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An Aggregation Problem : The Demand for Unskilled Labour

E. MALINVAUD[1]

[1] CREST – INSEE.

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Edmond MALINVAUD

CREST-INSEE

Abstract

A common conjecture concerns the effect of changes in the real unit cost of unskilled labour on the long-run demand for this labour : application of macroeconomic models, calibrated thanks to estimates of elasticities of substitution between productive factors, would under-estimate the effect in question, because such models ignore substitutions among consumption goods or services, which follow from induced changes in the relative prices of these goods. The under-estimation would be an aggregation bias due to microeconomic heterogeneity. The bias is here studied within a specification allowing for two sources of heterogeneity : in the unskilled-labour-input requirement for the production of various goods, in the demand functions for these goods. Two crucial characteristics of heterogeneity across goods enter the formula for the bias : a variance of an unskilled-labour-input intensity, a covariance between this intensity and the value of a characteristic of the demand function.

Résumé

Une conjecture fréquemment énoncée concerne l'effet des variations du coût du travail non-qualifié sur la demande à long terme de ce travail : l'application de modèles macroéconomiques, calibrés à l'aide d'estimations des élasticités de substitution entre facteurs de production, devrait sous-estimer l'effet en cause, car de tels modèles ignorent les substitutions entre biens de consommation, elles-mêmes induites par les modifications dans les prix relatifs des divers biens et services ; cette sous-estimation constituerait un biais d'agrégation dû à l'hétérogénéité microéconomique. Le biais est étudié ici dans le cadre d'une spécification où interviennent deux sources d'hétérogénéité, la première concernant les besoins en travail non-qualifié pour la production des différents biens, la seconde les fonctions de demande de ces biens. Deux mesures de l'hétérogénéité figurent dans la formule donnant la valeur du biais : la variance, entre les divers biens, d'un paramètre lié à leur contenu en travail non-qualifié, la covariance de ce paramètre et d'une caractéristique de la fonction de demande pour le bien correspondant.

Keywords. : Aggregation, labour demand, unskilled labour, system of consumption demand functions, heterogeneity.

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1. A conjecture

By how much will a permanent decrease in the real cost of unskilled labour increase in the long run the demand for this labour ? The question makes sense, particularly with respect to political debates in several Western European countries. Most often attempts at answering it concentrate on evaluating substitutabilities among factors of production. But the conjecture is also made that, beyond the effect directly resulting from factor substitutions, there is also a sizable indirect effect coming from changes in the composition of the demand for goods. The contemplated decrease in the real cost of unskilled labour would generate a decrease in the relative price of goods which are particularly unskilled-labour intensive, hence an increase in the demand for those goods, which would reinforce the demand for unskilled labour. In other words, macroeconomic models calibrated from estimates of elasticities of substitution between factors in production units would under-estimate the effect in question. They would be subject to a systematic aggregation bias coming from the heterogeneity of productive techniques with respect to their factor-input requirements.

A precise study of this supposed aggregation bias is a bit complex, because it has to make simultaneously explicit three sides of the main phenomenon : not only factor demands, but also the formation of relative prices and shifts in the demand for goods whose relative prices vary. This study reveals the existence of cases in which the aggregation bias vanishes, or even becomes counter-intuitive, the feedback from relative prices then going opposite to the direct effect from factor substitutions. At the end of the day research seems to find good reasons for us to be confident in the value of the conjecture, but with more provisos than expected, thus calling for extensive econometric investigation.

For the analytical exploration, which is presented in Malinvaud (2001.a) and will be substantially complemented here, the following framework looks to be suitable, even though it clearly simplifies in several respects.

(i) There are just two productive factors, here called skilled labour and unskilled labour. The market for skilled labour is competitive. The exogenous supply $\overline{L}$ of this labour is then fully employed at a real wage rate *w*, which will alternatively be called the skilled-labour real unit cost. On the market for unskilled labour the real unit cost *v* is exogenous. At this unit cost the demand *M* for unskilled labour leaves an unemployed excess supply.

(ii) There are *n* sectors of production, each one producing a specific good *i* (*i* = 1, 2...*n*) directly from the two factors. Sectors are represented as elementary units sector *i* producing output y_i, sold on a competitive market at price p_i, from inputs L_i and M_i

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according to a specific production function with decreasing marginal productivities and non-increasing returns to scale.

(iii) The consumption sector is treated as a single price-taking unit maximizing a utility $U(x_1,...,x_n)$ subject to a budget constraint. Equilibrium of the markets for goods requires $x_i = y_i$ for all *i*.

(iv) The analysis is static, being meant to be appropriate for a long-run of one decade or more.

The list of these hypotheses is meant to simplify as much as possible while allowing for an endogenous determination of relative prices and of the composition of the demand for goods. Only the market for unskilled labour experiences a disequilibrium, and one with excess-supply. But this level of disaggregation and this configuration of market equilibria seem to be relevant for our purpose, namely to focus on a long run to which would apply the diagnosis that, on the one hand, minimum wages will be permanently maintained above the market-clearing unskilled-labour wage, but on the other hand, firms will not be permanently exposed to a lack of demand. In other words, the study is meant to be orthogonal to one which would focus on Keynesian unemployment.

