \operatorname{ess\,inf}\left\{u(c(a),a)\mathbb{1}_{s(a)=1} + v(c(a),a)\mathbb{1}_{s(a)=0}\right\}.$$

The Rawlsian welfare W_R is obtained from the distribution of utilities in the population by taking its essential infimum, i.e. the lower bound of its support.

The participation problem does not have a lot of structure, in spite of the assumptions that we have made, so that we cannot expect that the solution has many properties of interest. Following tradition, we study the social planner choice in stages, starting with the case of complete information of the planner (first best), following with the situation where the planner only observes the productivity w of the workers (second best).

2 First best allocations

The first best allocations are obtained when the planner observes the agents' types a and has the power to order agent a to work or not to work and to attribute her an income $c(a)$ satisfying the feasibility condition.

We first study optimality with respect to the utilitarian criterion and argue that this criterion is not convenient for our purpose. In the second subsection (and in the remainder of the paper), we focus on the Rawlsian case.

2.1 First best utilitarian allocations

When the social planner knows the agents' characteristics a, the maximization separates by type. If λ denotes the Lagrange multiplier attached to the feasibility constraint, the maximization reduces to that of

$$\{\Psi[u(c(a);a)] + \lambda(w(a) - c(a))\}\mathbb{1}_{s(a)=1} + \{\Psi[v(c(a);a)] - \lambda c(a)\}\mathbb{1}_{s(a)=0},$$

for each value of a, where the unknowns are the participation status $s(a)$ and the income profile $c(a)$. They can be solved as a function of λ which then is found from the feasibility constraint. Optimization with respect to c yields, for the employed agents

$$\Psi'[u(c;a)]u'_c(c;a) = \lambda \text{ or, equivalently, } c = \rho_u(\lambda;a),$$

and for the unemployed

$$\Psi'[v(c;a)]v'_c(c;a) = \lambda \text{ or, equivalently, } c = \rho_v(\lambda;a).$$

The persons that are to be put to work are those for which it is socially profitable to do so, i.e. such that

$$\Psi[u(\rho_u(\lambda;a);a)] - \Psi[v(\rho_v(\lambda;a);a)] + \lambda(w - \rho_u(\lambda;a) + \rho_v(\lambda;a)) \geq 0.$$

Finally, λ can be solved from the feasibility constraint.

The utilitarian criterion makes sure that the marginal social utilities of income of all members of the economy, be they employed or unemployed, are equal. If the utilities function are of the form $u(c) - x(a)$ and $v(c)$, u'_c and v'_c are independent of a, and the first best incomes of the employed, ρ_u, and of the unemployed, ρ_v, are independent of their types. Then there is a (usually positive?) cutoff productivity $\bar{w}$, $\bar{w} = \rho_u - \rho_v + \{\Psi[v(\rho_v)] - \Psi[u(\rho_u)]\}/\lambda$, which separates the employed (productivity above the cutoff) from the unemployed (productivity below the cutoff).

Consider the particular case where Ψ is the identity mapping and $u'_c(c)$ is equal to $v'_c(c)$, say $v = u + x(a)$, where $x(a)$ is a positive scalar. In that case, workers and nonworkers receive the same income $c = (u')^{-1}(\lambda)$ and the cutoff point is given by $\bar{w}(a) = x(a)/\lambda$. From the government budget constraint (1), it follows that the first best allocation corresponds to the unique value of λ such that

$$u'_c\left(\int w(a)\mathbb{1}_{\lambda w(a)\geq x(a)}\mathrm{d}F(a)\right) = \lambda.$$

The lucky types with low productivities get the same income as the rest of society but are left out of the work force, so that their utilities are higher than that of the workers! This result is quite general as stated below.

Theorem 1 *Consider two agents a and a' with the same utility functions, $u(.;a) = u(.,a')$ and $v(.,a) = v(.,a')$, but different productivities $w(a) < w(a')$, so that at the utilitarian optimum agent a is unemployed while agent a' is employed.*

Under Assumption 2 when $\Delta_c > 0$ on $\mathbb{R}^+ \times A$, the utility of agent a is strictly larger than that of a' at the optimum.

Proof We assume that agent a is unemployed and receives $c(a) = \rho_v(\lambda;a)$ and that agent a' works and receives $c(a') = \rho_u(\lambda;a')$. We want to check that agent a is better off

$$v(c(a);a) > u(c(a');a'). \tag{2}$$

Since agents a and a' have the same utility functions ($u(.;a) = u(.;a')$), we have: $c(a') = \rho_u(\lambda;a') = \rho_u(\lambda;a)$. It follows that (2) is equivalent to

$$v(\rho_v(\lambda;a);a) > u(\rho_u(\lambda;a);a),$$

or

$$\rho_v(\lambda;a) + \Delta(\rho_v(\lambda;a);a) > \rho_u(\lambda;a). \tag{3}$$

Therefore we have to check that (3) holds for all a. We proceed by contradiction. Suppose that

$$\rho_u(\lambda;a) \geq \rho_v(\lambda;a) + \Delta(\rho_v(\lambda;a);a) \tag{4}$$

holds for some agent a. Then we would have $u(\rho_u;a) \geq v(\rho_v;a)$, so that, by concavity of Ψ

$$\Psi'[u(\rho_u;a)] \leq \Psi'[v(\rho_v;a)].$$

Now remark that, by definition of Δ,

$$(1 + \Delta'_c(\rho_v;a))u'_c(\rho_v + \Delta(\rho_v;a);a) = v'_c(\rho_v;a),$$

so that, thanks to our assumption $\Delta_c' > 0$: $u_c'(\rho_v + \Delta(\rho_v; a); a) < v_c'(\rho_v; a)$. From (4), we deduce

$$u_c'(\rho_u; a) < v_c'(\rho_v; a). \tag{5}$$

Using equations (4) and (5), it follows that

$$\Psi'[u(\rho_u; a)]u_c'(\rho_u; a) = \lambda < \Psi'[v(\rho_v; a)]v_c'(\rho_v; a) = \lambda,$$

which yields the desired contradiction. ∎

By giving the same *marginal* social utility to all the agents, the planner gives a higher *level of* utility to nonworkers (agents with low productivities) than to workers (agents with high productivities). We consider that this general property of the utilitarian criterion makes it unsuitable for our purpose. We do not believe that there is any *a priori* reason to favor the agents with low productivities based on ethical grounds, which disqualifies utilitarianism in our specific setup. The fact that a second best analysis, with the need to create incentives to work, leads to more 'acceptable' outcomes, is not, in our opinion, a justification to go on with utilitarianism: the criterion should mainly be judged on its first best recommendations. Hereafter, we focus on the Rawlsian case.

2.2 First best Rawlsian allocations

With a Rawlsian criterion, all efforts are made so that everybody gets the same *level of* utility. It then is worthwhile putting someone to work if and only if her productivity is larger than the extra income necessary to compensate her for the penibility of work.

To determine the first best Rawlsian allocation $(c(a), s(a))$, we proceed in two steps. First, we take the value of the welfare W_R as a given parameter and characterize the allocation $(c(a), s(a))$. The characterization (given in Proposition 2 below) is based on the following intuition:

- For each agent, we define $R(a)$ (resp. $r(a)$) as the *minimum* nonnegative income that ensures that agent a's utility is at least as large as W_R when she works (resp. does not work). Since all transfers between agents are possible, it is clear that, at the optimum, agent a collects $c(a) = R(a)$ when she works and $c(a) = r(a)$ when she does not;
- The working rule $s(a)$ then maximizes the government revenue (it is impossible to obtain a higher government revenue without deteriorating the welfare, otherwise one could use this gain to improve the welfare).

In a second step, we determine the value of W_R (which, of course, is endogenous). This is done by expressing the fact that the government budget constraint is binding: the government net revenue must be zero at the optimum.

2.2.1 Characterization of the allocation

The proof of the following proposition can be found in Appendix.

Proposition 2 *Under Assumption 1, consider an optimal allocation* $((c(a), s(a))$ *in the sense of Rawls, leading to a social utility level* W_R*. Define the partition of the type's set:* $A = A_1 \cup A_2 \cup A_3$ *by*

$$\begin{aligned} A_1 &= \{a \in A, u(0,a) \leq v(0,a) \leq W_R\} \\ A_2 &= \{a \in A, u(0,a) \leq W_R < v(0,a)\} \\ A_3 &= \{a \in A, W_R < u(0,a) \leq v(0,a)\} \end{aligned}$$

Then the allocation $((c(a), s(a))$ *is given in each set* A_i *by* $c(a) = R(a)\mathbb{1}_{s(a)=1} + r(a)\mathbb{1}_{s(a)=0}$ *and*

- *Agents* $a \in A_1$ *receive the utility* W_R *whatever their work status. Their incomes,* $R(a)$ *when they work and* $r(a)$ *when they do not, are given by* $u(R(a);a) = v(r(a);a) = W_R$. *The employment status* $s(a)$ *is given by*

$$\begin{array}{l} w(a) > \Delta(r(a);a) \Longrightarrow s(a) = 1 \\ w(a) < \Delta(r(a);a) \Longrightarrow s(a) = 0. \end{array} \tag{6}$$

  The work status and allocation are indeterminate on the border line, when $w(a) = \Delta(r(a);a)$. *Then society is indifferent between having agent a working with income* $R(a)$ *or not working with income* $r(a)$.

