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Information Sharing, Liquidity and Transaction Costs in Floor-Based Trading Systems*

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Abstract

Information Sharing, Liquidity and Transaction Costs in Floor-Based Trading Systems.

We consider information sharing between traders who possess different types of information, namely information on the value of a risky security or information on the volume of liquidity trading in this security. We interpret the traders as dual-capacity brokers on the floor of an Exchange. We identify conditions under which the traders are better off sharing information. We also show that information sharing improves price discovery, reduces volatility and lowers expected trading costs. Information sharing can improve or impair the depth of the market, depending on the values of the parameters. Overall our analysis suggests that information sharing among floor brokers improves the performance of floor-based trading systems.

Keywords : Market Microstructure, Floor-Based Trading Systems, Open Outcry, Information Sharing, Information Sales.
JEL Classification Numbers : G10, D82.

Résumé

Echange d'information, liquidité et coûts de transaction sur les marchés de parquet.

Nous considérons un marché de parquet sur lequel les courtiers ont accès à deux types d'informations privées : (i) des informations 'fondamentales' sur la valeur de l'actif risqué échangé ou (ii) des informations 'non fondamentales' concernant le volume des chocs de liquidité affectant le flux d'ordres. Nous analysons alors les conditions d'existence d'échanges d'informations 'fondamentales' versus 'non fondamentales' entre courtiers et leurs conséquences sur la qualité du marché. En termes de liquidité du marché, l'impact des échanges informationnels est ambigu (il y a amélioration ou détérioration de la liquidité suivant les valeurs des paramètres). Toutefois, leur émergence permet d'améliorer la performance du marché puisque ces échanges d'informations provoquent une diminution de la volatilité et des coûts de transaction, ainsi que l'accroissement de l'efficience informationnelle.

1 Introduction

The organization of trading on the NYSE has been remarkably stable since its creation in 1817. Trading is conducted through open outcry of bids and offers of brokers acting on behalf of their clients or for their own account. This trading mechanism is not unique to the NYSE (the Frankfurt Stock Exchange, the AMEX, the CBOT and the CBOE are other examples). However floor-based trading mechanisms are endangered species as they are progressively replaced by fully automated trading systems[1]. Given this trend toward automation, it is natural to ask whether floor-based trading systems can provide greater liquidity and lower execution costs than automated trading systems. This question is of paramount importance for market organizers and traders. Actually floor-based trading mechanisms can expect to survive only if they outperform automated trading systems along some dimensions.[2]

On many grounds, automated trading systems dominate floor-based trading systems. Trading commissions in these systems can be lower since they are cheaper to operate (Domowitz and Steil (1999)). For this reason explicit trading costs are smaller in automated trading systems (see Domowitz (2001)). Furthermore automated trading systems level the playing field since they put virtually no constraints on the number of traders and they are often more transparent. By design, floor-based markets foster person-to-person contacts. Hence the ability of market participants to share information is greater in these markets.[3] This feature of floor trading systems is often stressed as being one advantage, if not the unique one, of floor-based trading systems.[4] For instance Harris (2000), p.8, points out that

> ‘Floor-based trading systems dominate electronic trading systems when brokers need to exchange information about their clients to arrange their trades.’

There are two questions which are unanswered, however. First, is it optimal for floor bro-

[1]The Toronto Stock Exchange shut down its floor in 1997. The London International Financial Futures and Options Exchange (LIFFE) closed its floor in 2000.

[2]Competitors of the NYSE (Nasdaq, ECNs’ etc..) operate automated trading systems in most cases.

[3]Information sharing is much more difficult in electronic trading systems. First these systems usually restrict the set of messages that can be sent by traders (generally traders can only post prices and quantities). Furthermore trading in these systems is in most cases anonymous. This feature prevents traders from developing the reputation of honestly sharing information.

[4]Coval and Shumway (1998)shows that the level of noise on the floor of CBOT’s 30 year Treasury Bond futures affects price volatility. This also suggests that person to person contacts on the floor have an impact on price formation.

kers to share information? Second, what is the effect of information sharing on the overall performance of the market? For instance, is it beneficial only to those sharing information (the floor brokers) or can investors without an access to the floor also benefit from information sharing? We provide answers to these questions. In particular we study the impact of pre-trade information sharing on standard measures of market quality, namely price volatility, price discovery, market liquidity and trading costs.

We model floor trading and information sharing using Kyle (1985)'s model as a workhorse. We assume that traders (floor brokers) have access to two types of information: (i) fundamental information which is information on the value of the security and (ii) non-fundamental information which is information on the volume of liquidity (non-informed) trading.[5] We consider the possibility for floor brokers to exchange fundamental vs. non fundamental information. There are anecdotal evidences that floor brokers routinely share information. For instance, Sofianos and Werner (1997), p.6 notice that

> 'In addition, by standing in the crowd, floor brokers may learn about additional broker-represented liquidity that is not reflected in the specialist quotes: floor brokers will often exchange information on their intentions and capabilities, especially with competitors with whom they have good working relationships.'

For tractability, we assume that floor brokers decide to enter into bilateral information sharing agreements *before* observing their information. An agreement specifies the precision with which each broker will report his or her information to the other broker. Hence, partial information sharing is possible. After receiving fundamental or non-fundamental information, the brokers pool their information according to the terms of their agreement just before submitting their orders for execution. We establish the following results.

- There is a wide range of parameters for which floor brokers increase their expected profits when they share information.
- Information sharing occurs at the expense of the floor brokers who are not part to the information sharing agreement.
- Information sharing can improve or impair the depth of the market, depending on the values of the parameters. However information sharing always reduces the trading costs for liquidity traders.

[5]This distinction has been introduced by Madrigal (1996). See also Cao and Lyons (1998).

- Information sharing improves price discovery and reduces market volatility.

Information sharing lowers the total expected profits of all floor brokers but also changes the allocation of trading profits among floor brokers. More specifically the floor brokers who share information capture a larger part of the total expected profits, at the expense of floor brokers who do not share information. These two effects explain why information sharing can simultaneously benefit liquidity traders and the floor brokers who share their information. Overall information sharing is an advantage for floor-based trading systems since it results in (a) lower trading costs, (b) faster price discovery and (c) lower price volatility. Interestingly, in line with our result, Venkatamaran (2000) finds that trading costs on the NYSE are lower than on the Paris Bourse (an automated trading system), controlling for differences in stocks characteristics.[6]

Benveniste et al. (1992) show that floor trading allow market-makers to distinguish informed and non-informed traders. Our focus is on information sharing among floor brokers. Our analysis is closely related to the literature on information sales (e.g. Admati and Pfleiderer (1986), (1988) and Fishman and Hagerty (1995)). In contrast with this literature, we assume that the medium for information exchange is information, not money. Hence we consider floor-based trading systems as markets for bartering information. Another important difference is that we consider communication of information on the volume of liquidity trading. We show that it may be optimal to 'sell' (barter) such an information and that sales of non-fundamental information have an impact on market quality.

The model is described in the next section. Section 3 shows when information sharing is possible in our model. Section 4 analyzes the impact of information sharing on various measures of market performance. Section 5 concludes. The proofs which do not appear in the text are in the Appendix.

[6]Theissen (1999) compares effective bid-ask spreads in an automated trading system (Xetra) and the floor of the Frankfurt Stock Exchange for stocks that trade in both systems. He finds that the average quoted spreads on the floor can be larger or smaller than in the automated trading system, depending on the stock characteristics. On average the quoted spreads are equal. This is consistent with our result that the impact of information sharing on liquidity is ambiguous.

2 The Model

2.1 Information Sharing Agreements

We consider a model of trading in the market for a risky security which is based on Kyle (1985). The final value of the security, which is denoted $\tilde{v}$, is normally distributed with mean μ and variance $\sigma_v^2 = 1$. This final value is publicly revealed at date 2. Trading in this security takes place at date 1. At this date, investors submit market orders to buy or to sell shares of the security.[7] The excess demand (supply) is cleared at the price posted by a competitive market maker.

There are 3 different types of agents who submit orders: (i) N fundamental speculators, (2) a pool of liquidity traders and (3) 1 non-fundamental speculator. Fundamental speculators perfectly observe the final value of the security, just before submitting their orders at date 1. Liquidity traders have a net demand equal to $\tilde{x} = \tilde{x}_0 + \tilde{x}_B$ shares as a whole. We assume that $\tilde{x}_0$ and $\tilde{x}_B$ are normally and independently distributed with means 0 and variances σ_0^2 and σ_B^2 respectively. The non fundamental speculator perfectly observes $\tilde{x}_B$.

We interpret speculators as floor brokers who can engage in proprietary trading.[8] In particular $\tilde{x}_B$ can be seen as the orders sent to broker B. Broker B acts both as an agent (she channels a fraction of the orders submitted by liquidity traders) and as a principal (she submits orders for her own account). This practice is known as 'dual-trading' and is authorized in securities markets (see Chakravarty and Sarkar (1996) for a discussion).[9] We normalize the variance of the order flow due to liquidity trading to 1, i.e.:

$$\sigma_B^2 + \sigma_0^2 = 1.$$

In this way, σ_B^2 can be interpreted as broker B's market share of the total order flow from liquidity traders. The remaining part of the order flow can be seen as being intermediated by other floor brokers or is routed electronically to the floor.[10]

[7] Market orders are orders which are not contingent on prices.

[8] In the U.S, full line brokerage houses engage in proprietary trading activities. Discount brokers do not, however.

[9] For simplicity, we assume that speculator B can identify without errors whether her clients are uninformed or not. A similar assumption is made in Röell (1990) or Madrigal (1996). In reality repeated relationships enable brokers to assess their clients trading motivations.