Malinvaud (2001.a) deals with a full CES specification of this framework, with two exogenous constant elasticities of substitution : σ in the consumption sector and σ_P having the same value in all productive sectors. It turns out that, when both σ and σ_P are equal to 1, so that we are facing a log-linear economy, the aggregation bias, whose definition will be precisely given, disappears. In contrast when there is strict complementarity in production $(\sigma_P = 0)$ or perfect substitutability in consumption $(\sigma = \infty)$, the aggregation bias is positive. More generally, the sign of the aggregation bias is most often the same as that of $(\sigma - \sigma_P)/\sigma_P$. Indeed, the extreme of strict complementarity in consumption $(\sigma = 0)$ permits to understand the nature of the cases which invalidate the conjecture stated at the beginning, cases in which the feedback through relative prices runs counter to the direct factor-substitution effect. However, numerical exercises suggest that negative aggregation biases are likely to have negligible absolute values, whereas substantial positive biases may be found for values of σ and σ_P which cannot be said to be obviously irrealistic. For instance with $\sigma = 3$ and $\sigma_P = 0.5,$ say, the bias is equal to 0.08 if the share β_i of the value of output due the unskilled labour is on average across sectors equal to $\overline{\beta} = 0.1$ and its variance equal to $Var\ (\beta) = 25\times10^{-4}$. With again $\sigma = 3$ and $\sigma_P = 0.5$ the bias is equal to 0.20 if $\overline{\beta} = 0.2$ and $Var\ (\beta) = 10^{-2}$.

The relevance of the full CES specification may be questioned on the ground that the system of demand functions for goods exhibits Engel curves which are all straight lines from the origin. This is why the present paper investigates the aggregation bias within a model allowing for two significant dimensions of heterogeneity, not only heterogeneity of productive techniques with respect to their factor-input requirements, but also heterogeneity of the demands for goods, ranging from necessities to luxuries. On the other hand, after the rather extensive discussion in Malinvaud (2001.a) of the role of values given to the two parameters σ and σ_P within the domain defined by the first quadrant, we shall now restrict attention to $\sigma_P = 1$, that is, to Cobb-Douglas techniques. We shall even assume constant returns to scale, again for simplicity after the discussion in annex 3 of the earlier paper.

Introduction of these two dimensions of heterogeneity makes application of the approach defined in Malinvaud (2001.a) less easy, but it also brings interesting results. Indeed, it exhibits the role of the correlation between the two sources of heterogeneity across the full range of sectors (and goods they respectively produce). In particular, it turns out that the aggregation bias is all the larger as luxuries rather than necessities are relatively more unskilled-labour intensive.

Section 2 defines an aggregate model which will serve as a reference for the definition of the aggregation bias and incorporates the hypotheses listed above except for the multiplicity of sectors and goods. Section 3 presents the multi-sector model to be investigated in sections 4 to 7. The sign and size of the aggregation bias is examined in section 8. The last section 9 discusses the econometric work that would be relevant after the theoretical analysis of this paper.

2. An aggregate model

With just one sector producing the single good in the economy, this good naturally serves as numeraire. The price system is defined by the two real wage rates, w for skilled labour, v for unskilled labour. A single production function relates output y to the two inputs, L for skilled and M for unskilled labour. The hypotheses listed in section 1 imply :

$$y = f(L, M) \qquad\qquad L = \overline{L} \tag{1}$$

$$w = f_L^{'}(L, M) \qquad\qquad v = f_M^{'}(L, M) \tag{2}$$

These four equations determine the four endogenous variables y, L, M, w from the two exogenous variables v and $\overline{L}$.

Studying how M varies as a function of v for given $\overline{L}$ is particularly easy. The last of the four equations suffices for this study after L is replaced by $\overline{L}$. In the neighbourhood of an equilibrium the elasticity of the demand for unskilled labour with respect to its exogenous real unit cost v is directly found :

$$\frac{dM}{M} = -\eta \frac{dv}{v} \tag{3}$$

where

$$\eta = \frac{-f_M^{'}(\overline{L}, M)}{M f_{M^2}^{''}(\overline{L}, M)} \tag{4}$$

The classical hypotheses about production functions, in particular their concavity, implies that η is positive : a decrease in the unskilled wage leads to an increase in the demand for unskilled labour. Simultaneously there is an increase in output.

It is trivial but significant to note that the elasticity η appearing in this simple model does not correspond only to a factor-substitution effect, which by definition would occur along an isoquant. It takes account also of the income effect following form the increase in output.

We may consider in particular the Cobb-Douglas production function :

$$f(L, M) = AL^{\alpha} M^{\beta} \tag{5}$$

leading directly to

$$\eta = \frac{1}{1-\beta} \tag{6}$$

Simultaneously, when $\bar{L}$ is maintained fixed :

$$\frac{dy}{y} = \frac{dw}{w} = \frac{-\beta}{1-\beta} . \frac{dv}{v} \tag{7}$$

The elasticity η is naturally decomposed into the part, equal to 1, coming from the substitution effect and the part equal to $\beta/(1-\beta),$ measuring the income effect.

On the basis of this aggregate model it is natural to look for estimates of derivatives of production functions identifying unskilled labour as an input, or still simpler, to look for estimates of the exponent of this input in Cobb-Douglas fits. Data may come, for instance, from a panel of firms. The value found on average for β would enter the right-hand side of equation (6). Would a bias result from such a simple transposition of microeconomic data to a macroeconomic model ? Answering the question requires that we place it within the context of a precisely specified microeconomic model.