- *For* $a \in A_2$, *the incomes when working and not working are respectively given by* $u(R(a);a) = W_R$ *and* $r(a) = 0$. *The working rule is*

$$\begin{array}{l} u(w(a);a) > W_R \Longrightarrow s(a) = 1 \\ u(w(a);a) < W_R \Longrightarrow s(a) = 0. \end{array} \tag{7}$$

  When $u(w(a);a) < W_R$, *agent a does not work, gets a zero income, but enjoys a larger utility than the social norm:* $v(0,a) > W_R$. *Again on the border line, society is indifferent between having agent a working and collecting an income equal to* $w(a)$ *or not working with a zero income, while here the agent prefers the latter.*

- *Agents* $a \in A_3$ *work* ($s(a) = 1$) *and receive no income* ($R(a) = 0$).

The sets A_i only depend on the values of the utility functions when the agents do not get any income. For a given value W_R of the welfare, the utility of agents $a \in A_2$ is greater than W_R when they do not work and *receive zero income.* Thus, in case they indeed do not work, they are happier than the social norm. Agents $a \in A_3$ are even more remarkable: their preferences are such that, even though they *do* work and receive zero income, they are still happier than the social norm! *Hereafter, we will concentrate on agents* $a \in A_1$: we expect that the welfare level attained in the economy is high enough so that most agents fall in the set A_1.

Before completing the characterization of the FB allocation (it remains to determine W_R), we illustrate the property of the income schedules implied by Proposition 2 for the agents in A_1.

2.2.2 Properties of the associated schedules

Fix the type x of the agent, and let her productivity w vary. Some of the agents work, when $s(a) = s(w,x) = 1$, the others do not. Note that the inequalities (6) and (7) are implicit in w: the set of productivities of the working agents is not necessarily a half line, of the form $w \geq \omega$, but can be a union of intervals. At the optimum, around any pivotal agent ω, income is discontinuous, while for the agents in A_1, utility is constant, equal to W_R. If the agents do not work for w smaller than ω, they receive $c(a) = r(\omega_-, x)$, while the agents who work with w larger than ω get $c(a) = R(\omega_+, x)$. The inequality (6) expresses the fact that the discontinuity $R(\omega_+, x) - r(\omega_-, x) = \Delta(r(a);a)$ is equal to the productivity ω of the pivotal agent.

Consider the particular case where the utility functions u and v do not depend on the productivity w, but only on the agents's types x. Then the work aversion Δ also only depends on x. From Proposition 2, we know that r and R depend on x, but not on w. In that case, inequality (6) gives explicitly the set of workers: the

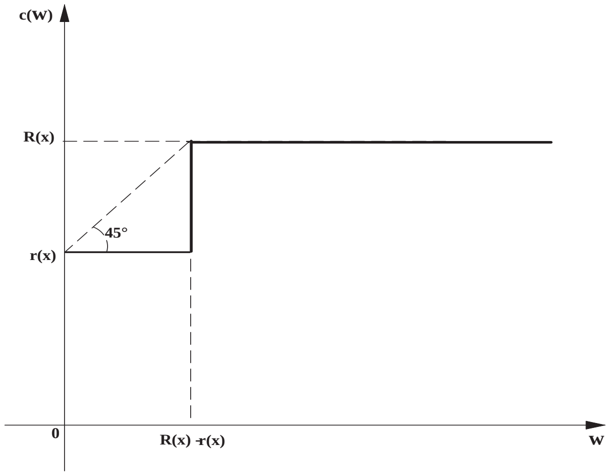

Figure 1: First best allocation: $R - r = w$ for the marginal agent

workers are all the agents with productivity w higher than the threshold $\Delta(r(x); x)$. Figure 1 represents the income collected by agent (w, x) as a function of w, for a fixed x (recall that, in the first best situation, the government observes x and can condition the schedules on x). In the picture, we have assumed that agents with productivity lower than $R(x) - r(x)$ do not work, while those with a larger productivity are in the labor force.

The workers are exactly compensated for the penibility of their labor and receive an income $R(a) = r + \Delta(r; a)$. There is zero taxation at the margin: the marginal agent who decides to work pockets all her productivity. Indeed, all agents such that $w(a) > \Delta(r; a)$ are working at the first best Rawlsian optimum, while all agents such that $w(a) < \Delta(r; a)$ are not working and receiving r. On the other hand, income is disconnected from productivity away from the margin, being equal either to the subsistence level, or to the reservation wage.

2.2.3 Determination of welfare at the optimum

In view of Proposition 2, it is easy to find the Rawlsian optimum. Let W be the unknown social utility level, and for all a such that $W > v(0, a)$, let $\rho(W; a)$ be the unique positive real number which satisfies $v(\rho(W; a); a) = W$. Also, for $W \leq v(0, a)$ let $R(W; a)$ be the solution of $u(R(W; a); a) = W$ when $u(0; a) < W$, or zero otherwise. Then the excess of commodity, or government budget surplus, if one were to implement a Rawlsian allocation satisfying Theorem 2 for this level of W, is

$$T(W) = \int_{v(0,a)>W} [w(a) - R(W; a)] \mathbb{1}_{u(w(a);a)\geq W} \mathrm{d}F(a)$$
$$+ \int_{v(0;a)\leq W} \left\{[w(a) - \Delta(\rho(W; a); a)] \mathbb{1}_{w(a)\geq\Delta(\rho(W;a);a)} - \rho(W; a)\right\} \mathrm{d}F(a). \quad (8)$$

Theorem 3 *The government net revenue $T(W)$ is a continuous and decreasing function of W. There exists a unique value $W^\star$ such that $T(W^\star) = 0$. This value determines the first best Rawlsian welfare. The corresponding allocation is given by Proposition 2 above, with $W_R = W^\star$.*

Proof First note that the functions under the sign $\int$ are continuous with respect to W. Furthermore, when $W = v(0; a)$, $\rho(W; a) = 0$ and $\Delta(\rho(W; a); a) = R(W; a)$.

Since both ρ and R are increasing in W by construction and Δ is increasing in its first argument by Assumption 2, $T(W)$ is decreasing in W. For $W = \operatorname{ess\,inf} u(0; a)$, $T(W) = \int w(a) \mathrm{d}F(a)$ is positive. For $W = \operatorname{ess\,sup} v(w(a); a)$, $\rho(W; a)$ is larger than $w(a)$, so that $T(W)$ is negative. The Rawlsian optimum corresponds to the unique solution of the equation $T(W) = 0$. ■

2.2.4 Introducing a normative assumption

The existence of agents with a utility larger than the social warranty W_R is directly linked to the heterogeneity of the utilities of the agents when unemployed. More specifically, the undesirable property of optimal utilitarian allocations, namely the fact that agents differing only through their productivity are better off when they do not work, also appears with the Rawlsian criterion, but only in the region A_2: agents $a \in A_2$ indeed prefer not working (and receiving zero income) than working (their utility when working is W_R, while it is greater than W_R when they do not work). This leads us to introduce the following assumption.

Assumption 3: *The utility out of work is fixed, independent of the agent type*

$$v(c; a) = v(c).$$

Under Assumption 3, the Rawlsian outcome is at least equal to $v(0)$ (it is feasible to leave everyone on the beach), so that the sets A_2 and A_3 of Proposition 2 are empty. The undesirable property mentioned above is thus ruled out in the Rawlsian first best. Furthermore, at the first best optimum, all unemployed persons get the same income r, defined by $W = v(r)$, while the workers receive $r + \Delta(r; a)$.

Let us insist on the normative nature of Assumption 3. For any well defined $v(c)$, we can always redefine the utility when employed[2] leaving the participation decision unchanged through

$$u(c + \Delta(c; a); a) = v(c).$$

Thus Assumption 3 is a value judgment. It says that the social planner does not make any difference between the unemployed persons. All the unemployed individuals have the same contribution to the social welfare.

2.2.5 Some comparative statics

The purpose of this section is to compare the optimal first best allocations in economies which only differ by their distributions of productivities.

Theorem 4 *Under Assumptions 1 and 2, consider two identical economies except for their productivities, the productivity of the prime economy being larger*

$$w'(a) \geq w(a) \text{ for all } a \text{ in } A.$$

Then the Rawlsian optimum of the prime economy yields a higher utility than that of the second

$$W'_R \geq W_R.$$

[2] This only defines u on $[\Delta(0; a), +\infty)$. As far as optimal Rawlsian allocations are concerned, the shape of u on the rest of its domain $[0, \Delta(0; a))$ is of no importance, since under Assumption 3 the allocations never assign a consumption lower than $\Delta(0; a)$ to someone who works.