[10] For instance, on the NYSE, orders can reach a market-maker through floor brokers or electronically through a system called SuperDot.

In reality floor brokers privately observe the orders that are submitted by their clients. It is reasonable to assume that the order flow from liquidity traders is independent across brokers. In contrast, signals on the fundamental value of the security are correlated. For these reasons, we assumed that only one floor broker observes the non-fundamental information, $\tilde{x}_B$, whereas several floor brokers observe the fundamental information, $\tilde{v}$.[11]

This framework has been extensively analyzed in the literature on informed trading in financial markets.[12] Information sharing between fundamental and non fundamental investors has been ruled out, however. Our purpose is to study the possibility and the effects of this activity. As argued in the introduction, this type of information exchange is a distinctive feature of floor markets. We model information sharing as follows. We assume that the non fundamental speculator, B, has an agreement to share information with one fundamental speculator, S. According to this agreement, before trading at date 1, the non-fundamental speculator sends a signal

$$\hat{x} = \tilde{x}_B + \tilde{\eta},$$

to the fundamental speculator. In exchange, the fundamental speculator sends a signal

$$\hat{v} = \tilde{v} + \tilde{\varepsilon},$$

to the non fundamental investor. The random variables $\tilde{\eta}$ and $\tilde{\varepsilon}$ are independently and normally distributed with mean zero and variances σ_η^2 and σ_ε^2, respectively. The larger is σ_η^2 (σ_ε^2), the less precise is the signal sent by speculator B (speculator S) and hence the lower is its informative value. Notice that we allow traders to only partially reveal their information. As in the literature on information sales, this possibility plays an important role in our analysis.[13] There is no information exchange at all when $\sigma_\eta^2 = \sigma_\varepsilon^2 = \infty$. The information sets of speculators B and S at date 1 are denoted $y_B = (\tilde{x}_B, \hat{x}, \hat{v})$ and $y_S = (\tilde{v}, \hat{x}, \hat{v})$, respectively.

For tractability, we assume that the speculators must decide to share information at

[11] The assumption that fundamental speculators perfectly observe the final value considerably simplifies the computations without affecting the results.

[12] In the seminal paper of Kyle (1985), investors are assumed to have no information on the orders submitted by liquidity traders. This assumption is relaxed in papers by Röell (1990), Sarkar (1995) or Madrigal (1996), among others. We borrow the distinction between 'fundamental' vs. 'non-fundamental' speculators from Madrigal (1996).

[13] Admati and Pfleiderer (1986) show that it can be optimal for a monopolistic seller of information to add noise to the information she sells.

date 0, i.e. prior to receiving information.[14] We say that information sharing is possible if there exists a pair (σ_η^2,σ_ε^2) such that the expected profits of the fundamental speculator *and* the non fundamental speculator are larger when there is information exchange than when there is not. In section 3, we identify parameters' values for which an information sharing agreement raises speculators B and S's expected profits.

It is worth stressing that we focus on the possibility of an information sharing agreement but not on its implementation. In particular, we do not address the issue of how such an information agreement can be enforced. In that, we follow the literature on information sale where the quality of the information which is sold is assumed to be contractible.[15] We also assume that the information sharing agreement and its characteristics ($\sigma_\eta^2, \sigma_\varepsilon^2$) are known by all participants (including the market-maker). This common knowledge assumption is also standard in the literature on information sales.

2.2 The equilibrium of the Floor Market

In this section, we derive the equilibrium of the trading stage at date 1, given the characteristics of the information sharing agreement between speculators B and S.

We denote by $Q^S(y_S)$ and $Q^B(y_B)$, the orders submitted by speculators S and B, respectively. In the set of fundamental speculators, we assign index 1 to speculator S. An order submitted by the speculators who do not share information is denoted $Q^i(\tilde{v}), i = 2, ..., N$. The total excess demand that must be cleared by the competitive market maker is therefore

$$O = \sum_{i=2}^{i=N} Q^i(\tilde{v}) + Q^S(y_S) + Q^B(y_B) + \tilde{x}.$$

As the market maker is assumed to be competitive, he sets a price $p(O)$ equal to the asset expected value conditional on the net order flow, i.e.

$$p(O) = E(\tilde{v} \mid O). \tag{1}$$

[14]The model would be significantly more complex to analyze without this assumption. For instance, the precisions with which speculators exchange their signals would depend on the realizations of these signals.

[15]Some papers have shown how incentives contracts can be used to induce an information provider to truthfully reveal the quality of his signal (see Allen (1990) or Bhattacharya and Pfleiderer (1985)). Reputation effects may also help to sustain information sharing agreements (see Benabou and Laroque (1992)).

An equilibrium consists of trading strategies $Q^S(.)$, $Q^B(.)$, $Q^i(.), i = 2, ..., N$ and a competitive price function $p(.)$ such that (i) each trader's trading strategy is a best response to other traders' strategies and (ii) the dealer's bidding strategy is given by Equation (1).[16] For a given information sharing agreement $(\sigma_\eta^2, \sigma_\varepsilon^2)$, the next lemma describes the unique linear equilibrium of the trading game.

Lemma 1 *: The trading stage has a unique linear equilibrium which is given by*

$$\begin{aligned} p(O) &= \mu + \lambda O, && (2) \\ Q^S(y_S) &= a_1(\tilde{v} - \mu) + a_2(\hat{v} - \mu) + a_3\hat{x}, && (3) \\ Q^i(\tilde{v}) &= a'(\tilde{v} - \mu), i = 2, ..., N && (4) \\ Q^B(y_B) &= b_1\tilde{x}_B + b_2\hat{x} + b_3(\hat{v} - \mu), && (5) \end{aligned}$$

where coefficients $a_1, a_2, a_3, a', b_1, b_2, b_3$ and λ are

$$\begin{aligned} a_1 &= \frac{3\left(\sigma_v^2 + \sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)}, \\ a_2 &= -\frac{\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)}, \\ a_3 &= -\frac{\sigma_B^2}{3\left(\sigma_B^2 + \sigma_\eta^2\right)}, \\ a' &= \frac{2\sigma_v^2 + 3\sigma_\varepsilon^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)}, \\ b_1 &= -\frac{1}{2}, \\ b_2 &= \frac{\sigma_B^2}{6\left(\sigma_B^2 + \sigma_\eta^2\right)}, \\ b_3 &= \frac{2\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)}, \end{aligned}$$

and

$$\lambda(\sigma_\varepsilon^2, \sigma_\eta^2) = \frac{6\sqrt{\sigma_v^2\left(\sigma_B^2 + \sigma_\eta^2\right)\left(4\left(N+1\right)\sigma_v^4 + \left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2 + 9N\sigma_\varepsilon^4\right)}}{\left(2\left(N+2\right)\sigma_v^2 + 3\left(N+1\right)\sigma_\varepsilon^2\right)\sqrt{\sigma_B^2\left(4\sigma_B^2 + 9\sigma_\eta^2\right) + 36\sigma_0^2\left(\sigma_B^2 + \sigma_\eta^2\right)}}.$$

[16] More precisely, we consider the Perfect Bayesian Equilibria of the trading game.

Intuitively, traders purchase (sell) the security when their estimation of the asset expected value is above (below) the unconditional expected value. Hence, the coefficients a_1, a' and b_3 are positive. Non fundamental information is also a source of profit. The intuition is as follows. Suppose that the fundamental speculators (but not the market maker) observe $\tilde{v} = \mu$. Suppose also that B and S perfectly share information and that liquidity traders submit buy orders. These orders push the price upward because the market maker can not distinguish liquidity orders from informed orders. Speculators B and S however know that the correct value of the security is μ. In anticipation of the upward pressure on the clearing price, they submit sell orders. By symmetry, they would submit buy orders when liquidity traders submit sell orders. This explains why coefficients b_1 and a_3 are negative. A similar effect is obtained in Röell (1990). This means that floor brokers B and S partly accommodate the orders coming from liquidity traders and reduce the order flow imbalance that must be executed by the market-maker.

The previous discussion show how speculators can profit both from fundamental and non fundamental information. This creates a motivation for sharing information. However, speculators S and B depreciate the value of their own information when they share it. Consider speculator B for instance. She alone has access to information on $\tilde{x}_B$. If she does not share information ($\sigma_\eta^2 = +\infty$), she accommodates half of the order flow she receives (since $b_1 = -1/2$). If she shares information perfectly ($\sigma_\eta^2 = 0$) then she accommodates only one third of the orders she receives ($b_1 + b_2 = -1/3$). But this is also the case for speculator S. Eventually floor brokers (B and S) accommodate a larger share (2/3) of the order flow received by broker B. Hence, *for a fixed value* of λ, prices react less to the order flow and speculator's B profits on non-fundamental information decrease.[17] In fact, when she shares information, speculator B acts as an oligopolist: she restricts her trade size so that her order and the order placed by speculator S do not push the clearing price too close to her estimation of the asset expected value ('the marginal cost'). This explains why b_2 has a sign opposite the sign of b_1.

A similar argument holds for speculator S. He depreciates the value of fundamental information when he shares it with speculator B. In order to mitigate this effect, he adjusts his trading strategy to the message he sends to speculator B. This explains why a_2 has a sign opposite a_1.

To sum up, information sharing has benefits and costs. Information sharing is a source

[17]The slope of the price schedule is affected by information sharing. Hence, in equilibrium, λ is not fixed and the argument is more complex. See Section 3.

of profits since it allows each broker to trade on a new type of private information. But the brokers obtain new information only if they disclose all or part of their information. This is costly since it reduces the trading profits that can be made on the information originally possessed by a broker.

3 Is Information Sharing Possible?

In this section, we identify cases in which speculators B and S are better off when they share information. We start by considering the effect of the precisions with which the speculators B and S share their information on the market depth (measured by λ^{-1}).[18] It turns out that these effects are important to interpret the results in this section.