3. The basic model

We now turn to the case of *n* goods (*i* = 1, 2...*n*) each one being produced in its sector *i*, output being denoted as y_i. Since market-clearing is assumed for goods, we do not distinguish in the notation the demand for good *i* from output y_i. Good *i* has a price p_i and aggregate income *Y* is given by :

$$\sum_{i=1}^{n} p_i y_i = Y \tag{8}$$

We specify as follows the utility function *U* from which the demands for goods will be derived :

$$U^{(\sigma-1)/\sigma} = \sum_{i=1}^{n} \gamma_i^{1/\sigma} (y_i - \chi_i)^{(\sigma-1)/\sigma} \qquad \text{if } \sigma \neq 1 \tag{9}$$

$$\log U = \sum_{i=1}^{n} \gamma_i \log(y_i - \chi_i) \qquad \text{if} \quad \sigma = 1 \qquad (9')$$

where the parameters γ_i and σ, an elasticity of substitution, are positive with

$$\sum_{i=1}^{n} \gamma_i = 1 \qquad (10)$$

where moreover appear the parameters χ_i. As explained in section 1, the *n* demands y_i are jointly determined so as to maximize *U* subject to the budget constraint (8) in which the prices p_i and aggregate income *Y* are taken as given. This maximization leads to :

$$y_i = \chi_i + \gamma_i \left[\frac{p_i}{P}\right]^{-\sigma} \frac{Y_e}{P} \qquad (11)$$

where the two auxiliary variables Y_e and *P* are given by :

$$Y_e = Y - \sum_{j=1}^{n} p_j \chi_j \qquad P^{1-\sigma} = \sum_{j=1}^{n} \gamma_j p_j^{1-\sigma} \qquad \text{if} \quad \sigma \neq 1 \qquad (12)$$

$$\log\ P = \sum_{j=1}^{n} \gamma_j \log p_j \qquad \text{if} \quad \sigma = 1 \qquad (12')$$

For obvious reason the variable Y_e will be called excess-income and the variable *P* appears as a price index. It is a natural idea to take *P* as the numeraire, which amounts to impose on the prices p_i the normalization :

$$\sum_{i=1}^{n} \gamma_i p_i^{1-\sigma} = 1 \qquad \text{if} \quad \sigma \neq 1 \qquad (13)$$

$$\sum_{i=1}^{n} \gamma_i \log p_i = 0 \qquad \text{if} \quad \sigma = 1 \qquad (13')$$

The system of demand laws defined by (11) is a simple generalization to an arbitrary elasticity of substitution σ of what is known, with $\sigma = 1$, as the Geary-Stone system, the first such system to have been used in econometric analysis of consumption data (see in particular R. Stone, 1954). Necessities are characterized by positive parameters χ_i, whereas $\chi_i \leq 0$ when *i* is a luxury.

For what follows it is often convenient to write system (11) as

$$p_i y_i = p_i \chi_i + k_i Y_e \qquad (14)$$

with

$$k_i = \gamma_i p_i^{1-\sigma} \quad (15)$$

$$Y_e = \sum_{j=1}^{n} p_j (y_j - \chi_j) \quad (16)$$

Variables k_i are positive and sum up to 1 because of the normalization (13) if $\sigma \neq 1$ and of (10) if $\sigma = 1$.

With Cobb-Douglas production functions the activity of sector *i* in its competitive environment is characterized by three equations corresponding to its production function and to equality of the marginal productivity of each input to its unit cost :

$$y_i = A_i L_i^{1-\beta_i} M_i^{\beta_i} \quad (17)$$

$$wL_i = (1-\beta_i) p_i y_i \qquad vM_i = \beta_i p_i y_i \quad (18)$$

A_i and β_i being parameters.

We may characterize the model as having 4*n* microeconomic endogenous variables (y_i, L_i, M_i, p_i) and 3 macroeconomic endogenous variables (Y_e, w, M) for two exogenous variables (*v*, $\overline{L}$). The corresponding system of 4*n* + 3 independent equations is given by (14) with k_i expressed by (15), (17), (18), (13) and :

$$\sum_{j=1}^{n} L_j = \overline{L} \qquad \sum_{j=1}^{n} M_j = M \quad (19)$$

Equation (16) is a direct consequence of (14) since the k_i sum up to 1. It does not provide an additional independent equation.

Since our aim is to determine the elasticity of *M* with respect to *v*, when $\overline{L}$ is maintained fixed, we shall work with this system of 4*n* + 3 endogenous variables and 4*n* + 3 equations. More precisely, starting from an equilibrium of the system, we shall study how the equilibrium changes when *v* varies by an infinitesimal change *dv*. This will give us in particular the corresponding infinitesimal change *dM* brought to *M*.

4. Equations within a neighbourhood of an equilibrium. First reduction

It will be convenient to work with relative changes such as *dv/v* and *dM/M*. Let us then start with writing most equations in logarithmic form. Equation (14) with k_i replaced by (15) gives :

$$\log y_i + \log\left[1 - \frac{\chi_i}{y_i}\right] = -\sigma \log p_i + \log Y_e + \log \gamma_i \quad (20)$$

Equations (17) and (18) give :

$$\log y_i = (1-\beta_i)\log L_i + \beta_i \log M_i + \log A_i \tag{21}$$

$$\log w = \log p_i + \log y_i - \log L_i + \log(1-\beta_i) \tag{22}$$

$$\log v = \log p_i + \log y_i - \log M_i + \log \beta_i \tag{23}$$

Equation (13) or (13') may remain in its present form. For the time being we have to work on this system of 4*n* + 1 equations. Equations (19) will appear much later on in the argument.