Proof The government revenue in the prime economy is given by formula (8), where $w(a)$ is replaced by $w(a')$. Note in particular that the quantities $\rho(W;a)$, $R(W;a)$ and $\Delta(\rho(W;a);a)$ are identical in both economies (this follows from the assumption that the economies are identical: $u(c;a) = u'(c;a)$ and $v(c;a) = v'(c;a)$ for all a). The government revenue is therefore higher in the prime economy: $T'(W) \geq T(W)$ for all W, which gives the result. ∎

Does unemployment and income support disappear at the Rawlsian optimum when society becomes richer and richer? The answer depends, not too surprisingly, on the evolution of the dispersions of both productivities and disutilities for work.

We consider a sequence of economies indexed by k where the agents have the same work aversion $\Delta(c;a)$ and productivity equal to $w_k(a)$. We assume that the average productivity in the economy tends to $+\infty$

$$\int w_k(a) \, \mathrm{d}F(a) \to +\infty$$

as $k \to +\infty$. The index k can be seen as an index of aggregate productivity. We are interested in the asymptotic behavior of the employment rate $\bar{G}_k^{\mathrm{FB}}$ at the first best optimum

$$\bar{G}_k^{\mathrm{FB}} = \mathrm{P}(w_k(a) - \Delta(r_k;a) \geq 0).$$

More precisely, we investigate the following question: does the employment rate tends to 1 when the economy becomes richer and richer? We first check that the welfare at the optimum goes to infinity with k.

Lemma 1 *Under assumptions 1 to 3, the subsistence revenue r_k at the first best optimum increases and tends to $+\infty$ as $k \to +\infty$.*

Proof It is easy to check that the government net revenue increases with k for all r, which implies that r_k increases with k. Now use the budget constraint to get

$$r_k = \int_{w_k(a)-\Delta(r_k;a)\geq 0} (w_k(a) - \Delta(r_k;a)) \, \mathrm{d}F(a) \geq \int (w_k(a) - \Delta(r_k;a)) \, \mathrm{d}F(a).$$

It follows that

$$r_k + \int \Delta(r_k;a) \, \mathrm{d}F(a) \geq \int w_k(a) \, \mathrm{d}F(a),$$

which gives the result (using Assumption 2). ∎

It turns out that the behavior of $\bar{G}_k^{\mathrm{FB}}$ depends on the difference of $\Delta - w$ from its mean, denoted α

$$\alpha_k(c;a) = \Delta(c;a) - w_k(a) - \int (\Delta(c;a) - w_k(a)) \, \mathrm{d}F(a).$$

Theorem 5 *A necessary condition for the employment rate $\bar{G}_k^{\mathrm{FB}}$ to tend to 1 as $k \to +\infty$ is that $\mathrm{P}(\alpha_k(r_k;a) \leq r_k)$ tends to 1 as $k \to +\infty$.*

A sufficient condition is that $\mathrm{P}(\alpha_k(r_k;a) \leq r_k) = 1$ for k large enough.

In the additive case ($w_k(a) = w(a) + k$), the quantity α_k depends on k only through r_k. Therefore the condition only involves the dispersion of $\Delta(c;a) - w(a)$ around its mean as $c \to +\infty$.

To illustrate the above theorem, consider the following economy. The agents are characterized by $a = (w + k, x)$, their utility when not working is $v(c) = \ln c$ while their utility at work is $u(c;x) = v(c) - \ln x$, where x is larger than 1. w and x are independently distributed, with mean in the population respectively equal to Ew and Ex. The work aversion of the typical agent is $\Delta(c;x) = c(x-1)$. We have in that case

$$\mathrm{P}(\alpha(r_k) \leq r_k) = \mathrm{P}\left[r_k\,(x - Ex) \leq r_k + w - Ew\right].$$

Now a direct application of Theorem 5 yields in this particular case :

1. everybody is employed for k large enough if the maximum value of x is strictly smaller than $Ex+1$ or if it is just equal to $Ex+1$ when w is non random;
2. the probability of employment does not tend to 1 when k goes to infinity if the maximum value of x is strictly larger than $Ex+1$, or if it is equal to $Ex+1$ and the distribution of w is not degenerate.

There is no presumption that the employment rate goes to one when the economy gets richer and richer.

3 Second best allocations

When all the characteristics of the agents, productivities and disutilities for work, are known to the government, the preceding section has shown that the optimal taxation scheme is very abrupt: give maximum incentives at the margin, i.e. income up to $w + r$, to everyone that is worth having in the labor force, when w is larger than the work aversion. This amounts to a marginal tax rate equal to $-\infty$ at that point, a large downward tax discontinuity. Such incentives to work are not implementable in practice. In France, apart from a temporary subsidy, there is a 100% marginal tax rate on earnings when one takes a job; in the US, the earned income tax credit creates some incentives to work, which vary with the composition of the family. The EITC can amount to 40% of earnings, i.e. each dollar earned yields 1.4 dollar for the wage earner (the marginal overall tax rate however is larger than -40%, because of social contributions, the phasing out of some social benefits and the phasing in of the income tax in this income range). Still, this is far from the kind of discontinuities shown by the optimal subsidy scheme of the last section. It is worth investigating features that would bring the theoretical prescriptions of the model closer to the observed facts. One dimension particularly seems worth of attention: typically work aversions are unobserved by the fiscal authorities and this fact is likely to smooth the shape of the optimal subsidy scheme.

To formalize this idea, we assume that agent a's productivity $w(a)$ is observed by the government *only when agent a work*, but that no other individual characteristics of the agent can be used to base the tax-subsidy scheme. The government, however, knows the distribution of individual characteristics in the economy.

3.1 Characterization of second best Rawlsian allocations

In the remainder of the paper, Assumptions 1, 2 and 3 are supposed to hold.

3.1.1 From incentive compatibility to schedules

In the game between the government and the agents, the government acts as a Stackelberg leader (the government is the 'principal'). The informational structure is as described above: the government observes $w(a)$ only when a works. We begin by defining the incentive compatible allocations $(c(a), s(a))$ in that game.

We first define the function $U(c, s; a)$ on $\mathbb{R}_+ \times \{0, 1\} \times A$ by

$$U(c, 1; a) = u(c; a) \text{ and } U(c, 0; a) = v(c; a).$$

Then an allocation $(c(a), s(a))$ is incentive compatible if

1. $c(a)$ depends only on $w(a)$ when $s(a) = 1$,
2. $c(a)$ is a constant c_0, independent of a, when $s(a) = 0$,
3. for all a' such that either $s(a') = 0$ or $w(a') = w(a)$

$$U(c(a), s(a); a) \geq U(c(a'), s(a'); a). \tag{9}$$

Equation (9) expresses the fact that agent a can mimic any agent a' who either does not work or has the same productivity as a.

Suppose that the government posts a menu $(R(w), r)$ of income schedules, respectively for the (potential) workers and for the (potential) unemployed. Facing such a menu, an agent a chooses either to work and receive $R(w(a))$ or not to work and receive r. This gives rise to an allocation $(c(a), s(a))$. It is very easy to check that this allocation is incentive compatible. The following lemma states the converse.

Lemma 2 *Let $(c(a), s(a))$ be an incentive compatible allocation such that*

$$v(0) \leq W_R = \operatorname{ess\,inf} U(c(a), s(a); a). \tag{10}$$

We let r be such that

$$W_R = \operatorname{ess\,inf} U(c(a), s(a); a)) = v(r). \tag{11}$$

We note E_w the set of workers with productivity w: $E_w = \{a \in A, s(a) = 1$ and $w(a) = w\}$. For all $w \geq 0$, we define $R(w)$ by

$$R(w) = \begin{cases} c(a) & \textit{for } a \in E_w \quad \textit{if } E_w \neq \emptyset \\ 0 & \qquad\qquad\quad\;\, \textit{if } E_w = \emptyset. \end{cases} \tag{12}$$

Then the menu of schedules $(r, R(w))$ implements the allocation $(c(a), s(a))$.

Proof of Lemma 2 Consider first a worker a $(s(a) = 1)$. We have to show that this agent, when she faces the schedule $(r, R(w))$, is willing to work and collect $c(a)$. In case she chooses to work, she collects $R(w(a)) = c(a)$. Then the problem is reduced to check that $u(R(w(a)); a) \geq v(r)$. This inequality follows from the definition of r.

Now suppose a does not work $(s(a) = 0)$ and receives c_0. It follows from the incentive compatibility conditions that: $v(c_0) = W_R$, therefore $c_0 = r$. In other words, all unemployed individuals (if there are any) receive utility W_R. Now we have to check $v(c_0) = v(r) \geq u(R(w(a)); a)$. This inequality follows from (9) if $E_w \neq \emptyset$ and from Assumption 1 in the other case. ∎

It is always possible to attain a value of the Rawlsian welfare greater than (or equal to) $v(0)$ (take the allocation $(c(a) = 0, s(a) = 0)$ for all a): all the allocations of interest satisfy (10). Therefore Lemma 2 allows us to work with schedules $(r, R(w))$ rather than with allocations $(c(a), s(a))$.