Lemma 2 *: The depth of the market (i.e. λ^{-1}) is affected by the precisions with which the fundamental and the non fundamental speculators share their information. Namely*

$$\frac{\partial \lambda}{\partial \sigma_{\eta}^{2}} < 0, \quad \text{and} \quad \frac{\partial \lambda}{\partial \sigma_{\varepsilon}^{2}} > 0.$$

Notice that an increase in the quality of the information provided by B to S enlarges λ, that is it decreases the depth of the market. The intuition for this result is as follows. Exchange of non-fundamental information increases the role of floor brokers (B and S) in the provision of liquidity. To see this point, let $Q^T = Q^B + Q^S$ be the total trade size of speculators B and S and consider their expected total trade size contingent on $\tilde{x}_B = x_B$. We obtain

$$E(Q^T \mid \tilde{x}_B = x_B) = (b_1 + b_2 + a_3)(x_B) = -(\frac{1}{2} + \frac{\sigma_B^2}{6(\sigma_B^2 + \sigma_{\eta}^2)})(x_B). \tag{6}$$

The smaller is σ_{η}^2, the larger is the fraction ($\mid b_1 + b_2 + a_3 \mid$) of the order imbalance x_B which is accommodated by speculators S and B. The exchange of non-fundamental information 'siphons' part of the orders coming from uninformed traders. This siphon effect increases the exposure of the market-maker to informed trading and the price schedule becomes steeper.

[18]The market depth is the order flow necessary to change the price by 1 unit. The larger is the market depth, the greater is the liquidity of the market. Actually, when λ is small, the market-maker accommodates large order imbalances without substantial changes in prices.

Interestingly an increase in the quality of the information provided by S to B has exactly the opposite effect: it improves the depth of the market. In this case, the effect of information sharing is to increase competition among fundamental traders. Hence they scale back their order size (a_1 and $a^{'}$ decrease when σ_ε^2 decreases). This competition effect reduces the market-maker's exposure to informed trading and thereby makes the price schedule less steep.

We denote by $\Pi^j(\sigma_\eta^2, \sigma_\varepsilon^2, N)$ speculator j's expected profit at date 0, that is before observing information. Using Lemma 1, we obtain the following result.

Lemma 3 *: For given values of σ_ε^2 and σ_η^2, the expected trading profits for speculators B and S are*

$$\begin{aligned}
\Pi^S(\sigma_\eta^2, \sigma_\epsilon^2, N) &= \left(\frac{\sigma_v^2 \left(\sigma_v^2 + \sigma_\varepsilon^2\right) \left(4\sigma_v^2 + 9\sigma_\varepsilon^2\right)}{\lambda \left(2 \left(N+2\right) \sigma_v^2 + 3 \left(N+1\right) \sigma_\varepsilon^2\right)^2} + \frac{\lambda \sigma_B^4}{9 \left(\sigma_B^2 + \sigma_\eta^2\right)} \right) \\
&= \Pi_f^S(\sigma_\eta^2, \sigma_\varepsilon^2, N) + \Pi_{nf}^S(\sigma_\eta^2, \sigma_\varepsilon^2, N), \\
\textit{and}, & \\
\Pi^B(\sigma_\eta^2, \sigma_\varepsilon^2, N) &= \left(\frac{4\sigma_v^4 \left(\sigma_v^2 + \sigma_\varepsilon^2\right)}{\lambda \left(2 \left(N+2\right) \sigma_v^2 + 3 \left(N+1\right) \sigma_\varepsilon^2\right)^2} + \frac{\lambda \sigma_B^2 \left(4\sigma_B^2 + 9\sigma_\eta^2\right)}{36 \left(\sigma_B^2 + \sigma_\eta^2\right)} \right) \\
&= \Pi_f^B(\sigma_\eta^2, \sigma_\varepsilon^2, N) + \Pi_{nf}^B(\sigma_\eta^2, \sigma_\varepsilon^2, N)
\end{aligned}$$

Each speculator's expected profits has two components: (i) the expected profit she or he obtains by trading fundamental information (Π_f^j) and (ii) the expected profit she or he obtains by trading on non-fundamental information (Π_{nf}^j). An information sharing agreement is viable if and only if both speculators B and S are better off when they share information. Hence an information sharing agreement is possible if and only if there exists a pair $(\sigma_\eta^2, \sigma_\varepsilon^2)$ such that

$$\Gamma_B \left(\sigma_\eta^2, \sigma_\varepsilon^2, N\right) \stackrel{def}{=} \Pi^B(\sigma_\eta^2, \sigma_\varepsilon^2, N) - \Pi^B(\infty, \infty, N) > 0,$$

and

$$\Gamma_S \left(\sigma_\eta^2, \sigma_\varepsilon^2, N\right) \stackrel{def}{=} \Pi^S(\sigma_\eta^2, \sigma_\varepsilon^2, N) - \Pi^S(\infty, \infty, N) > 0.$$

The Γs' measure the expected surplus associated with the information sharing agreement for speculators B and S.

Proposition 1 *The set of parameters for which speculators B and S share information is non-empty.*

Proof: We establish the result by providing 3 numerical examples. For each example, we report in Tables 1, 2 and 3 below the break-down of the trading profits for the different participants with and without information sharing. We also compare the market depth with and without information sharing.

Example 1: $\sigma_B^2 = \sigma_v^2 = 1$, $\sigma_0^2 = 0$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 2/3$, $N = 2$.

Type of Traders	**Information Sharing**	**No Information Sharing**
$(N-1) \times \Pi_f^i \quad i \neq S, B$	0.0589	0.1178
Π_f^S	0.0589	0.1178
Π_{nf}^S	0.0707	0
Π_f^B	0.0589	0
Π_{nf}^B	0.1767	0.2357
Market Depth (λ)	1.0607	0.9428
Total Expected Profits	0.4242	0.4714

Table 1

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0589 + 0.0707 - 0.1178 = 0.0118,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.0589 + 0.1767 - 0.2357 = 0.$$

Observe that the total surplus for speculators B and S is positive and equal to

$$\Gamma_S + \Gamma_B = 0.0118,$$

but that the total surplus for all speculators is negative and equal to

$$(N-1) \times \Pi_f^i + \Gamma_S + \Gamma_B = 0.0589 + 0.0118 - 0.1178 = -0.0471.$$

Example 2: $\sigma_B^2 = 0.8$, $\sigma_v^2 = 1$, $\sigma_0^2 = 0.2$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 0.45$, $N = 4$.

Type of Traders	Information Sharing	No Information Sharing
$(N-1) \times \Pi_f^i \quad i \neq S, B$	0.0427	0.0632
Π_f^S	0.0427	0.0632
Π_{nf}^S	0.0370	0
Π_f^B	0.0427	0
Π_{nf}^B	0.0838	0.1265
Market Depth (λ)	0.6498	0.6325
Total Expected Profits	0.2062	0.2529

Table 2

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0165,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.$$

Observe that the total surplus for speculators B and S is positive (0.0165) but that the total surplus for all speculators is negative (-0.0467.)

Example 3: $\sigma_B^2 = 0.8$, $\sigma_v^2 = 1$, $\sigma_0^2 = 0.2$, $\sigma_\varepsilon^2 = 0$, $\sigma_\eta^2 = 1.555$, $N = 4$.

Type of Traders	Information Sharing	No Information Sharing
$(N-1) \times \Pi_f^i \quad i \neq s, B$	0.0449	0.0632
Π_f^S	0.0449	0.0632
Π_{nf}^S	0.00187	0
Π_f^B	0.0449	0
Π_{nf}^B	0.1005	0.1265
Market Depth (λ)	0.6192	0.6325
Total Expected Profits	0.4242	0.4714

Table 3

In this case we obtain that

$$\Gamma_S = \Pi_f^S + \Pi_{nf}^S - \Pi^S(\infty, \infty) = 0.0003,$$

and

$$\Gamma_B = \Pi_f^B + \Pi_{nf}^B - \Pi^B(\infty, \infty) = 0.0188$$

Observe that the total surplus for speculators B and S is positive and equal to $\Gamma_S + \Gamma_B = 0.2089$. The total surplus for all speculators is negative and equal to -0.0467. ∎

Observe that in all the examples, the joint expected profits of speculators B and S increase when they share information. However at the same time, there is a decline in the joint expected profits of the speculators who do not share information. Eventually the total expected profits for all the speculators are lower. Information sharing is a way for speculators B and S to secure a larger part of a smaller 'cake'. The fall in total profits is not surprising. This decline reflects the fact that information sharing among floor brokers increases competition. The surprising part is that the joint expected profits of speculators B and S can increase despite the decline in the total trading profits for the speculators. We now provide an explanation of this result. The explanation is quite complex because several effects interplay.

Consider for a moment the case in which the fundamental speculator perfectly discloses his information ($\sigma_\varepsilon^2 = 0$) whereas the non fundamental speculator provides no information ($\sigma_\eta^2 = \infty$). Now we compare the expected total trade size of speculators S and B conditional on fundamental information with and without information sharing. To this end we compute the following ratio

$$r_1(0, \infty) \stackrel{def}{=} \frac{E(Q^T \mid \tilde{v} = v, \sigma_\varepsilon^2 = 0, \sigma_\eta^2 = \infty)}{E(Q^T \mid \tilde{v} = v, \sigma_\varepsilon^2 = \infty, \sigma_\eta^2 = \infty)} = \frac{2\lambda(\infty, \infty)(N+1)}{\lambda(0, \infty)(N+2)},$$

where the second equality follows from Lemma 3. As $\lambda(\infty, \infty) > \lambda(0, \infty)$ (Lemma 2), it is immediate that $r_1(0, \infty) > 1$. This means that *collectively* speculators B and S trades more aggressively on fundamental information when $\sigma_\varepsilon^2 = 0$ than when $\sigma_\varepsilon^2 = \infty$. This forces the speculators who do not share information to act less aggressively on their information. Actually we observe that

$$r_2(0, \infty) \stackrel{def}{=} \frac{E(Q^i \mid \tilde{v} = v, \sigma_\varepsilon^2 = 0, \sigma_\eta^2 = \infty)}{E(Q^i \mid \tilde{v} = v, \sigma_\varepsilon^2 = \infty, \sigma_\eta^2 = \infty)} = \frac{\lambda(\infty, \infty)(N+1)}{\lambda(0, \infty)(N+2)} = \frac{\sqrt{N}}{\sqrt{N+1}} < 1.[19]$$

[19] The last equality follows from the expressions of λ given in the proof of Lemma 2.