Differentiation of equations (20) to (23) and (13) or (13') leads to :

$$\frac{dy_i}{y_i} = \left[1-\frac{\chi_i}{y_i}\right]\left[-\sigma\frac{dp_i}{p_i} + \frac{dY_e}{Y_e}\right] \tag{24}$$

$$\frac{dy_i}{y_i} = (1-\beta_i)\frac{dL_i}{L_i} + \beta_i\frac{dM_i}{M_i} \tag{25}$$

$$\frac{dw}{w} = \frac{dp_i}{p_i} + \frac{dy_i}{y_i} - \frac{dL_i}{L_i} \tag{26}$$

$$\frac{dv}{v} = \frac{dp_i}{p_i} + \frac{dy_i}{y_i} - \frac{dM_i}{M_i} \tag{27}$$

$$\sum_j k_j \frac{dp_j}{p_j} = 0 \tag{28}$$

We immediately see from (26) and (27) that

$$\frac{dM_i}{M_i} - \frac{dL_i}{L_i} = -\left[\frac{dv}{v} - \frac{dw}{w}\right] \tag{29}$$

The intensity of factor substitution has to be the same in all sectors : they are indeed exposed to the same change in the relative cost of unskilled versus skilled labour and their production functions have the same elasticity of substitution $\sigma_P = 1$. Moreover examination of equations (24) to (27) suggests a natural reduction of this system of 4*n* equations to a simpler system of 2*n* equations from which the 2*n* variables dy_i / y_i and dp_i / p_i would have been eliminated.

Elimination leads to :

$$-\frac{dw}{w} = [1-\lambda_i + \lambda_i\beta_i]\frac{dL_i}{L_i} - \lambda_i\beta_i\frac{dM_i}{M_i} - \frac{1}{\sigma}\frac{dY_e}{Y_e} \tag{30}$$

$$-\frac{dv}{v} = -[\lambda_i - \lambda_i\beta_i]\frac{dL_i}{L_i} + [1-\lambda_i\beta_i]\frac{dM_i}{M_i} - \frac{1}{\sigma}\frac{dY_e}{Y_e} \tag{31}$$

where the new variable λ_i is defined by :

$$\lambda_i = 1 - \frac{1}{\sigma}\left[1 - \frac{\chi_i}{y_i}\right]^{-1} \tag{32}$$

Goods that are major necessities are characterized by high values of χ_i / y_i hence by high values of the inverse square bracket and by low values of λ_i .

The simplicity of system (30)-(31) immediately appears. We want to take advantage of it. But we may note in passing that elimination of dY_e / Y_e would be straightforward. From equations (24), each multiplied by $k_i[1-\chi_i / y_i]^{-1}$, and equation (28) it follows that dY_e / Y_e is a function of the *n* relative changes dy_j / y_j (see equation (46) below). Moreover (25) gives dy_j / y_j as a function of dL_j / L_j and dM_j / M_j. We could then write a system of 2*n* equations on the 2*n* + 2 relative changes dL_i / L_i, dM_i / M_i, *dw/w* and *dv/v*, the last of which is exogenous. Taking the two additional equations (19) would introduce just one additional relative change *dM/M*, which is particularly relevant in this paper. This is indeed close to the route we are going to follow in order to solve the system. But before doing so, let us take advantage of the simplicity of (30)-(31).

5. From two significant auxiliary macroeconomic variables to microeconomic changes

This simplicity suggests that we try and decompose each one of the microeconomic relative changes into a macroeconomic term and a microeconomic term according to :

$$\frac{dL_i}{L_i} = \varphi + \eta_i \qquad \frac{dM_i}{M_i} = \psi + \theta_i \tag{33}$$

in which moreover φ and ψ would fulfil two equations similar to average expressions of respectively (30) and (31). More precisely such equations are going to be :

$$-\frac{dw}{w} = [1 - \bar{\lambda} + m(\lambda\beta)]\varphi - m(\lambda\beta)\psi - \frac{1}{\sigma}\frac{dY_e}{Y_e} \tag{34}$$

$$-\frac{dv}{v} = [\bar{\lambda} - m(\lambda\beta)]\varphi + [1 - m(\lambda\beta)]\psi - \frac{1}{\sigma}\frac{dY_e}{Y_e} \tag{35}$$

where $\bar{\lambda}$ and $m(\lambda\beta)$ are convenient averages of respectively the λ_i and the products $\lambda_i\beta_i$:

$$\bar{\lambda} = \sum_j k_j\lambda_j \qquad m(\lambda\beta) = \sum_j k_j\lambda_j\beta_j \qquad \bar{\beta} = \sum_j k_j\beta_j \tag{36}$$

The last of these equations announces the definition we are going to use for the average $\overline{\beta}$ of the β_i. As we shall see, $\overline{\lambda}$ and $\overline{\beta}$ so defined will appear as providing, in this model, natural measures of the first order moments of the statistical distribution of (λ_i, β_i) across sectors. However, we shall have to pay particular attention in section 8 to the meaning of this definition of $\overline{\beta}$ with respect to the conjecture examined in this article. For the time being, let us go on with the determination of η_i and θ_i.