Actually, it will turn out to be more convenient to use the quantity $D(w) = R(w) - r$, which measures the financial incentives to work, rather than the net wage $R(w)$. Since $R \geq 0$, the incentive D is larger than $-r$. When she is in front of such a menu $(r, D(w))$, agent a decides to work when $\Delta(r, a) \leq D(w)$, with indifference in case of equality.

3.1.2 Rewriting the budget constraint

In the sequel, we note $G_{r,w}$ the c.d.f. of the distribution of work aversions $\Delta(r,a)$ conditional on the agent productivity w

$$G_{r,w}(D) = \Pr\ (\Delta(r,a) \leq D \mid w)$$

Note that Assumption 2 implies that $G_{r,w}(D)$ is a nonincreasing function of r.

Suppose now that the government posts a schedule $(r, D(w))$. Then the probability that an agent with productivity w works when she faces this schedule is $G_{r,w}(D(w))$. The government revenue under the scheme $(r, D(w))$ can be written as

$$\begin{aligned} T(r, D(.)) &= \int [w(a) - D(w(a))] \mathbb{1}_{\Delta(r;a) \leq D(w(a))} \mathrm{d}F(a) - r \\ &= \int [w - D(w)] G_{r,w}(D(w)) \mathrm{d}\tilde{F}(w) - r, \end{aligned} \tag{13}$$

where $\tilde{F}$ is the distribution of productivities in the population. The pair $(r, D(w))$ is feasible when it satisfies the budget constraint

$$T(r, D(.)) = 0. \tag{14}$$

By definition, a Rawlsian second best optimum maximizes W_R among all feasible incentive compatible allocations.

To characterize second best allocations, we follow the same path as in the first best case. We first take the value of the welfare W_R as given and determine the function $D(w)$. The schedule $D(w)$ must maximize the government revenue (otherwise an increase of revenue could be used to increase welfare). Then the value of r is chosen so that the budget constraint is binding: the government revenue must be zero at the optimum.

3.1.3 Characterization of the schedule $D(w)$

Suppose that the government tries to reach an allocation leading to a Rawlsian utility level equal to W_R. We let $\bar{d}(r|w)$ (possibly equal to $+\infty$) denote the maximum of the support of $G_{r,w}$.

The following lemma provides a characterization of the least costly incentive compatible allocation that guarantees a welfare level equal to W_R.

Theorem 6 *Let $W_R \geq 0$ be given. Let $(c(a), s(a))$ be the allocation that maximizes government revenue among all incentive compatible allocations such that $W_R = \operatorname{ess\,inf} U(c(a), s(a); a)$. Consider the schedule $(r, D(w))$ associated with $(c(a), s(a))$ by Lemma 1.*

Then $D(w)$ is an element of

$$\mathit{argmax}_{\ D}(w - D) G_{r,w}(D). \tag{15}$$

As a consequence, we have, for all w, $0 \leq D(w) \leq w$ and $D(w) \leq \bar{d}(r|w)$.

Proof The result follows directly from equation (13).

An interesting property of the optimal allocation is that, for each $w \geq 0$, there exists an agent with productivity w whose utility is equal to W_R. This is a consequence of Theorem 6.

Indeed, if there exists an unemployed agent with productivity w, we know that this agent's utility is W_R (from Lemma 1, all unemployed individuals receive utility W_R).

Otherwise, all agents of productivity w work, so that $G_{r,w}(D(w)) = 1$. Thus $D(w) \geq \bar{d}(r|w)$. Since Theorem 6 gives the inequality in the other direction, $D(w) = \bar{d}(r|w)$. Now an agent a with productivity w and work aversion $\Delta(r;a) = \bar{d}(r|w)$ consumes $c(a) = r + D(w) = r + \Delta(r;a)$, and has utility $u(c(a);a) = v(r) = W_R$[3].

It is important to remark that workers with productivity w and work aversion strictly lower than $D(w)$ get a rent, i.e. their utility is strictly larger than the social norm. In the second best environment, the government cannot extract all the rent from the agents. Hereafter, we denote $K_r(w)$ the value of the maximum

$$K_r(w) = \max_{D \leq w}(w - D)G_{r,w}(D). \tag{16}$$

The quantity $K_r(w)$ is related (but not equal) to the tax collected by the government on workers (recall $w - D(w) = w - R(w) + r$). It can be interpreted as the share of the total surplus w collected by the government on agents with productivity w (the government leaves $D(w)$ to the workers as an incentive to work). We can now complete the characterization by determining the value of r.

3.1.4 Determination of the subsistence revenue r

Theorem 6 gives a procedure to construct a Rawlsian optimum. For any r in $[0, \int wd\tilde{F}(w)]$, we note $T(r)$ (with a slight abuse of notation) the government net revenue

$$T(r) = \int_w K_r(w)\mathrm{d}\tilde{F}(w) - r. \tag{17}$$

Under Assumptions 1-3, we have

Lemma 3 *For all productivity w, the surplus $K_r(w)$ is a continuous and nonincreasing function of r.*

The proof of Lemma 3 is given in Appendix. The fact that $K_r(w)$ decreases in r is obvious (from Assumption 2). The continuity of $K_r(w)$ holds even though the c.d.f. $G_{r,w}$ is discontinuous (for instance, when the distribution of work aversions is discrete). The characterization of the subsistence revenue r easily follows from the lemma.

Theorem 7 *The government revenue $T(r)$ given by (17) is a continuous and decreasing function of r. The value of the subsistence revenue at the second best Rawlsian optimum is the unique solution to $T(r^\star) = 0$. The incentives to work $D(w) = R(w) - r$ are then given by Theorem 6.*

Proof The first assertion ($T(.)$ continuous and decreasing) follows directly from Lemma 3). It is positive when $r = 0$ and negative when $r = \int w\mathrm{d}\tilde{F}(w)$. Its unique zero corresponds to the Rawlsian optimum. ■

[3]It may happen that all agents work in the second best optimal allocation. A necessary condition for no unemployment at the optimum is $w \geq \bar{d}(r|w)$ for all w (this requires in particular that $\bar{d}(r|w) < +\infty$ for all w). The welfare is then given by $W_R = v(r)$, where r is the unique solution to the government budget constraint, which writes, in that case

$$\int w\mathrm{d}\tilde{F}(w) = r + \int \bar{d}(r|w)\mathrm{d}\tilde{F}(w).$$

3.1.5 Basic comparative statics

Some easy comparative statics results follow. Using the definition (16) of K_r, we see that the function K_r decreases when the distribution of work aversions first order stochastically increases ($G_{r,w}$ *decreases*). Therefore the government revenue also decreases for all r (see equation (17)). It follows that the optimal value of r (solution to $T(r) = 0$) also decreases. To establish a similar result for the distribution of productivities, we need an additional assumption.

Assumption 4: *The work aversion $\Delta(r, a)$ is independent of w.*

Under Assumption 4, the distribution of work aversions is independent of the productivity of the agents: $G_{r,w}$ does not depend on w, and can be written as G_r.

Using definition (16), we see that $K_r(w)$ is a nondecreasing function of w. It follows that the government revenue T (and consequently the optimum r) increases when the distribution of w stochastically increases (G_r being fixed and thus also the function K_r). Since r is equal to $v^{-1}(W_R)$, we have shown

Theorem 8 *Under Assumptions 1 to 4, the second best Rawlsian optimum utility level W_R*

- *decreases when the distribution of work aversions G_r first order stochastically increases in r for all w;*
- *increases when the distribution of productivities $\tilde{F}(w)$ first order stochastically increases.*

The same arguments[4] as above show that the employment rate in the economy

$$\bar{G}^{\mathrm{SB}} = \int G_r(D(w))\mathrm{d}\tilde{F}(w),$$

has the same behavior as the welfare W_R when the distributions of risk aversions and productivities first order stochastically increase.

3.2 Qualitative analysis

The Rawlsian second best optimum is particularly easy to characterize when assumption 4 holds. In the remainder of this section, suppose that Assumptions 1 to 4 hold.