Hence the speculators who share information appropriate a larger share of the total profits which derive from trading on fundamental information.[20] When $\sigma_\varepsilon^2 \geq 0$ and/or $\sigma_\eta^2 < \infty$, this effect remains present but is less strong. To see this point, recall that λ increases with σ_ε^2 and decreases with σ_η^2 (Lemma 2). This implies that

$$\frac{\lambda(\infty,\infty)}{\lambda(\sigma_\varepsilon^2,\sigma_\eta^2)} < \frac{\lambda(\infty,\infty)}{\lambda(0,\infty)},$$

or

$$r_1(\sigma_\varepsilon^2,\sigma_\eta^2) < r_1(0,\infty).$$

Hence the ratio r_1 is smaller when $\sigma_\varepsilon^2 \geq 0$ and/or $\sigma_\eta^2 < \infty$. But there are always values of σ_η^2 and σ_ε^2 such that information sharing increases the joint expected profits on fundamental information, i.e. such that

$$\Pi_f^S(\sigma_\varepsilon^2,\sigma_\eta^2,N)+\Pi_f^B(\sigma_\varepsilon^2,\sigma_\eta^2,N)-\Pi_f^S(\infty,\infty,N) > 0, \quad for \quad \sigma_\eta^2 < \infty, \quad \sigma_\varepsilon^2 < \infty \quad and \quad N > 1$$

This is the case for instance in Examples 2 and 3.

Now consider the effect of information sharing on the profits which derive from non-fundamental information. On the one hand, there are more speculators who accommodate the order flow brokered by B. This effect decreases the level of expected profit on non-fundamental information. On the other hand the exchange of non fundamental information decreases the market depth and this effect increases profits from non-fundamental speculation as can be seen from Lemma 3. It turns out that there are cases in which the second effect dominates and the joint expected trading profits of speculators S and B on non-fundamental information are larger when there is information sharing or

$$\Pi_{nf}^B(\sigma_\eta^2,\sigma_\varepsilon^2,N) + \Pi_{nf}^S(\sigma_\eta^2,\sigma_\varepsilon^2,N) - \Pi_{nf}^B(\infty,\infty,N) \geq 0, \quad for \quad \sigma_\eta^2 < \infty \quad and \quad \sigma_\varepsilon^2 < \infty$$

For instance in Example 1, the L.H.S of this inequality is equal to 0.0117. Observe that this can occur only when information sharing impairs market liquidity. In Examples 2 and 3, information sharing improves liquidity and the joint expected profit on non-fundamental

[20] Another way to interpret the result is to notice that speculators in our model are like Cournot competitors. In Cournot competition, each firm would like to commit to trade a larger size than it does in equilibrium. This commitment would force other firms to trade in smaller sizes. In this way the committed firm can capture a larger share of the total profits. Intuitively sharing fundamental information is a way to make this commitment credible. This effect has been pointed out by Fishman and Hagerty (1995) in a model of information sale.

information decreases.

To sum up, there are two reasons why information sharing can increase the joint expected profits of speculators B and S:

- Sharing fundamental information allows the coalition formed by brokers S and B to trade more aggressively on fundamental information and to capture thereby a larger share of the total profits.
- Sharing non-fundamental information reduces the market depth. Larger expected profits from speculation on non-fundamental information follows.

The precisions with which the speculators share their information plays two roles in the model. They determine (1) the size of the surplus created by information sharing and (2) the division of this surplus. For instance, consider Examples 2 and 3. The value of σ_η^2 is larger in Example 3, but otherwise the values of the parameters are identical in the two examples. The joint surplus for speculators B and S is smaller in Example 2. This shows that the size of the total surplus is affected by the terms of the information sharing agreement. Second the surplus for speculator $B(S)$ is larger (lower) in Example 3 than in Example 2. In line with the intuition, for a fixed value of σ_ε^2, speculator $B(S)$ prefers to provide (receive) an information of low (high) quality. Hence speculators B and S have conflicting views over the information sharing agreements which should be chosen.[21]

More generally, for a fixed value of σ_ε^2, the value of σ_η^2 must belong to an interval $[\bar{\sigma}_\eta^2, \hat{\sigma}_\eta^2]$ in order to sustain the information sharing agreement. When $\sigma_\eta^2 > \hat{\sigma}_\eta^2$, speculator S is better off not sharing information because his profit on non-fundamental information (his benefit from sharing information) is too low. When $\sigma_\eta^2 < \bar{\sigma}_\eta^2$, speculator B is better off not sharing information because speculator S captures a too large part of her profit on non-fundamental information (i.e. B's cost of sharing information is too high). The values of $\bar{\sigma}_\eta^2$ and $\hat{\sigma}_\eta^2$ depend on the values of all the other parameters, namely N, σ_ε^2 and σ_0^2. Figures 1, 2 and 3 illustrate the effect of these parameters on the interval $[\bar{\sigma}_\eta^2, \hat{\sigma}_\eta^2]$. We observe that

1. Speculator B provides less accurate information ($\bar{\sigma}_\eta^2$ and $\hat{\sigma}_\eta^2$ increase) when (1) the number of fundamental speculators increases (Figure 1) or (2) the precision with

[21] Our statement regarding market performance in Section 4 only depends on the existence of information sharing agreements, not on the values chosen for σ_ε^2 and σ_η^2. For this reason, they are not affected by the procedure used to select the precisions with which the speculators share information.

which speculator S transmits his information decreases (Figure 3). Actually, in these two cases, speculator B's expected profit on fundamental information (her benefit from sharing information) shrinks. Hence she keeps sharing information only if the cost to do so is reduced as well.

2. Speculator B provides more accurate information when the fraction of the order flow she intermediates decreases (Figure 2). Actually in this case, the value of non-fundamental information is lower. Accordingly speculator B must provide information of higher quality to induce S to share information.

It is worth stressing that the examples provided to support the claim of Proposition 1 are not isolated cases. In fact, for various values of the parameters σ_0^2 and $N \geq 2$, we have always been able to find values of σ_ε^2 and σ_η^2 such that speculators B and S are better off sharing information. The case $N = 1$ deserves special attention, however. In this case, we can show that information sharing never takes place.

Lemma 4 *: If $N = 1$ then no information sharing agreement can be sustained.*

The intuition for the result is the following. Observe that when $N = 1$, broker S has a monopolist access to fundamental information. When $N > 1$, we have argued that sharing information is a way to increase the joint expected profits on fundamental information for the coalition formed by brokers S and B. This is not the case when $N = 1$, since the joint expected profit on fundamental can not be larger than the expected profit for speculator S when she acts as a monopolist. Hence the joint expected profit on fundamental information decreases when there is information sharing and $N = 1$. The joint expected profit on non-fundamental information may increase with information sharing, even if $N = 1$ (because sharing non-fundamental information reduces the depth of the market) but this increase is never large enough to make both brokers B and S better off with information sharing.

4 Information Sharing and Market Performance

In this section, we analyze the effects of information sharing on traditional measures of market quality: (1) the informational efficiency of prices (measured by $Var(\tilde{v} \mid p)$), (2) price volatility (measured by $Var(\tilde{v} - p)$), (3) market depth (measured by λ) and (4) the expected trading costs borne by liquidity traders (i.e. the absolute value of their expected losses, $E(\tilde{x}(\tilde{v}-p))$). These aspects of market performance play a prominent role in the debates regarding the design of trading systems and have attracted considerable attention in the literature (see Madhavan (1996) or Vives (1995) for instance).

Proposition 2 *Prices are more informative and less volatile when there is information sharing.*

The intuition behind this result is simple. When speculators S and B share information, the number of speculators trading on fundamental information increases. It follows that the aggregate order flow is more informative. For this reason, prices are more accurate predictors of the final value of the security and price discovery is improved.

We now examine the impact of information sharing on the depth of the market. Examples 2 and 3 in the previous section show that we can find parameters values for which the depth of the market decreases (Example 2) or increases (Example 3) when there is information sharing. On the one hand an increase in the precision with which speculator S transmits his information improves the depth of the market. On the other hand, an increase in the precision with which speculator B transmits her information impairs the depth of the market (because of the siphon effect). Hence the impact of information sharing on the depth of the market can be positive or negative.

Proposition 3 *Information sharing has an ambiguous impact on market depth.*

Figure 4 illustrates the proposition. It depicts the evolution of the ratio of the market depth when there is no information sharing to the market depth when there is information sharing ($\frac{\lambda(\sigma_\eta^2,\sigma_\varepsilon^2)}{\lambda(\infty,\infty)}$) as a function of σ_η^2, for fixed values of the other parameters. Of course we limit our attention to the range of values for σ_η^2 such that information sharing is possible. As it can be seen the ratio decreases with σ_η^2. It is larger than 1 (information sharing is detrimental to market depth) for low values of σ_η^2 and smaller than 1 (information sharing improves market depth) for large values of σ_η^2.