Subtracting (34) from (30) and (35) from (31), taking account of (33) leads to :

$$(1-\lambda_i+\lambda_i\beta_i)\eta_i-\lambda_i\beta_i\theta_i=\rho_i \tag{37}$$

$$-(\lambda_i-\lambda_i\beta_i)\eta_i+(1-\lambda_i\beta_i)\theta_i=\rho_i \tag{38}$$

with :

$$\rho_i=[\lambda_i\beta_i-m(\lambda\beta)](\psi-\varphi)+[\lambda_i-\overline{\lambda}]\varphi \tag{39}$$

Subtracting (38) from (37) directly gives :

$$\eta_i=\theta_i \tag{40}$$

The two microeconomic terms in (33) are equal, so that :

$$\frac{dM_i}{M_i}-\frac{dL_i}{L_i}=\psi-\varphi \tag{41}$$

As we already observed with (29), the intensity of factor substitution depends only on macroeconomic variables. The fact that the two right-hand members of (29) and (41) are equal is a direct consequence of our definition of φ and ψ by the system (34)-(35), as is easily seen by subtraction of (34) from (35). But the fact that the microeconomic components of the two changes in the labour inputs of sector *i* are equal to θ_i does not imply that a microeconomic component of the relative change in output y_i is also equal to θ_i. Indeed (25) and (33) rather imply :

$$\frac{dy_i}{y_i}=\varphi+\beta_i(\psi-\varphi)+\theta_i \tag{42}$$

or equivalently :

$$\frac{dy_i}{y_i}=\psi+(1-\beta_i)(\varphi-\psi)+\theta_i \tag{43}$$

No matter how we decompose dy_i/y_i the microeconomic component differs from θ_i if the β_i in different sectors differ.

Since (40) applies, (37) can be simply written as :

$$(1-\lambda_i)\theta_i = \rho_i \tag{44}$$

The equation, together with (39), gives the value of θ_i. This value is an explicit function of the following elements : the macroeconomic infinitesimal terms φ and ψ, the parameter β_i and the variable λ_i of sector *i*, but also the parameters β_j together with variables λ_j and k_i of all sectors through the mean values $m(\lambda\beta)$ and $\overline{\lambda}$. These variables themselves depend on parameters χ_i and γ_j as well as on the values of y_j and p_j in the reference equilibrium. Clearly, relevant macroeconomic variables will similarly depend on all these elements.

For instance it is useful at this stage that we look at dY_e / Y_e in order to purge equations (34) and (35) from their last term. We first write (24) as :

$$\frac{1}{\sigma}\frac{dY_e}{Y_e} = (1-\lambda_i)\frac{dy_i}{y_i} + \frac{dp_i}{p_i} \tag{45}$$

Then (28) implies :

$$\frac{1}{\sigma}\frac{dY_e}{Y_e} = \sum_{j=1}^{n} k_j (1-\lambda_j)\frac{dy_j}{y_j} \tag{46}$$

Inserting (42), taking account of (44) and (39) we find :

$$\frac{1}{\sigma}\frac{dY_e}{Y_e} = (1-\overline{\lambda})\varphi + [\overline{\beta} - m(\lambda\beta)](\psi - \varphi) \tag{47}$$

Reporting in (34) – (35) we obtain the following two equations, which link φ and ψ to no other infinitesimal change than *dw/w* and *dv/v* :

$$-\frac{dw}{w} = \overline{\beta}(\varphi - \psi) \tag{48}$$

$$-\frac{dv}{v} = (1-\overline{\beta})(\psi - \varphi) \tag{49}$$

Clearly, these two simple equations will be useful for the full solution of our problem. We note in particular that they imply :

$$\frac{dw}{w} = \frac{-\overline{\beta}}{1-\overline{\beta}}.\frac{dv}{v} \tag{50}$$

which directly determines the endogenous change in the wage rate of skilled labour from the exogenous change in the unskilled wage. Comparison with equation (7) leads us to say that, according to our model, the elasticity of the skilled wage with respect to the unskilled wage suffers from no aggregation bias. This is a consequence of the constant returns to scale assumption. (The conclusion follows from the system (103)-(104) which, in Malinvaud (2001.a), corresponds to the system (48) – (49) of this paper).

Before leaving this section mainly devoted to finding how to compute microeconomic changes from φ and ψ, let us still note that equations (24), (39), (42), (44) and (47) imply a very simple equation for price changes :

$$\frac{dp_i}{p_i} = (\beta_i - \overline{\beta})(\varphi - \psi) \tag{51}$$

If for instance $\varphi - \psi$ is negative, which according to (49) means that *v* decreases, the price decreases for unskilled-labour intensive goods.

6. Aggregation of labour demands

Since we now know how to compute the microeconomic relative changes dL_i / L_i and dM_i / M_i, we are in a position to consider the corresponding aggregate changes *dL/L* and *dM/M*. We start from the equation :

$$\frac{dL}{L} = \sum_i \frac{L_i}{L}\frac{dL_i}{L_i} = \varphi + \sum_i \frac{L_i}{L}\theta_i \tag{52}$$

The weights L_i / L, with which the infinitesimal changes θ_i have to be combined, come from the reference equilibrium. We have to derive their expression in terms of the same parameters and variables as served for finding the expression of θ_i. This we can do from equations (14), (18) and (32), which imply :

$$wL_i = (1-\beta_i)\, p_i y_i = \sigma k_i (1-\beta_i)(1-\lambda_i) Y_e \tag{53}$$

Summing over sectors we reach :

$$wL = \sigma[1 - \overline{\beta} - \overline{\lambda} + m(\lambda\beta)]Y_e \tag{54}$$

Hence the weights are given by :

$$\frac{L_i}{L} = \frac{k_i(1-\beta_i)(1-\lambda_i)}{1 - \overline{\beta} - \overline{\lambda} + m(\lambda\beta)} \tag{55}$$