Theorem 9 *Consider a second best Rawlsian allocation. Under Assumption 4, we have*

1. *The surplus $K_r(w) = (w - D(w))G_r(D(w))$ raised by the government at the optimum is an increasing convex positive function of w, of slope at most equal to 1.*
2. *$D(w)$ is an increasing function of w, with $D(w) \leq w$. The proportion of agents of productivity w at work, $G_r(D(w))$, is also increasing in w.*

Proof From Theorem 5, $K(w)$ is the supremum of the set of linear mappings $(w - d)G_r(d)$, where d is any real number. It is positive ($d = w$ is possible), convex as the supremum of convex functions. $G_r(D(w))$ is a subgradient of $K(w)$, whose slope cannot thus exceed 1. Convexity implies that the subgradient is nondecreasing, which implies that $G_r(D(w))$ is nondecreasing in w, and $D(w)$ as well. ■

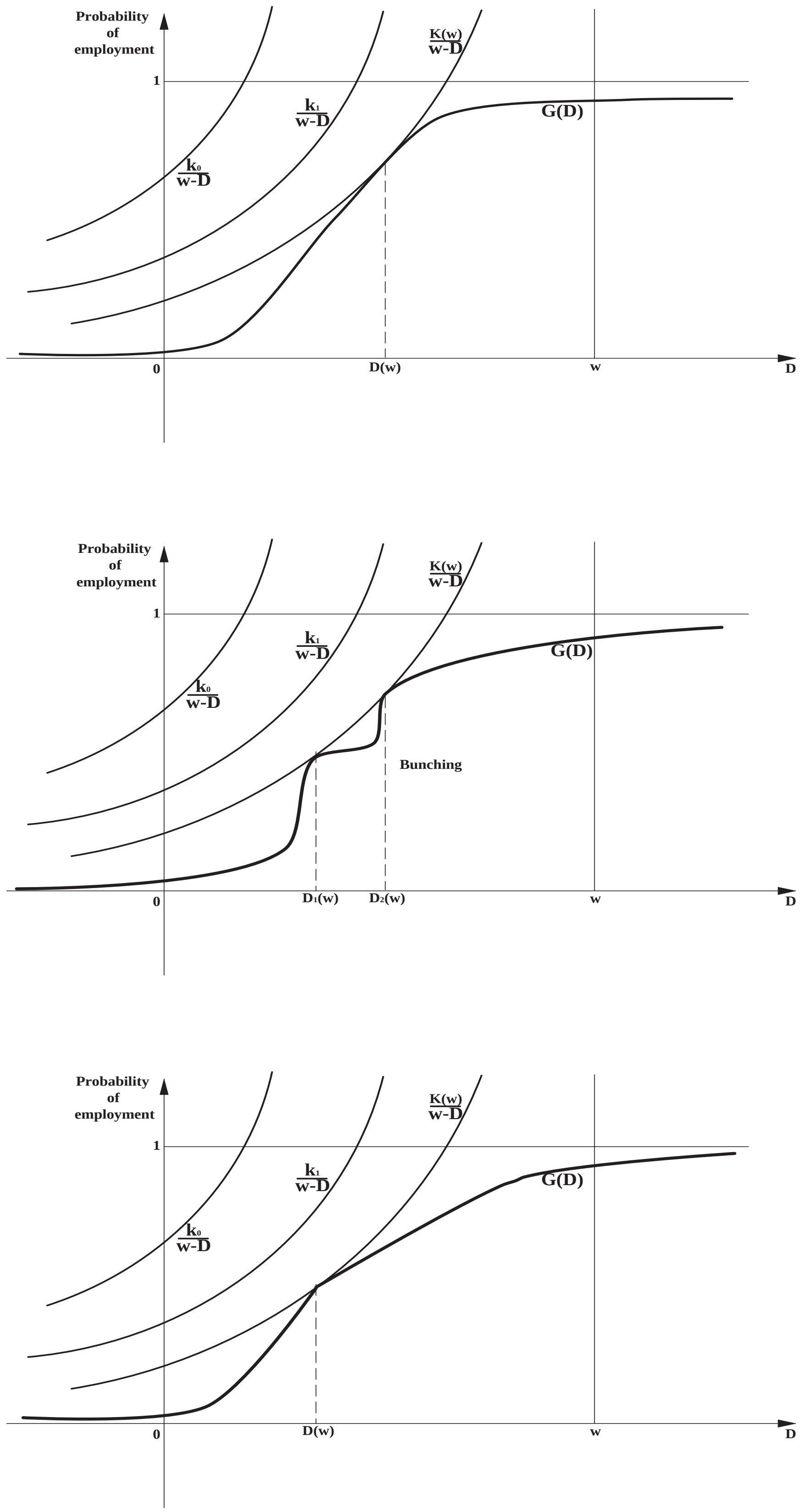

Figure 2: The optimization program

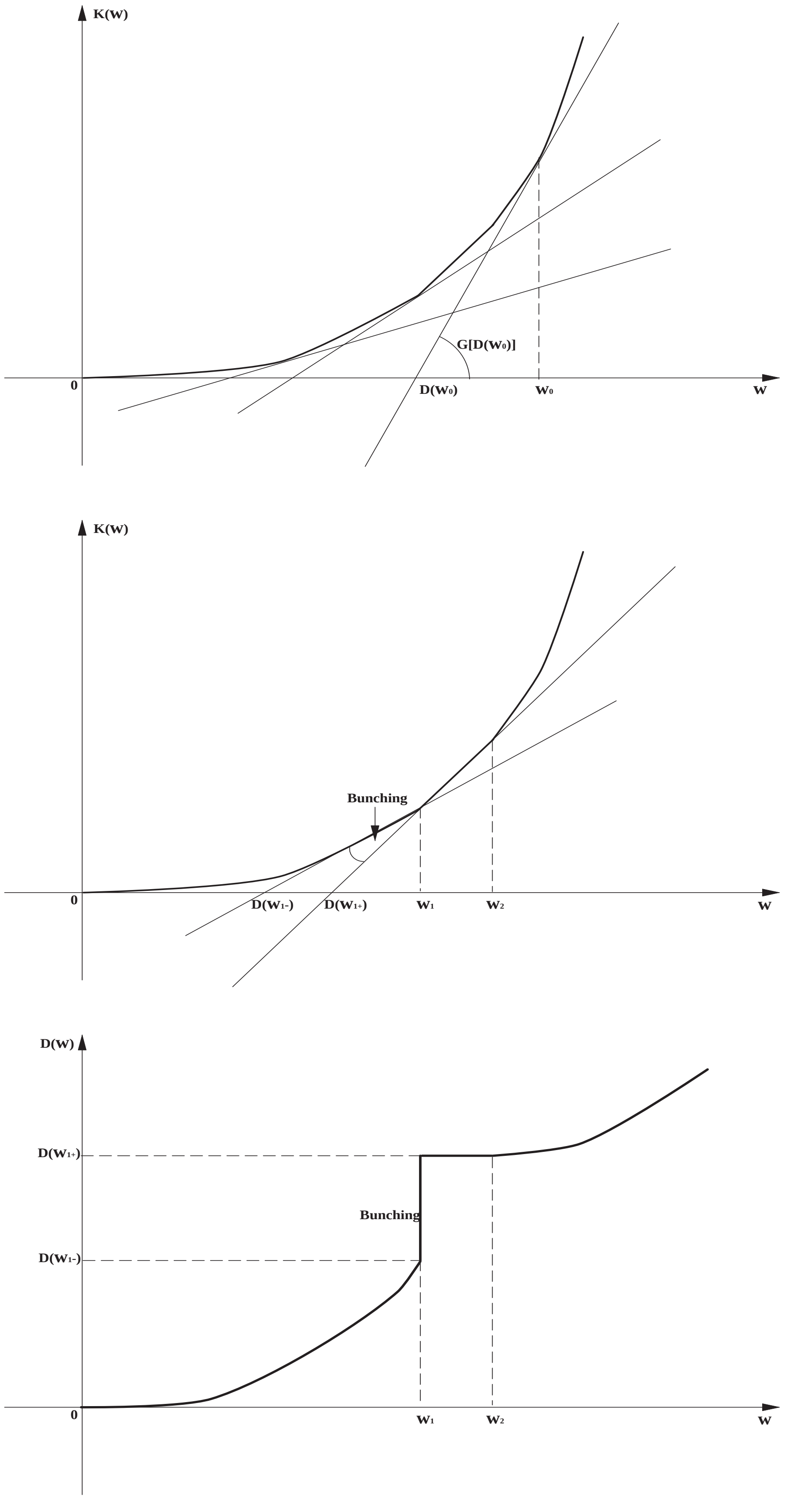

Figure 3: Government revenue and incentive scheme

The theorem shows that, under Assumption 4, the marginal tax rates $T'(w) = 1 - D'(w)$ are less than 1. The fact that $D(w)$ is increasing implies that it would not be in the interest of an agent to announce a productivity lower than hers, if this were allowed. The tax schedule is incentive proof to the mimicking of agents with lower productivities..

A graphical representation helps to understand the structure of the problem. On Figure 2, the c.d.f. $G_r(D)$ is plotted: if D is selected by the government, $G_r(D)$ is the proportion of agents that are willing to work. For a given value of w, the problem (see the definition of K_r (16)) is to find the maximum value of k such that $k/(w-D)$ intersects the graph of the c.d.f..

On Figure 2, therefore, for a given w, we draw a bunch of isoquants of the form $k/(w-D)$, all arcs of hyperbola whose asymptotes are the negative D axis and the vertical line of abscissa w. The solution is at the highest isoquant which is tangent to the c.d.f.. When w increases, the hyperbolas translate to the right, so that both $D(w)$ and $K_r(w)$ increase.