Notice that the market depth is related to the bid-ask spread. Actually a buy order of size q pushes the price upward by λq whereas a sell order of the same size pushes the price downward by λq. Hence

$$s(q) = p(q) - p(-q) = 2\lambda q,$$

measures the bid-ask spread for an order of size q in our model. The spread increases with λ. Accordingly the impact of information sharing on bid-ask spreads is ambiguous. Interestingly empirical studies which compare bid-ask spreads in floor-based trading systems and automated trading systems have not found that spreads were systematically lower in one trading venue. For instance, several studies (Kofman and Moser (1997), Pirrong (1996) and Shyy and Lee (1995)) have compared the bid-ask spreads on LIFFE (when it was a floor market) and DTB (an automated trading system) for the German Bund futures contract. Kofman and Moser (1997) find that spreads are equal in the two markets; Pirrong (1996) reports narrower spreads on DTB whereas Shyy and Lee (1995) find smaller spreads on LIFFE.

Finally we consider the effects of information sharing on the expected trading costs for the liquidity traders. These expected trading costs are

$$E(TC) = E(\tilde{x}(p - \tilde{v})) = \underbrace{E(\tilde{x}_B(p - \tilde{v}))}_{Orders\ channeled\ by\ B} + \underbrace{E(\tilde{x}_0(p - \tilde{v}))}_{Orders\ not\ channeled\ by\ B}.$$

In the last expression, we distinguish between the expected trading costs for the liquidity traders who send their orders to broker B and the expected trading costs for those who do not. Using Lemma 1, we rewrite the expected trading costs as

$$E(TC) = \underbrace{\lambda g(\sigma_\eta^2)\sigma_B^2}_{Orders\ channeled\ by\ B} + \lambda\sigma_0^2,$$

with $g(\sigma_\eta^2) = \left(\frac{2\sigma_B^2+3\sigma_\eta^2}{6\left(\sigma_B^2+\sigma_\eta^2\right)}\right)$. The ratio $g(\sigma_\eta^2)$ decreases with σ_η^2. Hence when information sharing improves market depth, it also decreases the expected trading costs for all liquidity traders : (1) the liquidity traders whose orders are channeled through broker B and (2) the other liquidity traders. When information sharing decreases market depth (increases λ), the expected trading costs of the liquidity traders who do not send their order to broker B increase. However the expected trading costs for the liquidity traders who use B's services decline despite the decrease in market depth. Actually information sharing increases competition among traders providing counter-parties to B's clients. Therefore a smaller fraction of the orders submitted by B's clients must be executed against the market-maker when speculators S and B share non fundamental information (see Equation (6)). The next proposition shows that the reduction in the expected trading costs for B's clients always dominates the increase in expected trading costs for the other liquidity traders.

Proposition 4 *The expected trading costs borne by the liquidity traders are always lower* *when there is information sharing.*

The trading game is a zero-sum game in this model. This implies that the expected trading costs borne by liquidity traders are equal to the speculators aggregate expected profits. Let $\Pi^a(\sigma_\eta^2, \sigma_\varepsilon^2, N)$ be speculators' aggregate expected profits. By definition

$$\Pi^a(\sigma_\eta^2, \sigma_\varepsilon^2, N) = \Pi^S + \Pi^B + (N-1)\Pi^i,$$

where $\Pi^i(\sigma_\eta^2, \sigma_\varepsilon^2, N)$ is the expected profit of a speculator who is not part to the information sharing agreement. Information sharing always increases the joint expected profits of speculators B and S, i.e. $\Pi^S + \Pi^B$. Therefore, the concomitant decrease in trading costs for liquidity traders and increase in total expected profits for speculators S and B occur at the expense of the speculators who do not share information.

Overall the results of this section show how information sharing on the floor can improve the quality of floor-based markets along several dimensions. Information sharing makes price more informative, less volatile and fosters competition between floor brokers, so that ultimately the trading costs borne by the traders without an access to the floor are lower.

5 Conclusion

In this paper we have analyzed pre-trade information sharing between two traders endowed with different types of information, namely fundamental or non-fundamental information. We find that there are cases in which the two traders are better off sharing their information. Information sharing improves price discovery and decreases volatility. We also show that information sharing decreases the expected trading costs borne by liquidity traders. Finally the effect of information sharing on market depth and bid-ask spreads is ambiguous.

Floor-based trading systems are designed in such a way that they greatly facilitate information sharing. Overall our results show that this feature improves their performance. An interesting question is whether the benefits brought up by information sharing are outweighted by inherent disadvantages of floor-based systems (such as lack of transparency or larger operating costs). This issue is left for future research.

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6 Appendix

Proof of Lemma 1

Step1: The optimal trading strategy for speculator S. Let $y_S = (\tilde{v}, \hat{v}, \hat{x})$ be the information set of speculator S. The latter chooses his market order, Q^S, so as to maximize his expected profit

$$\pi^S(y_S) = E(Q^S(\tilde{v} - p(\tilde{O})) \mid y_S).$$

The first order condition yields

$$Q^S(y_S) = \frac{(\tilde{v} - \mu) - \lambda \times E\left[Q^B(y_B) + \sum_{j=2}^{j=N} Q^i(\tilde{v}) + \tilde{x}_0 + \tilde{x}_B \mid y_S\right]}{2\lambda}. \tag{7}$$

Notice that

$$E\left(Q^i(\tilde{v}) \mid y_S\right) = a'(\tilde{v} - \mu),$$

and

$$E\left(Q^B(y_B) \mid y_S\right) = b_1 E\left(\tilde{x}_B \mid \hat{x}\right) + b_2\hat{x} + b_3(\hat{v} - \mu),$$

and

$$E\left(\tilde{x}_B \mid \hat{x}\right) = \frac{\sigma_B^2 \hat{x}}{\sigma_B^2 + \sigma_\eta^2}.$$

Substituting these expressions in Equation (7) yields

$$\begin{aligned} Q^S(y_S) &= \frac{(\tilde{v} - \mu)}{2\lambda} - \frac{1}{2} \times \left[(N-1)\, a'(\tilde{v} - \mu) + b_3(\hat{v} - \mu) + (b_1 + 1)\, \frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2}\hat{x} + b_2\hat{x}\right] \\ &= \left(\frac{1}{2\lambda} - \frac{(N-1)\, a'}{2}\right)(\tilde{v} - \mu) - \frac{b_3}{2}(\hat{v} - \mu) - \frac{1}{2}\left((b_1 + 1)\, \frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2} + b_2\right)\hat{x}. \end{aligned}$$

Hence,

$$\begin{aligned} a_1 &= \left(\frac{1}{2\lambda} - \frac{(N-1)\, a'}{2}\right) \\ a_2 &= -\frac{b_3}{2} \\ a_3 &= -\frac{1}{2}\left((b_1 + 1)\, \frac{\sigma_B^2}{\sigma_B^2 + \sigma_\eta^2} + b_2\right) \end{aligned}$$

Step 2: The optimal trading strategy for speculator i, $i \neq S$.

Speculator i chooses his market order, Q^i, so as to maximize his expected profit

$$\pi^i(v) = E(Q^i(\tilde{v} - p(\tilde{O})) \mid \tilde{v} = v).$$

The first order condition yields

$$Q^i(\tilde{v}) = \frac{(\tilde{v} - \mu) - \lambda \times E\left[Q^S(y_S) + \sum_{j \neq i}^{j=N} Q^j(\tilde{v}) + Q^B(y_B) + \sum x_i \mid \tilde{v} = v\right]}{2\lambda} \tag{8}$$

We focus on symmetric trading strategies for all the speculators $i \neq S$. This imposes $Q^j(\tilde{v}) = Q^i(\tilde{v}), \quad \forall j \neq i$. Eventually, Equation 8 yields

$$Q^i(\tilde{v}) = \frac{(\tilde{v} - \mu)}{N\lambda} - \frac{1}{N} \times \left(E\left[Q^S(y_S) \mid \tilde{v}\right] + E\left[Q^B(y_B) \mid \tilde{v} = v\right]\right). \tag{9}$$

Furthermore

$$E\left(Q^S(y_S) \mid \tilde{v} = v\right) = (a_1 + a_2)(\tilde{v} - \mu),$$

and

$$E\left(Q^B(y_B) \mid \tilde{v} = v\right) = b_3(\tilde{v} - \mu).$$

Consequently

$$Q^i(\tilde{v}) = \left(\frac{1}{N\lambda} - \frac{(a_1 + a_2 + b_3)}{N}\right)(\tilde{v} - \mu) \tag{10}$$

We deduce that

$$a' = \frac{1}{N\lambda} - \frac{(a_1 + a_2 + b_3)}{N}. \tag{11}$$

Step 3: The optimal trading strategy for speculator B. We denote $y_B = (\tilde{x}_B, \hat{v}, \hat{x})$, the information set of speculator B. She chooses her market order, Q^B, so as to maximize

$$\pi^B(y_B) = E(Q^B(\tilde{v} - p(\tilde{O})) \mid y_B).$$

The first order condition yields

$$\begin{aligned}
Q^B(y_B) &= \frac{E(\tilde{v} \mid \hat{v}) - \mu - \lambda \times E\left[Q^S(y_S) + \sum_{i=2}^{N} Q^i(\tilde{v}) + \sum x_i \mid y_B\right]}{2\lambda} \\
&= \frac{E(\tilde{v} \mid \hat{v}) - \mu - \lambda \times E\left[Q^S(y_S) + (N-1)\, Q^i(\tilde{v}) + \sum x_i \mid y_B\right]}{2\lambda}.
\end{aligned}$$