Equations (44) and (39) now show :

$$\sum_i k_i (1-\beta_i)(1-\lambda_i)\theta_i = \sum_i k_i (1-\beta_i)\rho_i \tag{56}$$
$$= -[m(\lambda\beta) - \overline{\lambda}\overline{\beta}]\varphi + [m(\lambda\beta^2) - \overline{\beta} m(\lambda\beta)](\varphi - \psi)$$

Together with (55) this last equation transforms (52) into :

$$\frac{dL}{L}=\frac{(1-\overline{\beta})(1-\overline{\lambda})\varphi+[m(\lambda\beta^2)-\overline{\beta}m(\lambda\beta)](\varphi-\psi)}{1-\overline{\beta}-\overline{\lambda}+m(\lambda\beta)} \tag{57}$$

Considering this expression we see that it contains mean values of the products $\lambda_i\beta_i$ and $\lambda_i\beta_i^2$, the last ones coming from the combination between the weight L_i/L and the microeconomic change variable θ_i. In interpretations it will be convenient to replace such mean values of products by their expressions in terms of moments, i.e. of mean values of products of deviations $\lambda_i-\overline{\lambda}$ and $\beta_i-\overline{\beta}$. We therefore introduce the notation $\overline{m}$ illustrated by :

$$\overline{m}(\lambda\beta^2)=\sum_i k_i(\lambda_i-\overline{\lambda})(\beta_i-\overline{\beta})^2 \tag{58}$$

We then note that :

$$m(\lambda\beta)=\overline{m}(\lambda\beta)+\overline{\lambda}\overline{\beta} \tag{59}$$

$$m(\lambda\beta^2)=\overline{m}(\lambda\beta^2)+\overline{\lambda}\overline{m}(\beta^2)+2\overline{\beta}\overline{m}(\lambda\beta)+\overline{\lambda}\overline{\beta}^2 \tag{60}$$

so that :

$$m(\lambda\beta^2)-\overline{\beta}m(\lambda\beta)=\overline{m}(\lambda\beta^2)+\overline{\lambda}\overline{m}(\beta^2)+\overline{m}(\lambda\beta) \tag{61}$$

Finally we may write (57) as :

$$\frac{dL}{L}=\frac{(1-\overline{\beta})(1-\overline{\lambda})\varphi+[\overline{m}(\lambda\beta^2)+\overline{\lambda}\overline{m}(\beta^2)+\overline{m}(\lambda\beta)](\varphi-\psi)}{(1-\overline{\beta})(1-\overline{\lambda})+\overline{m}(\lambda\beta)} \tag{62}$$

I shall later give reasons for $\overline{m}(\lambda\beta^2)$ to be taken as negligible. Two second order moments of the statistical distribution across sectors thus seem to be relevant : the variance of β_i, a natural measure of heterogeneity of productive techniques, and the covariance of β_i and λ_i which may be taken as an indicator of the concordance between heterogeneity in input requirements and heterogeneity in the demands for goods.

The same kind of argument applied to the aggregation of the demands for unskilled labour leads to :

$$\frac{dM}{M}=\frac{\overline{\beta}(1-\overline{\lambda})\psi+[\overline{m}(\lambda\beta^2)+\overline{\lambda}\overline{m}(\beta^2)-(1-\overline{\beta})m(\lambda\beta)](\psi-\varphi)}{\overline{\beta}(1-\overline{\lambda})-\overline{m}(\lambda\beta)} \tag{63}$$

Heterogeneity again appears through three moments of the statistical distribution, the two most relevant being the variance of β_i and the covariance between β_i and λ_i.

Let us note in passing that the relative change in the aggregate volume of output, i.e. in the average of the dy_i / y_i computed with weights $p_i y_i / Y$, could also be written as a function of φ, ψ and characteristics of the statistical distribution. Elimination of φ and ψ from the system made of that equation together with (62) and (63) would provide a local approximation of "the aggregate production function". But our purpose in this article is not to contribute to the theory of aggregation of production functions. So, we do not insist on this remark.

7. Full solution

At this point we are in the position to derive the full implications of our model as far as the macroeconomic infinitesimal changes in the neighbourhood of an equilibrium are concerned. The four equations (48), (49), (62) and (63) contain one such exogenous change dv/v and exactly four endogenous changes *dw/w*, φ, ψ, *dM/M*, because we know from the equilibrium of the skilled-labour market (19) that in (62) the left-hand member has to be equal to zero. We even see that the two equations (49) and (62) determine φ and ψ as functions of *dv/v*. The elasticity of *M* with respect to *v* will be read from (63) when φ and ψ will there be replaced by expressions of these functions. The elasticity in question will depend on $\overline{\beta}, \overline{\lambda}, \overline{m}(\lambda\beta), \overline{m}(\beta^2)$ and $\overline{m}(\lambda\beta^2)$, which are characteristics of the joint statistical distribution of (β_i, λ_i) across sectors. Microeconomic heterogeneity will so enter the formula we are now looking for.