The upper panel of Figure 2 is the 'regular' case, where there is a nice unique tangency point. But the program needs not be so well behaved. The middle panel shows a situation where there are two tangency points. At this particular value of w, both $D_1(w)$ and $D_2(w)$ yield the optimum $K_r(w)$. All the agents with work aversions between these two values are always treated in the same way, either non employed (for productivities smaller than w) or employed (for productivities larger then w): they are bunched together. When productivity varies, the incentive scheme has a discontinuity at w: $D(w_-) < D(w_+)$. Formally, the bunches associated with a discontinuity of the schedule at w are

$$B(w, \bar{w}) = \{a \in A, w(a) = \bar{w} \text{ and } D(w_-) \leq \Delta(r; a) \leq D(w_+)\}.$$

When $\bar{w} > w$, agents $a \in B(w, \bar{w})$ work and receive $R(w) = r + D(w)$. When $\bar{w} < w$, they do not work and receive r. When $\bar{w} = w$, their (common) work status depend on whether the government chooses $D(w) = D(w_-)$ or $D(w) = D(w_+)$ (the government is indifferent between the two possibilities). Note that such discontinuities have nothing pathological: they will occur as soon as the c.d.f. has pieces that are flatter than the arc of hyperbola going through them, for instance for discrete distributions.

The bottom panel of Figure 2 shows another possibility, where $D(w)$ and $G_r(D(w))$ stay constant on a range of productivities. In that case, agents a with the same work aversion $\Delta(r; a) = d$ and productivity $w_1 \leq w(a) \leq w_2$ are treated the same way. When $d < D(w_1) = D(w_2)$, they work and receive $r + D(w_1)$. When $d > D(w_1) = D(w_2)$, they do not work and receive r. This occurs when the optimum is at a kink of the graph of the c.d.f..

Even though the point wise optimization problem can be badly behaved, the overall optimization is simple, as shown on Figure 3. The two upper panels show the plan $(w, K(w))$. The maximization involves taking the upper envelope $K(w)$ of a set of straight lines of equation $(w - D)G_r(D)$, with varying D's. As shown in the top panel, the typical line intersect the w axis at D, and has slope $G_r(D)$, a number between 0 and 1. The function $K_r(w)$ is increasing convex (and therefore continuous). Linear portions of K correspond to flat portions of the schedule, while kinks in K correspond to a discontinuity of D.

[4] We will see in the next section that, under Assumptions 4, $D(w)$ and $G_r(D(w))$ are nondecreasing functions of w.

Proposition 10 *When G is log concave, there is no bunching and the marginal tax rate is nonnegative.*

Proof The problem (15) can be rewritten $\max_{D \leq w} \ln(w-D)+H(D)$, with $H(D) = \ln G$. Since H is concave, the function $D \to \ln(w-D)+H(D)$ is strictly concave and has a unique maximum, characterized by the first order conditions[5]

$$H'(D) = \frac{1}{w-D}.$$

Since D is nondecreasing and H' is nonincreasing, it follows that $w - D(w)$ increases in w, which gives the result. ∎

As mentioned in the introduction, one feature of Mirrlees framework is that marginal tax rates are nonnegative at the optimum. We recover this property in our setup whenever the distribution of work aversions is log-concave. In this circumstance, the use of negative tax rates (like with EITC in the US or WFTC in the UK) is not justified by incentive purposes. Recall, however, that when the distribution is *not* log-concave, this property needs not to hold. In particular, bunching gives rise to negative tax rate. Indeed, at a point w such that $D(w_-) < D(w_+)$, the marginal tax rate is negative (it is actually $-\infty$, since $w - D(w)$ has a downwards discontinuity).

3.3 Examples

3.3.1 A single dimension of heterogeneity

Suppose that the only heterogeneity parameter is the productivity of the agents, which also influences their utilities ($a = w$). Suppose also that the work aversion depends only on w (not on r). In other words, the utility functions satisfy: $u(c; w) = v(c - \Delta(w))$.

Under Assumption 4, at the first best, r is defined through $W_R = v(r)$. Every agent is indifferent between working or not: $R(w) = r + \Delta(w)$. The work rule is given by $s(w) = \mathbb{1}_{w \geq \Delta(w)}$. Finally, the unknown subsistence income (and therefore the social utility level) is determined by the budget constraint

$$r = \int [w - \Delta(w)] \mathbb{1}_{w \geq \Delta(w)} \mathrm{d}\tilde{F}(w).$$

In the first best, the central planner knows the productivity of every agent. In the second best, he only observes the productivity of the agents that are employed. Nevertheless, since the first best allocation gives the same utility to all agents, it is incentive compatible, and is equal to the second best. Indeed, formally, the distribution $G_{r,w}$ is the Dirac mass at $\Delta(w)$. Then

$$K_{r,w} = [w - \Delta(w)] \mathbb{1}_{w \geq \Delta(w)},$$

so that all the agents with $w > \Delta(w)$ work.

3.3.2 Discrete distribution of work aversions

Assume that, independently of w, the work aversion $\Delta(c; a)$ takes two values $\Delta_j(c)$, with probability p_j, $j = 1, 2$, $p_1 + p_2 = 1$ and $\Delta_1(c) \leq \Delta_2(c)$ for all c. Assumptions 2 and 3 are supposed to hold: Δ_1 and Δ_2 are nondecreasing functions of c on the one hand, the utility v when not working does not depend on j on the other hand. Since Δ_j and p_j do not depend on w, note that Assumption 4 is also satisfied.

[5] When G has a kink, the first order condition is that 0 is in the subgradient of $\ln(w-D)+H(D)$.

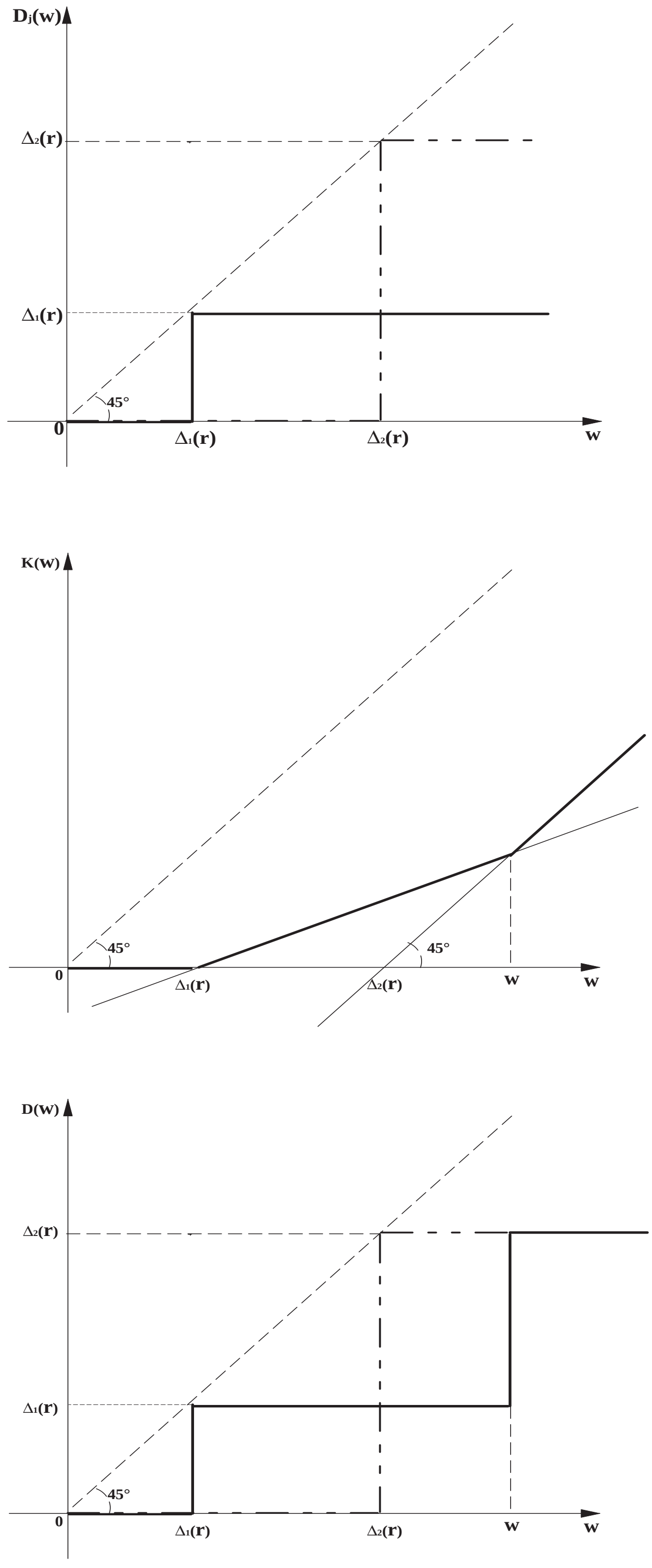

Figure 4: Example 2. First best and second best

In the first best situation, the government observes w and j. For a given j, we know that the income schedule is given by $D_j(w) = \Delta_j(r)\mathbb{1}_{w\geq\Delta_j(r)}$, as shown on the upper panel of Figure 4. The government revenue is given by

$$T^{\mathrm{FB}}(r) = \int \left[p_1(w-\Delta_1(r))\mathbb{1}_{w\geq\Delta_1(r)} + p_2(w-\Delta_2(r))\mathbb{1}_{w\geq\Delta_2(r)}\right] \mathrm{d}\tilde{F}(w) - r.$$