We notice that

$$E\left(Q^S(y_S) \mid y_B\right) = a_1 E(\tilde{v} - \mu \mid \hat{v}) + a_2(\hat{v} - \mu) + a_3\hat{x},$$

and

$$E\left(Q^i(\tilde{v}) \mid y_B\right) = a' E(\tilde{v} - \mu \mid \hat{v})$$

and that

$$E(\tilde{v} - \mu \mid \hat{v}) = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(\hat{v} - \mu)$$

Substituting these expressions in the first order condition for Speculator B yields (after some algebra)

$$Q^B(y_B) = -\frac{\tilde{x}_B}{2} - \frac{a_3}{2}\hat{x} + \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} - \lambda a_2 - \lambda\left(a_1 + (N-1)\, a'\right)\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right)(\hat{v} - \mu). \tag{12}$$

Hence,

$$\begin{aligned}
b_1 &= -\frac{1}{2} \\
b_2 &= -\frac{a_3}{2} \\
b_3 &= \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} - \lambda a_2 - \lambda\left(a_1 + (N-1)\, a'\right)\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right)
\end{aligned}$$

Steps 1 to 3 give us 9 equations with 9 unknowns (a_1, a_2 etc...). Solving this system of equations yield

$$
\begin{aligned}
a_1 &= \frac{3\left(\sigma_v^2+\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)} \\
a_2 &= -\frac{\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)} \\
a_3 &= -\frac{\sigma_B^2}{3\left(\sigma_B^2+\sigma_\eta^2\right)} \\
a' &= \frac{2\sigma_v^2+3\sigma_\varepsilon^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)} \\
b_1 &= -\frac{1}{2} \\
b_2 &= \frac{\sigma_B^2}{6\left(\sigma_B^2+\sigma_\eta^2\right)} \\
b_3 &= \frac{2\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}
\end{aligned}
$$

Step 4. Computation of λ. Recall that

$$p(O) = E(\tilde{v} \mid \tilde{O} = O).$$

Given speculators' trading rules,

$$O = Q^S\left(y_S\right) + \left(N-1\right)Q^i(\tilde{v}) + Q^B\left(y_B\right) + \sum x_i$$

$$= \left(a_1 + \left(N-1\right)a'\right)\left(\tilde{v}-\mu\right) + \left(a_2+b_3\right)\left(\hat{v}-\mu\right) + \left(a_3+b_2\right)\hat{x} + \left(b_1+1\right)\tilde{x}_B + \tilde{x}_0.$$

Hence $\tilde{O}$ is normally distributed, with mean zero. Consequently

$$p(O) = \mu + \lambda O,$$

with

$$\lambda = \frac{Cov(\tilde{v},\tilde{O})}{Var(\tilde{O})}. \tag{13}$$

Now

$$cov\left(\tilde{v},\tilde{O}\right) = \left(a_1 + \left(N-1\right)a' + a_2 + b_3\right)\sigma_v^2 = \frac{\left(2\sigma_v^2\left(N+1\right)+3N\sigma_\varepsilon^2\right)\sigma_v^2}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}, \tag{14}$$

and

$$
\begin{aligned}
Var(O) &= \left(a_1+(N-1)a'\right)^2\sigma_v^2+(a_2+b_3)^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)+2\left(a_1+(N-1)a'\right)(a_2+b_3)\sigma_v^2\\
&\quad+(a_3+b_2)^2\left(\sigma_B^2+\sigma_\eta^2\right)+(b_1+1)^2\sigma_B^2+2(a_3+b_2)(b_1+1)\sigma_B^2+\sigma_0^2\\
&= \left(a_1+(N-1)a'+a_2+b_3\right)^2\sigma_v^2+\left(\frac{b_3}{2}\right)^2\sigma_\varepsilon^2+\left(\frac{a_3}{2}+\frac{1}{2}\right)^2\sigma_B^2+\left(\frac{a_3}{2}\right)^2\sigma_\eta^2+\sigma_0^2\\
&= \sigma_v^2\frac{\left(2\sigma_v^2(N+1)+3N\sigma_\varepsilon^2\right)^2+\sigma_v^2\sigma_\varepsilon^2}{\left(\lambda\left(2(N+2)\sigma_v^2+3(N+1)\sigma_\varepsilon^2\right)\right)^2}-\frac{5\sigma_B^4}{36\left(\sigma_B^2+\sigma_\eta^2\right)}+\frac{\sigma_B^2}{4}+\sigma_0^2.
\end{aligned}
$$

We deduce that

$$
\lambda=\frac{6\sqrt{\sigma_v^2\left(\sigma_B^2+\sigma_\eta^2\right)\left(4(N+1)\sigma_v^4+(12N+5)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}}{\left(2(N+2)\sigma_v^2+3(N+1)\sigma_\varepsilon^2\right)\sqrt{\left(\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right)}}.\blacksquare \tag{15}
$$

Proof of Lemma 2

We write the equilibrium value of λ in the following way:

$$
\begin{aligned}
\lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right) &= 6\frac{\sqrt{\sigma_v^2\left(4(N+1)\sigma_v^4+(12N+5)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}}{\left(2(N+2)\sigma_v^2+3(N+1)\sigma_\varepsilon^2\right)}\times\frac{\sqrt{\sigma_B^2+\sigma_\eta^2}}{\sqrt{\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)}}\\
&= 6\times\lambda_1\left(\sigma_\varepsilon^2\right)\times\lambda_2\left(\sigma_\eta^2\right).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
\frac{\partial\lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right)}{\partial\sigma_\eta^2} &= 6\times\lambda_1\left(\sigma_\varepsilon^2\right)\times\frac{\partial\lambda_2\left(\sigma_\eta^2\right)}{\partial\sigma_\eta^2}\\
&= -\frac{15\times\lambda_1\left(\sigma_\varepsilon^2\right)\times\left(\sigma_B^2\right)^2}{\sqrt{\sigma_B^2+\sigma_\eta^2}\left(\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right)^{\frac{3}{2}}}<0,\\
\frac{\partial\lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right)}{\partial\sigma_\varepsilon^2} &= 6\times\lambda_2\left(\sigma_\eta^2\right)\times\frac{\partial\lambda_1\left(\sigma_\varepsilon^2\right)}{\partial\sigma_\varepsilon^2}\\
&= \frac{6\times\lambda_2\left(\sigma_\eta^2\right)\times\sigma_v^4\left(3(7N-5)\sigma_\varepsilon^2+2(5N-2)\sigma_v^2\right)}{2\left(2(N+2)\sigma_v^2+3(N+1)\sigma_\varepsilon^2\right)^2\sqrt{\sigma_v^2\left(4(N+1)\sigma_v^4+(12N+5)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}}>0.
\end{aligned}
$$

We also observe that

$$\begin{aligned}\lim_{\sigma_\varepsilon^2\to\infty} \lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right) &= 6\times\lambda_2\left(\sigma_\eta^2\right)\times\frac{\sqrt{N\sigma_v^2}}{(N+1)},\\ \lim_{\sigma_\eta^2\to\infty} \lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right) &= 2\times\lambda_1\left(\sigma_\varepsilon^2\right)\times\frac{1}{\sqrt{\sigma_B^2+4\sigma_0^2}}.\end{aligned}$$

Consequently,

$$\lambda(\infty,\infty)=\lim_{\substack{\sigma_\eta^2\to\infty\\ \sigma_\varepsilon^2\to\infty}}\lambda\left(\sigma_\varepsilon^2,\sigma_\eta^2\right)=6\times\frac{1}{\sqrt{9\sigma_B^2+36\sigma_0^2}}\times\lim_{\sigma_\varepsilon^2\to\infty}\lambda_1\left(\sigma_\varepsilon^2\right)=\frac{2\sqrt{N\sigma_v^2}}{(N+1)\sqrt{\sigma_B^2+4\sigma_0^2}}.\blacksquare$$

Proof of Lemma 3

We denote by $\pi^j(y_j)$, speculator j's expected profit given his information set y_j prior to trading at date 1 and by $\Pi^j(\sigma_\eta^2,\sigma_\varepsilon^2,N)$, his expected profit, at date 0, that is before observing information. Notice that

$$\pi^j(y_j)=Q^j\times E(\tilde{v}-\mu-\lambda\tilde{x}-\lambda Q^{-j}-\lambda Q^j\mid y_j).$$

The first order condition for speculator j imposes that

$$2\lambda Q^j=E(\tilde{v}-\mu-\lambda\tilde{x}-\lambda Q^{-j}\mid y_j).$$

Hence, $\pi^j(y_i)=\lambda(Q^j)^2$ and

$$\Pi^j=E(\pi^j(y_j))=\lambda\times Var(Q^j).$$

It follows that

$$\Pi^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)=\lambda\left(a_1^2Var\tilde{v}+a_3^2Var\hat{x}_1+a_2^2Var\hat{v}+2a_1a_2cov\left(\tilde{v},\hat{v}\right)\right),$$

which yield (using the expressions for a_1, a_2 and a_3)

$$\Pi^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)=\left(\frac{\sigma_v^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(4\sigma_v^2+9\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}+\frac{\lambda\sigma_B^4}{9\left(\sigma_B^2+\sigma_\eta^2\right)}\right).$$

We define

$$\Pi_{nf}^{S} \overset{def}{=} \frac{\lambda\sigma_B^4}{9\left(\sigma_B^2+\sigma_\eta^2\right)},$$

and

$$\Pi_{f}^{S} \overset{def}{=} \left(\frac{\sigma_v^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\left(4\sigma_v^2+9\sigma_\varepsilon^2\right)}{\lambda\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)^2}\right).$$