With *dL* = 0 in (62) and with equation (49) we directly obtain :

$$(1-\overline{\beta})^2(1-\overline{\lambda})\varphi = -[\overline{m}(\lambda\beta^2) + \overline{\lambda}\overline{m}(\beta^2) + \overline{m}(\lambda\beta)]\frac{dv}{v} \qquad (64)$$

and simultaneously :

$$(1-\overline{\beta})^2(1-\overline{\lambda})\psi = -[\overline{m}(\lambda\beta^2) + \overline{\lambda}\overline{m}(\beta^2) + \overline{m}(\lambda\beta) + (1-\overline{\beta})(1-\overline{\lambda})]\frac{dv}{v} \qquad (65)$$

Introducing these values of φ and ψ in (63) we reach :

$$(1-\overline{\beta})^2[\overline{\beta}(1-\overline{\lambda}) - \overline{m}(\lambda\beta)]\frac{dM}{M} = -[\overline{m}(\overline{\lambda}\overline{\beta}^2) + \overline{\lambda}\overline{m}(\beta^2) + \overline{m}(\lambda\beta) + \overline{\beta}(1-\overline{\beta})(1-\overline{\lambda})]\frac{dv}{v} \qquad (66)$$

Finding this formula was our objective. The result may look a bit frightening. Thus, let us try to make it as transparent as possible. First, let us identify what may be called the aggregation bias *B*. A simple transposition of the aggregate model of section 2 would have argued from (3) and replaced β by $\overline{\beta}$ in (6). It is thus natural to interpret (66) thanks to its identification with :

$$\frac{dM}{M} = -\left[\frac{1}{1-\overline{\beta}} + B\right]\frac{dv}{v} \qquad (67)$$

This gives the following formula for the bias :

$$(1-\overline{\beta})^2[\overline{\beta}(1-\overline{\lambda})-\overline{m}(\lambda\beta)]B=\overline{m}(\lambda\beta^2)+\overline{\lambda}\overline{m}(\beta^2)+\overline{\beta}\overline{m}(\lambda\beta) \quad (68)$$

We know that, if the bivariate statistical distribution of (λ_i, β_i) was Gaussian, the third-order moment $\overline{m}(\lambda\beta^2)$ would be equal to zero. The same would be true with all bivariate distributions having the required symmetry. Moreover, there is little chance that econometric evidence will soon reveal significant departure from such a symmetry. Let us then neglect this third-order moment. Using a probably more transparent notation for second-order moments, we shall focus on the following formula :

$$B=\frac{\overline{\lambda}Var(\beta)+\overline{\beta}Cov(\lambda,\beta)}{(1-\overline{\beta})^2[\overline{\beta}(1-\overline{\lambda})-Cov(\lambda,\beta)]} \quad (69)$$

8. Meaning, sign and size of the aggregation bias

As we are now going to interpret the above formula, we must first pay attention to the definition chosen for the average $\overline{\beta}$ of the β_i. Whereas equation (66) applies unambiguously with the definition given by (36), interpreting *B* in (67) as "the aggregation bias" critically depends on the persuasiveness of the definition chosen for $\overline{\beta}$. Is this definition appropriate for representing by $-[1-\overline{\beta}]^{-1}$ the evaluation that users of the direct macroeconomic model of section 2 would give when asked to estimate the elasticity of the long-run demand for unskilled labour with respect to its real unit cost ? Are the weights k_i defined by (15) appropriate for the transposition made here from our microeconomic model to an evaluation of the bias that is likely to result from macroeconomic practice ?

We cannot answer these questions without making explicit our ideas about the source from which macroeconomists using the aggregate model will draw their estimate of β. Many possibilities exist between two extremes : macroeconomists may limit attention to fully aggregated data directly fitted to their model ; alternatively they may rely on econometric estimates from data for a representative sample of firms. With respect to the resulting aggregation bias the results turn out to look similar, judging from our microeconomic model.

For a direct macroeconometric fit users are likely to mainly consider the share of the value of output *Y* spent on unskilled labour, namely *vM/Y*. This is easily computed in our model from the second equation (18) together with (14), which lead to :

$$\frac{vM}{Y}=\overline{\beta}+\frac{1}{Y}\sum_i(\beta_i-\overline{\beta})p_i\chi_i \quad (70)$$

From a representative sample of microeconomic data users are likely to draw an estimate of the average $\tilde{\beta}$ of the β_i respectively weighted by the value of output of firm *i*, hence

$$\tilde{\beta}=\frac{1}{Y}\sum_i p_i y_i\beta_i \quad (71)$$

According to (18) again, this is precisely equal to *vM/Y*, hence also to the right-hand member of (70).

In the case of homothetic demand functions, i.e. when all χ_i are nil, $\tilde{\beta}$ given by (70) is identical to $\bar{\beta}$. Whatever which of the two econometric sources was chosen, the aggregate model should provide for the elasticity η an estimate close to $[1-\bar{\beta}]^{-1}$, so that *B* defined by (67) or (69) should be an appropriate measure of the aggregation bias. On the other hand, in the general case of non-homothetic demand functions, focusing on $\tilde{\beta}$ rather than $\bar{\beta}$ would lead users of the aggregate model to estimate η by $[1\text{-}\tilde{\beta}]^{-1}$. A different aggregation bias $\tilde{B}$ would result.

From (70) we may write :

$$\tilde{\beta} - \bar{\beta} = \frac{1}{Y} Cov(\beta,\ p\chi) \tag{72}$$

The estimate $\tilde{\beta}$ will be smaller than $\bar{\beta}$, hence the bias $\tilde{B}$ larger than *B,* when the above covariance will be negative. This is likely to be the case if luxuries are more unskilled-labour intensive than are necessities. This remark being made, let us however examine the value of *B* given by (69).

The case in which all λ_i given by (32) would be equal provides an attractive benchmark for our discussion because the bias would then be proportional to the variance of β_i and would have the sign of the common value λ. This was the case studied in Malinvaud (2001.a) where all χ_i were assumed equal to zero. As was reported in section 1, large absolute values of the bias were unlikely except when the elasticity of substitution σ between the demands for different goods was substantially higher than the elasticity of substitution between factor inputs, and then the bias meant that neglecting heterogeneity led to underestimation of the effect of changes in the unskilled-labour unit cost. A positive common value of the ratios χ_i / y_i would mean that a still higher value of σ would be necessary for obtaining a given positive level of *B*.