In the second best situation, the government does not observe j and the schedule can only depend on w. The government solves

$$K_r(w) = \max[0, p_1(w-\Delta_1(r)), w-\Delta_2(r)],$$

as shown in the middle panel of Figure 4. The government revenue is given by

$$T^{\mathrm{SB}}(r) = \int \left[p_1(w-\Delta_1(r))\mathbb{1}_{\Delta_1(r)\leq w\leq\bar{w}(r)} + (w-\Delta_2(r))\mathbb{1}_{w\geq\bar{w}(r)}\right] \mathrm{d}\tilde{F}(w) - r.$$

It is easy to check that the government revenue is always greater in the first best than in the second best: $T^{\mathrm{FB}}(r) \geq T^{\mathrm{SB}}(r)$ for all r. Therefore the subsistence revenue and the welfare are higher in the first best than in the second best: $r^{\mathrm{FB}} \geq r^{\mathrm{SB}}$. The schedule, represented on the lower panel of Figure 4, is given by

$$D(w) = \begin{cases} \text{any value } < \Delta_1(r) & \text{if } w \leq \Delta_1(r) \\ \Delta_1(r) & \text{if } \Delta_1(r) \leq w \leq \bar{w} \\ \Delta_2(r) & \text{if } \bar{w} \leq w, \end{cases}$$

where $\bar{w}(r) = (\Delta_2(r) - p_1\Delta_1(r))/(1-p_1)$. Note that $\bar{w}(r) \geq \Delta_2(r)$. The corresponding values of the probability $G_r(D(w))$ are $0, p_1$ and 1. Note that the productivity of the pivotal agent $\bar{w}$ is strictly larger than the difference $\Delta_2(r) - \Delta_1(r)$.

3.4 The inverse problem

In the first best case, the distribution G degenerates into a Dirac mass at some point d: $D(w)$ is zero for w smaller than d, and equal to d for w larger than d. The second best situation obviously yields more general shapes for social transfers. Indeed, essentially any increasing function of w, smaller than w, is the incentive schedule of a well chosen economy.

In this section, we investigate the inverse problem: a schedule $(r, D(w))$ being given, is it possible to find an economy (i.e. utility functions u and v and distributions $G_{r,w}$ and $\tilde{F}$) such that $(r, D(w))$ is a Rawlsian second best optimal scheme? To give a precise answer to this question, it is useful to introduce the following integral

$$I(y) = \int_y^{+\infty} \frac{dx}{D^{-1}(x) - x}.$$

This integral is positive, possibly equal to $+\infty$.

Theorem 11 *Let $(r, D(w))$ be a scheme such that $D(w)$ is nondecreasing on $[0, +\infty[$, satisfies $D(w) \leq w$ for all w and there exists some y with $I(y)$ finite.*

Let y_0 be the smallest number, $y_0 \geq D(0)$, such that $I(y)$ is finite for y larger than y_0. Define the distribution G, with support in $[y_0, +\infty)$ by

$$G(d) = \exp(-I(d)). \tag{18}$$

Let $\tilde{F}$ be any distribution on $\mathbb{R}^+$ satisfying the budget constraint $T(r) = 0$.

Consider the following economy. Let the space of agents $a = (w, x)$ be a subspace of $\mathbb{R}^2$, such that w and x are independently distributed according to respectively $\tilde{F}$

and G. Take any increasing concave differentiable function $v(c)$ on $\mathbb{R}_+$, and define $u(c;a)$ on $[y_0, +\infty)$ through $u(c+x;a) = v(c)$ ($u(c;a)$ can be any number smaller than $v(0)$ for c smaller than y_0). Then the scheme $(r, D(w))$ implements the second best Rawlsian optimum of this economy.

The proof can be found in Appendix. It is interesting to remark that G given by (18) is log-concave *if and only if* the marginal tax rate is nonnegative for all w. Indeed the function $\ln G = -I(d)$ is concave if and only if $D^{-1}(x) - x$ is a nondecreasing function of x or, equivalently, $w - D(w)$ is a nondecreasing function of w.

Note also that, in the above economy, the distribution of work aversions does not depend on w nor on r.

References

Beaudry, P., and C. Blackorby (1997): "Taxes and Employment Subsidies in Optimal Redistribution Programs," Discussion Paper 22, University of British Columbia.

Besley, T., and S. Coate (1995): "The Design of Income Maintenance Programmes," *Review of Economic Studies*, 62, 187–221.

Diamond, P. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88, 83–95.

Mirrlees, J. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175–208.

Piketty, T. (1997): "La redistribution fiscale face au chômage," *Revue Française d'Économie*, XII(1), 158–201.

Saez, E. (2000): "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses," Discussion Paper 7708, NBER.

——— (2001): "Using Elasticities to derive Optimal Income Tax Rates," *Review of Economic Studies*, 68.

Salanié, B. (1998): "A Note on Optimal Income Taxation," mimeo CREST.

Appendix

Proof of Proposition 2

First consider the set A_1 of agents such that $W_R \geq v(0;a)$. All the agents in A_1 get a utility level equal to W_R at the optimum (otherwise, if a non null set had more, it would be possible to reduce their income, and use the proceeds to increase the social optimum). Let

$$T_1 = \int_{A_1} \{[w(a) - R(a)]\mathbb{1}_{s(a)=1} - r(a)\mathbb{1}_{s(a)=0}\}\mathrm{d}F(a)$$

denote total government revenue, after transfers, from the agents in A_1. This quantity can be rewritten as

$$T_1 = \int_{A_1} \{[w(a) - \Delta(r(a);a)]\mathbb{1}_{s(a)=1} - r(a)\}\mathrm{d}F(a).$$

Keeping the agents in A_1 at the same utility level, one maximizes T_1 by choosing the employment status according to 6.

Second, consider the set A_2 of agents such that $u(0;a) \leq W_R < v(0;a)$. Define $R(a)$ by the equality $u(R(a);a) = W_R$. Then the government receipts from these agents are

$$T_2 = \int_{A_2} [w(a) - R(a)] \mathbb{1}_{s(a)=1} \mathrm{d}F(a).$$

Keeping them at a utility level at least as large as W_R, it is optimal to put to work the agents such that $w(a) > R(a)$, i.e. $u(w(a);a) > W_R$, and to leave the others out of work, with a zero income.

Finally, consider the agents such that $W_R < u(0;a)$. One cannot lower their utility. Obviously, it is optimal to put them to work and to collect their wage to redistribute it to the rest of society.

■

The proof of Theorem 5 relies on two preliminary lemmas.

Lemma 4 *Suppose* $\mathrm{P}(\alpha_k(r_k;a) \leq r_k) = 1$. *Then everybody works.*

Proof We take r_k such that

$$\int (\Delta(r_k;a) - w_k(a)) \, \mathrm{d}F(a) = r_k,$$

and check that it corresponds to the first best. By assumption, we have

$$\begin{aligned} \Delta(r_k;a) - w_k(a) &= \Delta(r_k;a) - w_k(a) - \int (\Delta(r_k;a) - w_k(a)) \, \mathrm{d}F(a) - r_k \\ &= \alpha_k(a) - r_k \leq 0. \end{aligned}$$

The result follows from the characterization of the FB optimum (Theorem 3): the allocation where everybody works and receive $r_k + \Delta(r_k;a)$ is optimal. ■

We now study the case where the dispersion of $\Delta - w$ is high when r increases. We begin with a simple inequality.

Lemma 5 *The employment rate at the first best optimum satisfies*

$$\bar{G}_k^{\mathrm{FB}} \leq \mathrm{P}\left(\alpha_k(r_k;a) \leq r_k\right), \tag{19}$$

with equality if and only if everybody works at the optimum ($\bar{G}^{\mathrm{FB}} = 1$).

Proof By using the budget constraint, we get (dropping the index k)

$$w - \Delta - \int (w - \Delta) \, \mathrm{d}F \geq w - \Delta - \int_{w - \Delta \geq 0} (w - \Delta) \, \mathrm{d}F = w - \Delta - r. \tag{20}$$

It follows that

$$w - \Delta \geq 0 \Longrightarrow w - \Delta - \int (w - \Delta) \, \mathrm{d}F + r \geq 0$$

which gives (19). Equality in (19) and in (20) are both equivalent to $w - \Delta \geq 0$ for almost all a. ■

Proof of Theorem 5 The necessary part follows directly from Lemmas 1 and 5. The sufficient part is Lemma 4.