We proceed exactly in the same way for speculator B.∎

Proof of Lemma 4

Assume that $N=1$. Then, using Lemma 3, we obtain that

$$\begin{aligned}
\sum(\sigma_\eta^2,\sigma_\varepsilon^2) &= \Pi_f^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)+\Pi_f^B(\sigma_\eta^2,\sigma_\varepsilon^2,N)+\Pi_{nf}^S(\sigma_\eta^2,\sigma_\varepsilon^2,N)+\Pi_f^B(\sigma_\eta^2,\sigma_\varepsilon^2,N)\\
&= \frac{\sqrt{\sigma_v^2\left(8\sigma_v^2+9\sigma_\varepsilon^2\right)}}{36\sqrt{\left(\sigma_v^2+\sigma_\varepsilon^2\right)}}\times\sigma_B^2\times\left(\frac{\left(8\sigma_B^2+9\sigma_\eta^2\right)+\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)}{\sqrt{\left(\sigma_B^2+\sigma_\eta^2\right)\left(\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right)}}\right)\\
&= \sum\nolimits_1(\sigma_\varepsilon^2)\times\sum\nolimits_2(\sigma_\eta^2)
\end{aligned}$$

We observe that

$$\begin{aligned}
\forall\sigma_\eta^2,\sigma_\varepsilon^2,\frac{\partial\sum(\sigma_\eta^2,\sigma_\varepsilon^2)}{\partial\sigma_\varepsilon^2} &= \sum\nolimits_2(\sigma_\eta^2)\times\frac{\partial\sum_1(\sigma_\varepsilon^2)}{\partial\sigma_\varepsilon^2}\\
&= \sum\nolimits_2(\sigma_\eta^2)\times\left(\frac{\sigma_v^4}{\left(\sqrt{\left(\sigma_v^2+\sigma_\varepsilon^2\right)}\right)^3\sqrt{\left(8\sigma_v^2+9\sigma_\varepsilon^2\right)}}\right)>0.
\end{aligned}$$

Furthermore

$$\begin{aligned}
\frac{\partial\sum(\sigma_\eta^2,\sigma_\varepsilon^2)}{\partial\sigma_\eta^2} &= \sum\nolimits_1(\sigma_\varepsilon^2)\times\sigma_B^2\times\frac{\partial}{\partial\sigma_\eta^2}\left(\frac{\left(8\sigma_B^2+9\sigma_\eta^2\right)+\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)}{\sqrt{\left(\sigma_B^2+\sigma_\eta^2\right)\left(\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right)}}\right)\\
&= \sum\nolimits_1(\sigma_\varepsilon^2)\times\sigma_B^4\times\frac{\left(-160\sigma_B^6-\sigma_B^4\left(-76+135\sigma_\eta^2\right)+9\sigma_B^2\left(8+9\sigma_\eta^2\right)+72\sigma_\eta^2\right)}{2\left(\sqrt{\left(\sigma_B^2+\sigma_\eta^2\right)\left(\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right)}\right)^3}.
\end{aligned}$$

We deduce that $\frac{\partial\sum(\sigma_\eta^2,\sigma_\varepsilon^2)}{\partial\sigma_\eta^2}>0$ iff

$$\sigma_\eta^2 > -\frac{4\sigma_B^2\left(40\sigma_B^4-19\sigma_B^2-18\right)}{9\left(15\sigma_B^4-9\sigma_B^2-8\right)},$$

which always hold true for $\sigma_B^2 \leq 1$ since in this case the R.H.S of the previous inequality is negative.

We conclude that Σ increases with σ_ε^2 and σ_η^2. This implies that

$$\Sigma(\sigma_\varepsilon^2, \sigma_\eta^2) \leq \Sigma(\infty, \infty).$$

Hence information sharing can not increase the total expected profits of speculators B and S. As this is a necessary condition for information sharing to take place, we conclude that when $N = 1$, information sharing never takes place.$\blacksquare$

Proof of Proposition 2

Step 1: Prices are more informative when there is information sharing. Recall that $\tilde{v}$ and $\tilde{p}$ are normally distributed and that $\tilde{p}(O) = \mu + \lambda O$. Therefore

$$Var(\tilde{v} \mid \tilde{p}(O) = p) = \sigma_v^2 - \frac{Cov^2(\tilde{v}, \tilde{O})}{Var(\tilde{O})}.$$

Using Equations (13) and (14) which appear in the proof of Lemma 1, we obtain that

$$Var(\tilde{v} \mid \tilde{p}(O) = p) = \sigma_v^2 - \lambda Cov(\tilde{v}, \tilde{O}) = \sigma_v^2 - \frac{(2\sigma_v^2 (N+1) + 3N\sigma_\varepsilon^2)\, \sigma_v^2}{(2\,(N+2)\,\sigma_v^2 + 3\,(N+1)\,\sigma_\varepsilon^2)}.$$

It is immediate that $Var(\tilde{v} \mid \tilde{p}(O) = p)$ increases with σ_ε^2 and does not depend on σ_η^2. This means that information sharing (a decrease in σ_ε^2 and σ_η^2) makes equilibrium prices more informative.

Step 2: Prices are less volatile when there is information sharing.

Observe that

$$Var(\tilde{v} - p) = E(E((\tilde{v} - p)^2 \mid \tilde{p} = p)).$$

As $\tilde{p} = E(\tilde{v} \mid \tilde{p})$, the previous equality implies that

$$Var(\tilde{v} - p) = E(Var(\tilde{v} \mid \tilde{p} = p)).$$

Finally since $\tilde{v}$ and $\tilde{p}$ are normally distributed, $Var(\tilde{v} \mid \tilde{p} = p)$ is constant so that

$$Var(\tilde{v} - p) = Var(\tilde{v} \mid \tilde{p} = p).$$

Hence prices are less volatile when there is information sharing since prices are more informative in this case.∎

Proof of Proposition 4

The expected trading costs for the liquidity traders when there is information sharing are

$$E\left(CT^{e}\right)=\lambda\left(\frac{6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2}{6\left(\sigma_B^2+\sigma_\eta^2\right)}\right).$$

Using the expression for λ, we rewrite this equation as

$$E\left(CT^{e}\right)=\frac{\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4+\left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}\left(6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2\right)}{\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)\sqrt{\left(\sigma_B^2+\sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right]}}.$$

When the brokers do not share their information, then

$$\begin{aligned}E\left(CT^{ne}\right)&=E\left[\left(P\left(O\right)-\tilde{v}\right)\times\tilde{x}\right]=\lambda^{ne}\left(\sigma_0^2+\frac{1}{2}\sigma_B^2\right)\\&=\frac{\sqrt{\sigma_v^2N}}{\left(N+1\right)}\times\frac{\left(2\sigma_0^2+\sigma_B^2\right)}{\sqrt{\sigma_B^2+4\sigma_0^2}}.\end{aligned}$$

We denote Φ the difference between the expected trading costs when there is information sharing and when there is no information sharing. Hence

$$\Phi\left(N,\sigma_\varepsilon^2,\sigma_\eta^2\right)=E\left(CT^{e}\right)-E\left(CT^{ne}\right)$$

Straightforward manipulations show that

$$\frac{\sqrt{\sigma_v^2\left(4\left(N+1\right)\sigma_v^4+\left(12N+5\right)\sigma_v^2\sigma_\varepsilon^2+9N\sigma_\varepsilon^4\right)}}{\left(2\left(N+2\right)\sigma_v^2+3\left(N+1\right)\sigma_\varepsilon^2\right)}<\frac{\sqrt{\sigma_v^2N}}{\left(N+1\right)}\tag{16}$$

Now consider the following function

$$\psi\left(\sigma_\eta^2\right)=\frac{\left(6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2\right)^2}{\left(\sigma_B^2+\sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right]}-\frac{\left(2\sigma_0^2+\sigma_B^2\right)^2}{\sigma_B^2+4\sigma_0^2}.$$

As $\sigma_0^2=1-\sigma_B^2$, we rewrite the previous equation as

$$\psi\left(\sigma_\eta^2\right)=\frac{\left(6\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(4\sigma_B^2+3\sigma_\eta^2\right)\right)^2}{\left(\sigma_B^2+\sigma_\eta^2\right)\left[36\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(32\sigma_B^2+27\sigma_\eta^2\right)\right]}-\frac{\left(2-\sigma_B^2\right)^2}{4-3\sigma_B^2}.$$

Observe that

$$\psi(0)=\frac{\sigma_B^2\left(-7+11\sigma_B^2-4\sigma_B^4\right)}{\left(9-8\sigma_B^2\right)\left(4-3\sigma_B^2\right)}<0,\text{ since }\sigma_B^2\in[0,1]$$

and

$$\lim_{\sigma_\eta^2\to\infty}\psi\left(\sigma_\eta^2\right)=0.$$

and

$$\psi'\left(\sigma_\eta^2\right)=\frac{\sigma_B^4\left(176\sigma_B^8+144\sigma_B^6\left(2\sigma_\eta^2-3\right)-72\sigma_B^2\sigma_\eta^2\left(5\sigma_\eta^2-7\right)+9\sigma_B^4\left(13\sigma_\eta^4-88\sigma_\eta^2+28\right)+252\sigma_\eta^4\right)}{\left(\sigma_B^2+\sigma_\eta^2\right)^2\left[36\left(\sigma_B^2+\sigma_\eta^2\right)-\sigma_B^2\left(32\sigma_B^2+27\sigma_\eta^2\right)\right]^2} \tag{17}$$

Now we remark that if $\sigma_B^2\in\left[0,\frac{21}{22}\right]$, then $\psi'\left(\sigma_\eta^2\right)>0$ and therefore $\psi\left(\sigma_\eta^2\right)<0$. If $\sigma_B^2\in\left[\frac{21}{22},1\right]$, then there is a unique value of σ_η^2 such that $\psi'=0$. This value is

$$\bar{\sigma}_\eta^2=\frac{2\sigma_B^2\left(22\sigma_B^2-21\right)}{3\left(14-13\sigma_B^2\right)}.$$