On the other hand, when the λ_i differ across sectors, a positive correlation between λ_i and β_i would lead (for all practical purposes) to a higher value of *B*. Since goods that are necessities are characterized by low of values of λ_i, a positive correlation means that, in a sense, luxuries rather than necessities are unskilled-labour intensive. Numerical examples show that, in such a case, introduction of $Cov(\lambda, \beta)$ in the numerator and denominator of (69) may make a difference. For instance when $\bar{\beta} = 0.1, \bar{\lambda} = 0.55$ and $Var(\beta) = 25 \times 10^{-4}$, the value of *B* is equal to 0.038 if there is a zero correlation between λ_i and β_i, whereas it amounts to 0.060 if the standard deviation of λ_i is equal to 0.22 and the correlation coefficient between λ_i and β_i equal to 0.50. Although there is no simple correspondence between the two covariances serving respectively in (69) and (72), the remark made after (72) reinforces the conclusion just reached because $\tilde{B}$ is likely to be larger than B when cov (λ, β) is positive.

As we saw in section 1, numerical results about the aggregation bias have to be substantially raised if the microeconomic elasticity of substitution between the two labour inputs is actually smaller than 1, its value according to the Cobb-Douglas hypothesis. However, the rough conclusion stands that the bias cannot be large unless either there is a clear tendency for luxuries to be more labour-intensive than are necessities, or the substitution among the demands for different goods is more intense than most people would think. In particular, econometric results about systems of household consumption demand functions give a range of price elasticities which, compared to our specification (24), suggest that $\sigma = 3$ would be rather atypical. For instance R. Blundell and J.-M. Robin (2000) analysing British household budget data of the period 1974-1993 and distinguishing 22 consumption goods find own uncompensated price elasticities in the range 0.4 to 2.0 except for two outsiders with values 2.2 and 3.5.

But it would be very bold to take our fundamentally simple model of the economy, as specified in section 3, and to directly use its results after calibrations coming from just two sources : estimates of elasticities of substitution between a particular factor input and other primary inputs in production units, estimates of price elasticities of household consumption demands for still fairly large groups of goods and services. The actual interplay between the structure of the demand for factors of production and the structure of the demand system for the full range of produced goods and services is definitely more complex than assumed here. This paper cannot go at length into the study of the complexity in question. But in our concluding section we may briefly reflect on the reasons why our model would not be directly appropriate for econometric estimation. We shall so look at the conjecture discussed in this paper from another point of view.

9. Toward econometric evaluations of heterogeneity effects in the aggregate

In the first place, taking as given the fact that the econometric analysis of consumption demand will not distinguish more than one, two or three dozens of commodity groups, we wonder how the analysis could be supplemented so as to better account for the full spectrum of relevant substitutabilities. Two important additions to our model then come to mind : intra-industry substitutabilities between techniques of production, substitutabilities along the channels in input-output networks leading from primary factors to consumption goods.

According to our model each industry i produces a commodity i using a range of techniques that are perfectly represented by a specific production function. But since commodity i is really a group of many goods or services and industry i is really made of many production units, there is ample room for intra-industry substitutabilities between micro-goods or services, as well as between production units specialized in different ranges of techniques and serving somewhat different micro-markets. These low-level heterogeneities leading to low-level substitutabilities can be very significant. Part of the contemplated econometric work would have to gauge their importance. We can simply add here that the potential importance of such substitutabilities for our problem is revealed by the model here discussed if we directly apply it to an industry in which would be produced n goods j

perfectly substitutable to each other. We then have just to consider the limits for an infinitely large σ. In the specification of this paper, the bias *B* turns out to be infinitely large[2].

In actual fact, changes in relative prices and the resulting substitutions between goods and services do not concern only household consumption but also exports and imports, investments and more generally all intermediate inputs in production. Again, part of a full econometric project on our topic will have to wonder on how to deal with this multiplicity of cases. A relevant question is to know whether substitution effects are cumulative along the input-output chains leading to finished goods or services and eventually ending in final uses. This might even deserve a formal analysis, which would complement the one given here.

An econometric investigation would also have to face a serious practical difficulty in the selection of a proper data base. An appropriate classification would have to distinguish goods and services with respect not only to their uses but also to the unskilled-labour intensity of their production. Common classifications pay very little attention to this second criterion. Hence, the data base ought to be built from quite detailed statistical sources.

Finally, we must recognize that the equilibrium concept used in this paper is rather extreme : perfect competition except for the unskilled-labour market, where excess-supply prevails because of a relatively high controlled unit labour cost. The last section in Malinvaud (2001.a) briefly speculates on the corrections which might somewhat cope with this feature. The very tentative conclusions will not be repeated here.

References

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R. Stone (1954), « Linear expenditure systems and demand analysis : an application to the pattern of British demand », *Economic Journal*, vol 64, p. 511-27.

2 Actually the analysis of this paper assumes σ to be finite. How to transpose it to the case of an infinite σ is discussed in appendix 1 of Malinvaud (2001.a), where it appears that the limit bias is finite if returns to scale are decreasing (no doubt, replacing perfect by monopolistic competition would also lower the bias). However, formula (69) still stands for constant returns to scale and perfect competition when all λ_i tend to 1, implying that $\bar{\lambda}$ tends to 1 and $Cov(\lambda, \beta)$ to zero.