Proof of Lemma 3

We write $K_r(w)$ as

$$K_r(w) = \max_{(D,p)\in B(r)}(w-D)p$$

where $B(r)$ is the set

$$B(r) = \{(D,p) \mid 0 \le p \le G_{r,w}(D) \text{ and } -r \le D \le w\}.$$

We want to prove the continuity with respect to r. To simplify notations, we drop the index w in the expression $G_{r,w}(D)$. The proof follows from three preliminary properties.

<u>Claim 1:</u> For each r, the set $B(r)$ is compact.

<u>Claim 2:</u> The graph of the correspondence $r \to B(r)$ is closed, namely

$$(D_k, p_k, r_k) \to (D, p, r) \text{ and } (D_k, p_k) \in B(r_k) \Longrightarrow (D, p) \in B(r).$$

or, to be more specific

$$(D_k, p_k, r_k) \to (D, p, r) \text{ and } p_k \le G_{r_k}(D_k) \Longrightarrow p \le G_r(D). \tag{21}$$

<u>Claim 3:</u> For each sequence $r_k \to r$ and each $(D,p) \in B(r)$ with $D < w$, there exists a sequence $(D_k, p_k) \in B(r_k)$ such that $(D_k, p_k) \to (D, p)$.

<u>Proof of the continuity of $r \to K_r$:</u>

Pick a sequence $r_k \to r$. Take some $(D,p) \in B(r)$ such that $(w-D)p = K_r$. We can assume that $D < w$ (if $D = w$, then $K_r = 0$ and we can take any $D < w$ and $p = 0$). >From Claim 3, there exists $(D_k, p_k) \in B(r_k) \to (D, p)$. Passing to the limit in

$$(w - D_k)p_k \le K_{r_k}$$

we get $K_r \le \liminf K_{r_k}$.

Now consider another sequence (D_k, p_k) defined by

$$(D_k, p_k) \in B(r_k) \quad \text{and} \quad (w - D_k)p_k = K_{r_k}. \tag{22}$$

This sequence (D_k, p_k) is clearly bounded, then we can pick a convergent subsequence. We denote (D,p) the limit. >From Claim 2, we know that $(D,p) \in B(r)$. Passing to the limit in (22) gives $\limsup K_{r_k} \le K_r$, which completes the proof of continuity.

<u>Proof of Claim 1:</u> Since $D \to G_r(D)$ is nondecreasing and right continuous, the set $B(r)$ is closed. The fact that it is bounded is obvious.

<u>Proof of Claim 2:</u> The claim is obvious when G is continuous. In the general case, we use the properties of G: $G_r(D)$ is right continuous and nondecreasing with respect to D, and left continuous and nonincreasing with respect to r (since Δ is continuous and nondecreasing).

Any sequence can be separated into (at most) four subsequences.

1. A subsequence of k's such that $D_k \le D$ and $r_k \ge r$. Then $p_k \le G_{r_k}(D_k) \le G_r(D)$ for $k \ge G$, which gives $p \le G_r(D)$;
2. A subsequence of k's such that $D_k > D$ and $r_k < r$. Then we obtain (21) by taking the limit when $k \to +\infty$ in $p_k \le G_{r_k}(D_k)$, using the right (resp. left) continuity of $G_r(D)$ w.r.t. D (resp. r);

3. A subsequence of k's such that $r_k < r$ and $D_k \leq D$. Then $p_k \leq G_{r_k}(D_k) \leq G_{r_k}(D)$, which gives (21) by passing to the limit;

4. The last case is symmetric of case 3.

Proof of Claim 3: We have to check that for all $r_k \to r$ and $p \leq G_r(D)$ and each (D,p) such that $D < w$, there exists (D_k, p_k), such that $p_k \leq G_{r_k}(D_k)$ and $(D_k, p_k) \to (D,p)$.

We separate the r_k's that are below r from those that are above. First, consider the set of k's such that $r_k \leq r$. Then take $D_k = D$ and $p_k = \min(p, G_{r_k}(D))$. We have: $p_k \leq G_{r_k}(D_k)$, $D_k \to D$ and $p_k \to p$ thanks to the left continuity of $G_r(D)$ w.r.t. r.

Now consider the remaining k's, such that $r_k > r$. Let C_r be a finite upper bound for $\Delta'_c(s,a)$, $a \in A$, s in some interval $[r, r+\varepsilon]$, which exists under our regularity assumption. Take $D_k = D + C_r|r_k - r|$ and $p_k = p$. Then $D_k \leq w$ for k large enough. It is easy to check that, for all $a \in A$ and for k large enough,

$$\Delta(r,a) \leq D \Rightarrow \Delta(r_k, a) \leq D_k,$$

which implies $p_k \leq G_r(D) \leq G_{r_k}(D_k)$. It follows that $(D_k, p_k) \in B(r_k)$. The fact that $(D_k, p_k) \to (D,p)$ is obvious.

Proof of Theorem 11

Let D be a nondecreasing function on $[0, +\infty[$. For $y \geq D(0)$, we define

$$D^{-1}(y) = \inf\{w \geq 0, D(w) \geq y\}.$$

If the set $\{w \geq 0, D(w) \geq y\}$ is empty, we set: $D^{-1}(y) = +\infty$. It is easy to check that the function D^{-1} is nondecreasing. Therefore D^{-1} is measurable on its domain of definition.

Suppose that $D(w) \leq w$ for all w. Then it is easy to check that $y \leq D^{-1}(y)$ for all $y \geq D(0)$. Therefore the function

$$y \to \frac{1}{D^{-1}(y) - y}$$

is nonnegative with values in $[0, +\infty]$ and measurable on $[D(0), +\infty[$. It follows that for all $y \geq D(0)$ the integral

$$I(y) = \int_y^{+\infty} \frac{\mathrm{d}x}{D^{-1}(x) - x} \tag{23}$$

can be unambiguously defined (see Rudin, 1966, Real and Complex Analysis, McGraw-Hill, Chapter 1). Its value is either a nonnegative real number or $+\infty$.

Suppose that there exists $y_1 \geq D(0)$ such that $I(y_1) < +\infty$. It follows from Lebesgue dominated-convergence theorem (see Rudin, 1966) that $I(y)$ tends to zero as y goes to $+\infty$. Finally, we define y_0 by

$$y_0 = \inf\{y \geq D(0), I(y) < +\infty\}. \tag{24}$$

The function $y \to I(y)$ is finite and decreasing on $]y_0, +\infty[$ and it tends to zero as y goes to $+\infty$.

Lemma 6 *Let D be a nondecreasing function on $[0, +\infty[$ satisfying*

- *$D(w) \leq w$ for all w;*
- *there exists $y_1 \geq D(0)$ such that $I(y_1) < +\infty$, where I is given by (23).*

Then we can define the function G by

$$G(d) = \exp(-I(d)) \tag{25}$$

for all $d > y_0$, where y_0 is given by (24). The function G is the restriction on $[y_0, +\infty[$ of the cumulative distribution function of a probability measure on $\mathbb{R}$. Furthermore we have

$$\max_{d \geq D(0)}(w-d)G(d) = (w - D(w))G(D(w)) \tag{26}$$

for all $w \geq 0$.

Finally, suppose that D is discontinuous at some point w_0. Then for every d_1, d_2 in $[D(w_0^-), D^(w_0^+)]$, we have

$$(w-d_1)G(d_1) = (w-d_2)G(d_2), \tag{27}$$

where $w = D^{-1}(d_1) = D^{-1}(d_2)$.

Proof The first part of the result is obvious since the function G defined by (25) is nondecreasing and tends to zero as d goes to $+\infty$. Note that $G(y)$ tends either to 0 or to some $\alpha > 0$ when y goes to y_0. In the latter case, we set: $G(d) = 0$ for $d < y_0$ (which amounts to put the positive mass α on y_0).

We now check that equation (26) is satisfied. Let $w \geq 0$. Consider first the case $d \leq D(w)$. For all $x \leq D(w)$, we have $D^{-1}(x) \leq w$, then $D^{-1}(x) - x \leq w - x$ and

$$\ln \frac{w-d}{w-D(w)} = \int_d^{D(w)} \frac{\mathrm{d}x}{w-x} \leq \int_d^{D(w)} \frac{\mathrm{d}x}{D^{-1}(x)-x}.$$

which is equivalent to

$$(w-d)G(d) \leq (w-D(w))G(D(w)).$$

We show in a similar way that the above inequality holds for $d \geq D(w)$ as well, which completes the proof of (26).

Now consider w_0 such that $D(w_0^-) < D(w_0^+)$ and take $d_1 < d_2$ in $[D(w_0^-), D(w_0^+)]$. The function D^{-1} is constant on $[D(w_0^-), D(w_0^+)]$. We note $w = D^{-1}(d_1) = D^{-1}(d_2)$ (remark that $w = w_0^-$). We have

$$I(d_2) - I(d_1) = \int_{d_1}^{d_2} \frac{\mathrm{d}x}{w-x} = \ln \frac{w-d_1}{w-d_2},$$

which yields (27).

Note that $K_r(w)$ given by (16) tends to $+\infty$ as $w \to +\infty$. It follows that there exists many distributions satisfying the budget constraint $T(r) = 0$.