Hence ψ has only one extremum and this extremum is a minimum since

$$\psi''\left(\bar{\sigma}_\eta^2\right)=\frac{27\left(14-13\sigma_B^2\right)^4}{625\sigma_B^8\left(2\sigma_B^2-1\right)}>0,$$

We deduce that $\forall\sigma_\eta^2$ and $\forall\sigma_B^2$, $\psi\left(\sigma_\eta^2\right)<0$. We conclude that

$$\frac{\left(6\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)+\left(2\sigma_B^2+3\sigma_\eta^2\right)\sigma_B^2\right)}{\sqrt{\left(\sigma_B^2+\sigma_\eta^2\right)\left[\sigma_B^2\left(4\sigma_B^2+9\sigma_\eta^2\right)+36\sigma_0^2\left(\sigma_B^2+\sigma_\eta^2\right)\right]}}<\frac{\left(2\sigma_0^2+\sigma_B^2\right)}{\sqrt{\sigma_B^2+4\sigma_0^2}}. \tag{18}$$

Using Inequality (16) and Inequality (18), we deduce that $\Phi\left(N, \sigma_{\varepsilon}^{2}, \sigma_{\eta}^{2}\right) < 0$ which means that the expected trading costs are always lower when there is information sharing.$\blacksquare$

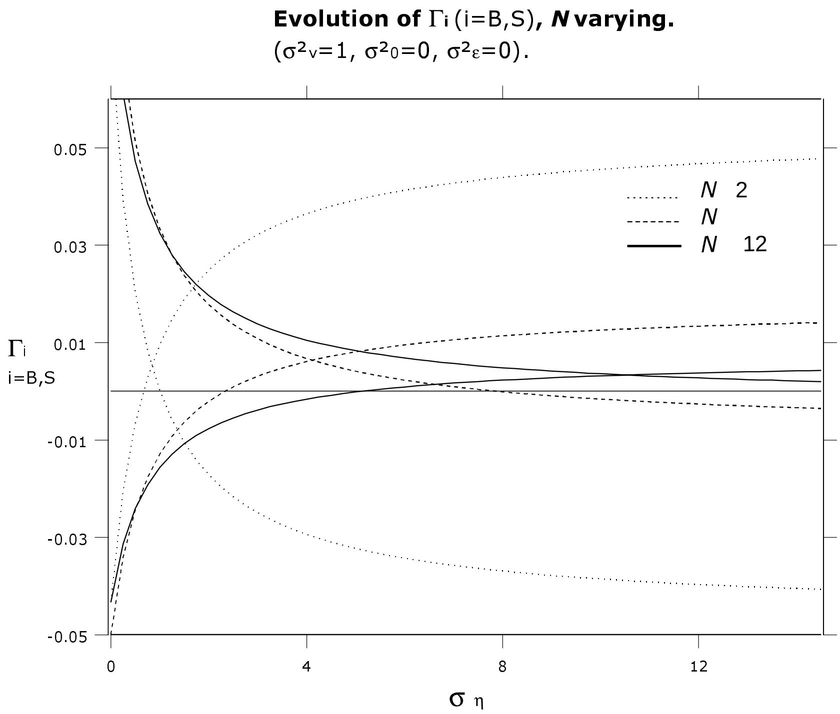

Figure 1

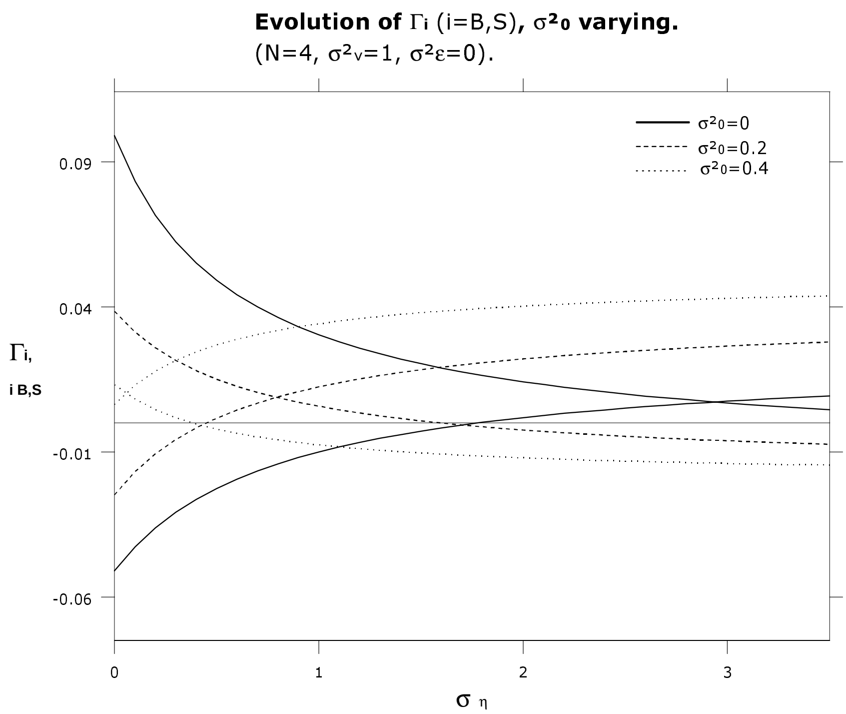

Figure 2

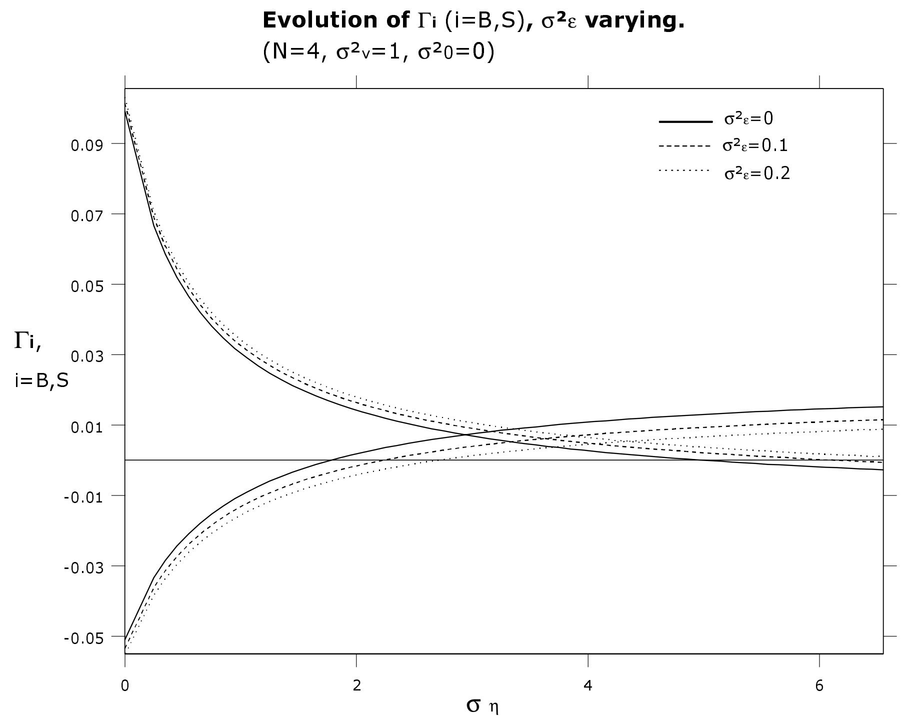

Figure 3

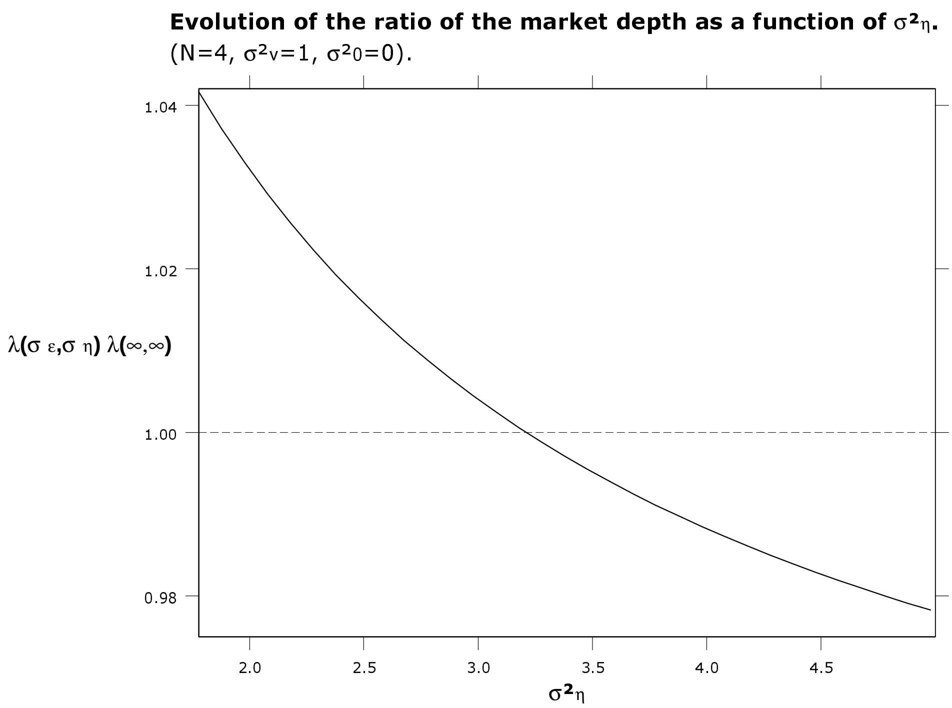

Figure 4